THIRD EDITION

THE THEORY AND PRACTICE OF REINFORCED CONCRETE

C. W. DUNHAM

Associate Professor of Civil Engineering Yale University

McGRAW-HILL BOOK COMPANY, INC. · KŌGAKUSHA COMPANY, LTD.

23.9.61
PWD B'lore

THE THEORY AND PRACTICE OF REINFORCED CONCRETE

McGraw-Hill Civil Engineering Series

HARMER E. DAVIS, *Consulting Editor*

BABBITT · Engineering in Public Health
BABBITT AND DOLAND · Water Supply Engineering
BENJAMIN · Statically Indeterminate Structures
CHOW · Open-Channel Hydraulics
DAVIS, TROXELL, AND WISKOCIL · The Testing and Inspection of Engineering Materials
DUNHAM · Foundations of Structures
DUNHAM · The Theory and Practice of Reinforced Concrete
DUNHAM AND YOUNG · Contracts, Specifications, and Law for Engineers
GAYLORD AND GAYLORD · Structural Design
HENNES AND EKSE · Fundamentals of Transportation Engineering
HENRY · Design and Construction of Engineering Foundations
KRYNINE AND JUDD · Principles of Engineering Geology and Geotechnics
LINSLEY AND FRANZINI · Elements of Hydraulic Engineering
LINSLEY, KOHLER, AND PAULHUS · Applied Hydrology
LINSLEY, KOHLER, AND PAULHUS · Hydrology for Engineers
LUEDER · Aerial Photographic Interpretation
MATSON, SMITH, AND HURD · Traffic Engineering
MEAD, MEAD, AND AKERMAN · Contracts, Specifications, and Engineering Relations
NORRIS, HANSEN, HOLLEY, BIGGS, NAMYET, AND MINAMI · Structural Design for Dynamic Loads
PEURIFOY · Construction Planning, Equipment, and Methods
PEURIFOY · Estimating Construction Costs
TROXELL AND DAVIS · Composition and Properties of Concrete
TSCHEBOTARIOFF · Soil Mechanics, Foundations, and Earth Structures
URQUHART, O'ROURKE, AND WINTER · Design of Concrete Structures
WANG AND ECKEL · Elementary Theory of Structures

THE THEORY AND PRACTICE OF REINFORCED CONCRETE

CLARENCE W. DUNHAM, C.E.

Associate Professor of Civil Engineering, Yale University; Consulting Structural Engineer for the New York Engineering Office of the Anaconda Copper Mining Co.; Formerly Assistant Engineer, Department of Engineering, The Port of New York Authority

THIRD EDITION

McGRAW-HILL BOOK COMPANY, INC.

New York Toronto London

KŌGAKUSHA COMPANY, LTD.

Tokyo

THE THEORY AND PRACTICE OF REINFORCED CONCRETE

ASIAN STUDENTS' EDITION

Library of Congress Catalog Card Number: 53–7117

PRINTING CO., LTD.
TOSHO INSATSU
TOKYO. JAPAN

PREFACE

In order to meet the needs of the student without undue expense to him, this third edition is being divided into two volumes. This one includes material that the undergraduate should master if he is to be well prepared to handle the design of ordinary reinforced-concrete structures with reasonable facility. Indeterminate structures and advanced material for the graduate student and the practicing engineer are to be included in a subsequent volume. Even in the first volume it has seemed to be desirable to include an introduction to prestressed concrete, precast concrete, ultimate-load design, and many of the details that are essential for an understanding of practical work and for the production of proper designs.

Much in the line of concrete construction is changing and developing and will undoubtedly continue to grow and to improve. This applies to theory as well as to practice. Many things are debatable even now and not all men will have the same ideas and opinions as those expressed here. Nevertheless, the author has attempted to present the theory and the art as he sees them.

In this volume he has tried to present fundamental principles and concepts. Admittedly, it is difficult to tell where to begin and where to stop in such a vast field as reinforced concrete.

The author hopes to teach the reader to visualize how each part of a structure acts; to design these parts so that each one will perform safely the service for which it is intended; and, finally, to plan the operations in the field so that the entire work will be a thing of which he is proud. Sound judgment and practical engineering sense are exceedingly important. These essentials are attained chiefly through hard work and long experience by the individual. However, the author hopes to expedite their attainment by presenting the subject from the viewpoint of the practicing engineer.

The reader will notice that, in many cases, this book does not go into extreme refinements of design and calculation. When one realizes that the assumed loads, their distribution, and the allowable unit stresses in the concrete, and even the ultimate strength of the concrete are often rather approximate and that they are based upon experiments, experience, and judgment, it seems to be inadvisable to carry subsequent computations to a degree of refinement that is not justified by the accuracy

of the fundamental data from which the calculations are started. Therefore, the use of the slide rule is sufficient for all work in this volume. In many cases, the numerical answers are rounded to the nearest important significant figure. The methods of analysis that are employed are designed to show fundamental principles and their application. They are believed to yield results that are on the side of safety and to be sufficiently accurate for all practical purposes.

The study of reinforced-concrete design should not be confined to making an acquaintance with a large number of formulas and to substituting quantities in those formulas in order to obtain a lot of numerical answers. This can lead to dangerous results because the structures built from the plans generally involve the safety of people and property as well as the wise or unwise use of money and materials. It is essential for an engineer to develop a thorough understanding of the action of structures, the behavior of materials, and the proper use of all the facilities at his command for both design and construction.

The tables and diagrams in the Appendix are useful for many purposes. References to them are made frequently in order to have the student appreciate their utility and learn when and how to use them. However, the author's chief emphasis is upon the basic principles of the design of various types of structures and upon the practical features which are so important in planning and building them.

Specific recommendations have been given for many theoretical and practical procedures. Where this has been done without reference to other authorities, the author merely attempts to provide the reader with some definite suggestions, but he does not pretend to set up unchangeable specifications.

It is desirable for a student to work on the creation of designs for complete structures, not merely the analysis or dimensioning of isolated members. For that reason a few plans of miscellaneous structures are presented in Chap. 15. Each one is based upon an actual structure, but some are simplified in detail. The instructor or the student alone can use as a backlog or project whatever problem or problems his interest and available time will permit.

The author is grateful to W. B. Sinnickson, Engineer of Tests, The Port of New York Authority, who has rewritten Chap. 1 for the purpose of clarifying several obscurities, to give greater emphasis to the practical aspects of the subject matter, and to include current concepts of some details of the subject. He wishes that it be stated that opinions and interpretations of fact contained there are his personal opinions and do not necessarily indicate or reflect the policies or opinions of the engineering staff of The Port of New York Authority.

The author is indebted to many associates and other friends for useful data and for helpful suggestions. He is especially grateful to The Port of New York Authority; to Samuel Potashnick, A. C. Seaman, Leon Kirsch, Walter Gadkowski, William J. Delaney, Paul F. Pape, L. A. Warner, and H. Gesund; and to Profs. William S. LaLonde, Jr., Leroy W. Clark, Bert B. Williams, Hardy Cross, Francis M. Baron, and Henry A. Pfisterer. These men and others, too, have assisted in various ways in connection with the first two editions or with this third one.

C. W. Dunham

CONTENTS

PREFACE v

LIST OF ABBREVIATIONS xii

1. PROPERTIES AND MANUFACTURE OF CONCRETE 1

Introduction—Definition and description of concrete—Cement—Aggregate—Admixtures—Compressive strength of concrete—Modulus of elasticity of concrete—Tensile strength of concrete—Shearing strength of concrete—Miscellaneous properties—Proportioning of concrete—Computations—Mixing concrete—Handling and placing of concrete—Curing—Forms—Reinforcing steel—Allowable unit stresses and safety factor—Importance of workmanship.

2. BEAMS SUBJECTED TO BENDING 64

Introduction—Table of symbols and their meanings—Distribution of stresses in a reinforced-concrete beam—Resistance to bending, straight-line theory—Problems in the analysis and design of beams, straight-line theory—Resistance to bending, ultimate-load theory—The transformed-section method—Beams with compressive reinforcement, straight-line theory—Beams with compressive reinforcement, ultimate-load theory—T beams—Irregular beams—Distribution of loads on beams—Practice problems.

3. BOND 113

Nature and magnitude of bond stress—Hooks—Bond of multiple layers of rods—Splices—Development of longitudinal reinforcement—Bond stresses on longitudinal bars in beams—Distribution of bond stresses in simply supported beams—Distribution of bond stresses in continuous beams—Bond under impact and reversal of stress—Bond of concrete to structural plates and shapes—Practice problems.

4. SHEAR AND WEB REINFORCEMENT IN BEAMS 146

Introduction—Determination of shearing stresses—Vertical stirrups—Spacing of vertical stirrups—Inclined stirrups—Spacing of inclined stirrups—Bent-up rods—Spacing of bent-up rods—Web reinforcement at supports of continuous beams—Combination of stirrups and bent-up rods—Web reinforcement in beams carrying moving loads—General considerations—Practice problems.

5. Composite Beams 196

Introduction—I beam and thin-slab construction—I beams completely encased in thick slabs—Fabricated trusses in concrete slabs—Practice problems.

6. Columns 212

Introduction—General discussion of reinforced-concrete columns—Design of tied columns—Design of spirally reinforced columns—Composite columns—Combination columns—Pipe columns filled with concrete—Long columns—Estimating column loads—Practice problems.

7. Combined Bending and Compression 238

Introduction—Construction procedure affecting columns—Combined compression and bending without resulting tension upon the section—Combined compression and bending with resulting tension upon the section—Approximate method for the design of rectangular columns with tension on part of the section—Analysis by ultimate-load theory—Practice problems.

8. Retaining Walls. 264

Introduction—Definitions of parts—Types of reinforced-concrete retaining walls—Stability and safety factor—Foundations—Lateral earth pressure—Surcharge—Water pressure—Local thrust at the top of a wall due to temperature—Preliminary steps in the design of a cantilever retaining wall—Design of the stem—Stability and foundation pressure—Design of the heel—Design of the toe—Design of the parapet—Arrangement of reinforcement and other practical details—Stone-faced retaining walls—Miscellaneous data—Practice problems.

9. Footings 304

Introduction—Fundamental principles—Concrete pedestals—Reinforced-concrete footing for a column—Column footings subjected to overturning moments—Combined footings—Practice problems.

10. Large Slabs 334

Introduction—Two-way slabs—Flat slabs—Practice problems.

11. Forms 351

Introduction—Strength of forms holding vertical loads—Lateral pressure on forms—Details and miscellaneous data.

12. Precast Concrete 368

Introduction—Economics—Design features—Small bridges—Building construction.

13. PRESTRESSED CONCRETE 382

Introduction—Fundamental principles—Methods for pretensioning—Methods for posttensioning—Shear and web reinforcement—Computations for a pretensioned beam—Computations for a posttensioned girder—Miscellaneous construction—Practice problems.

14. ARCHITECTURAL AND MISCELLANEOUS DETAILS 415

Basic principles underlying architectural treatment—Moldings and minor details of forms—Surface finish and appearance—Temperature reinforcement—Shrinkage—Construction joints—Expansion and contraction joints—Waterproofing—Drainage—Deflections—Encasement of structural steel—Reinforcement around openings—Torsion in concrete—Reinforced-concrete brackets—Planning reinforcement—Construction drawings—Miscellaneous details.

15. DESIGN PROBLEMS 450

Introduction—Assumptions—Deformed structures—Concept of design—Problems

APPENDIX 465

INDEX. 489

LIST OF ABBREVIATIONS

ft = foot
ft-lb = foot-pound
ft^2 = square foot
ft^3 = cubic foot
hr = hour
in. = inch
$in.^2$ = square inch
$in.^3$ = cubic inch or inches cubed
$in.^4$ = inches to fourth power
in.-lb = inch-pound
lb = pound
psi = pounds per square inch
psf = pounds per square foot
pcf = pounds per cubic foot
pli = pounds per linear inch
plf = pounds per linear foot
kip = 1,000 pounds
ksi = kips per square inch
ksf = kips per square foot
ft-k = foot-kip
in.-k = inch-kip
yd = yard
yd^2 = square yard
yd^3 = cubic yard

1

PROPERTIES AND MANUFACTURE OF CONCRETE[1]

1-1. Introduction. A concrete structure, either plain or reinforced, is unique among the many systems of modern construction. With few exceptions it is the only type of structure that is manufactured from its component materials on the site of the work. In most instances, the quality of its raw materials is decidedly variable. The compounding of its ingredients, the control of its chemical processes, and the arrangement of its parts are often performed by the least skilled of mechanical artisans. The inspection of its fabrication is sometimes delegated to the least experienced member of a supervisory force and may occasionally be neglected entirely.

The personal element—the care with which work is executed in the field—is of major importance in concrete construction. Structures built of steel, stone masonry, or various other materials are composed of elementary units which are partially or entirely prefabricated in factories or shops by skilled workmen. Such materials are fitted or assembled on the work by skilled mechanics, but concrete is often manufactured at the work site by whoever is available. The designer of reinforced-concrete structures should remember this. He must know the useful properties and practical limitations of the materials with which his plan will be constructed. With this knowledge he should plan the work in such a manner that desirable results are easily and correctly attained in the field. The study of concrete as a material is a complete subject by itself.[2] Only some of the most important aspects of the subject—the characteristics of concrete and the factors influencing its quality—can be given in this chapter.

1-2. Definition and description of concrete. Concrete is an artificial stone that is cast in place in a plastic condition. Its essential ingre-

[1] Contributed by W. B. Sinnickson, Engineer of Tests, The Port of New York Authority.

[2] Edward E. Bauer, "Plain Concrete," 3d ed., McGraw-Hill Book Company, Inc., New York, 1949.

dients are cement and water, which react with each other chemically to form another material having useful strength. A mixture of cement and water is termed *cement paste*. Such a mixture is expensive. To increase the volume of artificial stone produced from a prescribed amount of cement it is customary to add inert filler materials known as *aggregates*. When a large amount of cement paste is combined with a small amount of fine aggregate, and the combination is of fluid consistency, the mixture is termed *grout*. With the addition of somewhat more fine aggregate, such that the paste loses its fluidity and behaves as a cohesive plastic, the mixture is termed *mortar*. With the further addition of coarse aggregate, the mixture is called *concrete*.

It has long been customary to designate these mixtures in terms of the relative volumes of cement, fine aggregate, and coarse aggregate used in their preparation. For example, concrete proportions given as 1:2:4 mean a mixture of 1 ft^3 of cement, 2 ft^3 of fine aggregate, and 4 ft^3 of coarse aggregate. Another given as 1:3 is intended to mean a mixture of cement and fine aggregate, without any coarse aggregate. The latter would be classified as mortar. It should be observed that in each of these examples the amount of water to be used is undisclosed.

The foregoing system of indicating proportions of materials by volume is obsolescent but is still used on small projects or in connection with minor work. Now proportions are more often given by weight, and sometimes the total water to be used is also indicated. For example, proportions of materials for constructing the anchorages of the Bronx-Whitestone Bridge, in New York City, were determined by experiment and were given as 94:184:380 lb plus 5.6 gal of water. It should be remembered that, invariably when proportions are given, whether by weight or by volume, the ingredients are in the same order: cement first, fine aggregate next, and coarse aggregate last. Water is indicated separately, most often as gallons per bag of cement, sometimes as a ratio associated with the cement, and, occasionally, as the total amount in a unit volume of concrete.

Water, cement, and both fine and coarse aggregate, when mixed together in suitable proportions, produce concrete that is a plastic mass capable of being poured into molds. Concrete castings are made in this manner into objects of predetermined shape and size. The molds, which are actually called *forms*, must be built to restrain the plastic mass until it solidifies. Usually, the forms must be constructed in such a manner that the concrete, when it is poured, will be in its final position in the structure. This is not always necessary, however, and precast-concrete members made in forms on the ground or in specialized plants remote from the work are becoming increasingly important. Precast units are most economical when many members of identical size are required, in which case a single

form or a set of forms can be used repeatedly. Forms, in addition to their primary function of restraining concrete within dimensional limits until it solidifies, serve a less obvious purpose that should not be overlooked. They support the mass until it has attained sufficient strength to support itself without undue deflection or complete collapse.

Concrete does not solidify or attain useful strength quickly. The chemical reaction of cement and water is relatively slow and requires time and favorable temperatures for its completion. The reaction requires at least several days and may require several weeks for the production of worth-while results, and it continues thereafter for several years. It is customarily divided, for descriptive purposes, into three distinct phases. The first, designated the time of *initial set*, requires from 45 min to about 8 hr for completion. During this time the freshly mixed concrete gradually decreases in plasticity and develops pronounced resistance to flow. Disturbance of the mass or remixing during this time may cause serious damage to the concrete.

The second phase is an interval during which the concrete appears to be a relatively soft solid without surface hardness. It will support light loads without indentation; but it is easily abraded, and its surface can be scored, roughened, or otherwise marred without appreciable effort. This phase is termed the interval of *final set*. The time required for concrete to attain a condition that might be regarded as completely and finally set is very indefinite, inasmuch as the condition is in itself indefinable. However, within an interval of about 5 to 20 hr after the original mixing operation, the mass develops surface hardness to such a degree that its finish can no longer be manipulated or modified with ordinary hand tools such as trowels, floats, edgers, belts, and brooms, and when this condition prevails the concrete is said to have set.

The third phase is one of progressive hardening and increase in strength. For concrete of good quality this progressive improvement continues indefinitely. It is rapid during early ages until about 1 month after mixing, at which time the mass has attained the major portion of its potential hardness and strength. After the first month the improvement continues, but at a greatly reduced rate. This peculiar property of improvement with age will be discussed later in greater detail. It is graphically illustrated in Fig. 1-1.

1-3. Cement. For more than two thousand years man has used various cementaceous materials in the building of his important structures. All can be placed in one or the other of two distinct categories: those which do not, and those which *do* set and harden in the presence of appreciable amounts of water. The latter are of major importance and are said to have *hydraulic* properties. A general classification of hydraulic cements should include *pozzolanic material* of volcanic origin, so effectively used by

the Romans; *hydraulic lime* frequently used in France; *natural cement* such as that produced near Louisville, Ky., and Rosendale, N.Y., of the type used in constructing the Brooklyn Bridge; *alumina cement*, which is popular for sea-water construction in European countries; and *portland cement*.

Each of these types of hydraulic cement is in current use in some part of the world. The use of any particular type is a matter of engineering psychology and economic necessity. Engineering practice, in the United

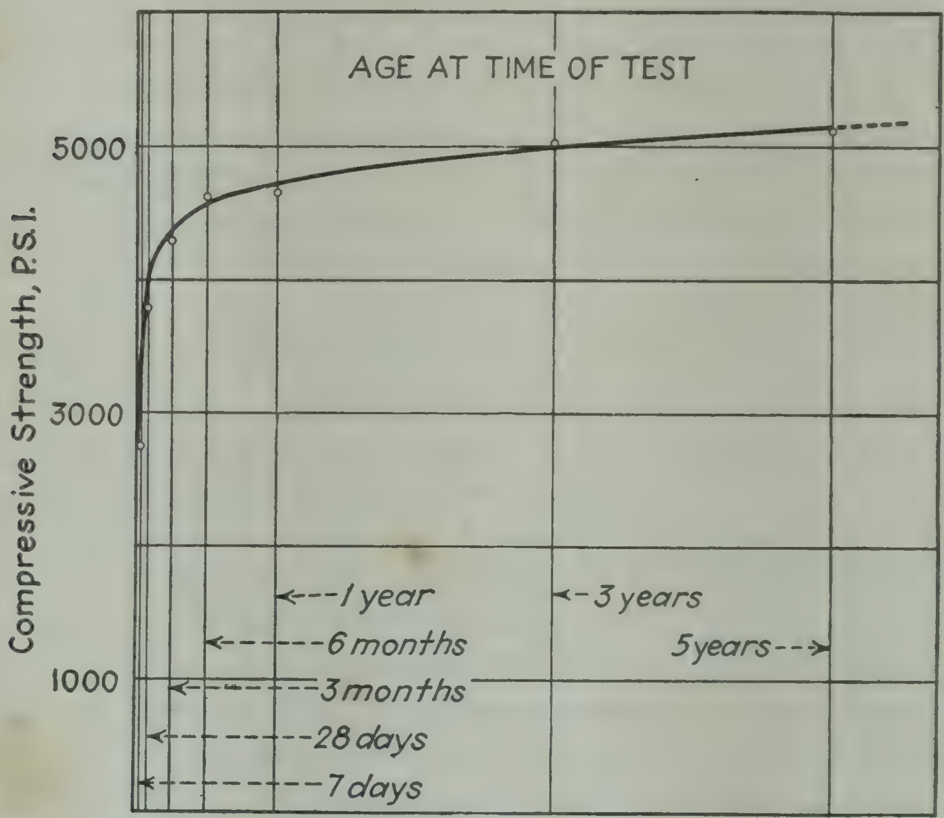


FIG. 1-1. Relation of compressive strength to age. George Washington Bridge, Riverside Drive connections to the New York approach. Each point is the average unit strength of twenty-five 6- by 12-in. cylinders mixed in the laboratory and cured in moist air at 70°F.

States, is inalterably linked to the rapid construction and early utilization of a structure, for which reason portland cement is almost universally used. Although aluminous and natural cements are sometimes used in this country for structures of a specialized nature, the further discussion of reinforced-concrete construction will be confined mainly to that in which portland cement is the binding agent.

During the year 1904 the American Society for Testing Materials adopted a standard specification for portland cement. Its requirements were definitely regulatory, but their range was such that manufacturers throughout the United States could meet them with little difficulty. During 1912, the Federal Specification Board also adopted a standard specification having requirements similar to the contemporary ASTM spec-

ification. These specifications remained similar in essential requirements even though they were revised at times. Because of their great latitude, manufacturers could solve their individual problems of composition and plant operation with little attention to specification restrictions. During the period 1914 to 1918 it became evident to many engineers and cement technologists that, although most cements conformed with then current specification requirements, they were often different in accomplishment. Differences in durability of concrete made with cements which seemed to be chemically similar and were of neighboring origin were observed particularly in structures built in sea water. Some cements attained unusual strength. Others reached adequate strength but later showed evidence of retrogression. Some produced unforeseen expansion of structures. Occasionally, some set quickly and generated much heat, while others were slow and lazy in their rate of gaining useful strength. Other less significant differences were also observed.

To understand and control this contradictive behavior, the Portland Cement Association in September, 1914, and many engineers and others about 1920 began intensive investigations of problems pertaining to concrete proportioning. Several years thereafter most of the manufacturers, by means of the Portland Cement Association Fellowship at the National Bureau of Standards, began coordinated scientific studies of the reactions occurring during manufacture, the constitution of the finished product, and the chemical and physical behavior incidental to the utilization of cement. Earlier chemical research had been carried on by governmental agencies[1] or by persons[2] not necessarily associated with cement manufacture, as well as by individual producers who were in most instances poorly equipped for pure research but were anxious to solve specific problems. Much of the early research was contradictory or inconclusive and sometimes extemely controversial.

During 1927, the International Cement Corp. placed on the market a so-called *high-early-strength* portland cement. This was the first of several important deviations from the all-purpose general-utility portland cement, the only type available until that time. There was, at the time, little apparent difference in the composition of this as compared with other portland cements, but the performance of the new material was remarkably different. Later, however, when the results of research on the constitution of cement clinker became known, its real difference became evident. Concrete made with this new cement exhibited similar plastic properties during mixing and placing, required almost identical

[1] U.S. Geological Survey; Carnegie Geophysical Laboratory; National Bureau of Standards.

[2] H. LeChatalier, W. Michaelis, and A. E. Tornebohm were noteworthy among scores of others as being so nearly in accord with current concept.

proportions of ingredients, and set and hardened at about the same rate as did concrete made with normal cement. Nevertheless, the premium material was amazingly quick in its development of strength and attained useful values within about one-fifth the time experienced with ordinary cement. The ASTM in 1930, and the Federal government in 1936, adopted specifications for this improved type of portlant cement.

Competition of other manufacturers soon provided other sources of supply of the premium material. As a consequence of this competitive activity the strength of ordinary portland cement was also improved by almost all manufacturers, and the strength differential between normal and high-early-strength cements became less pronounced. New disparities in the behavior of cement in concrete now became troublesome. Some cements were finely ground and others were coarse. Some required much more water than others to produce concrete of comparable consistency. Some produced cohesive plastic concrete of fatty texture while others encouraged segregation and bleeding of water to the surface of plastic mixtures.[1] Even though the compressive strength of ordinary concrete per pound of cement used was appreciably greater after 1926, a similar improvement of other attributes such as durability and impermeability was not evident. It became possible during the early 1930's to make concrete of good compressive strength but with insufficient cement to provide adequate resistance to the action of water, ice, and chemically destructive salts.

Great constructive activity during the 1930's on the part of state highway departments, Federal and other governmental agencies, and private industry emphasized differences in the behavior of apparently similar cements. Engineers engaged in building great irrigation and flood-control projects were anxious to minimize or control the heat evolved during the reaction of cement and water. Others responsible for sea-coast construction were active in searching for the inherent characteristic of cement that would assure durability of concrete exposed to tidal action and sea water. Highway engineers, and especially those of Northern states experiencing severe winter climates, were concerned about the spalling and surface disintegration of pavement slabs because repeated freezing and thawing and the chemical attack of snow-removal aids caused unpredictable deterioration of concrete pavements. During this decade many independent specifications were prepared by engineers for the purpose of controlling the quality of cement used in specialized structures.

Some of these independently prepared specifications contained logical and reasonable requirements. Others were founded largely on the faith of their proponents in some unusual testing procedure or some unstudied

[1] T. C. Powers, The Bleeding of Portland Cement Paste, Mortar and Concrete, *PCA Research Lab. Bull.* 2, July, 1939.

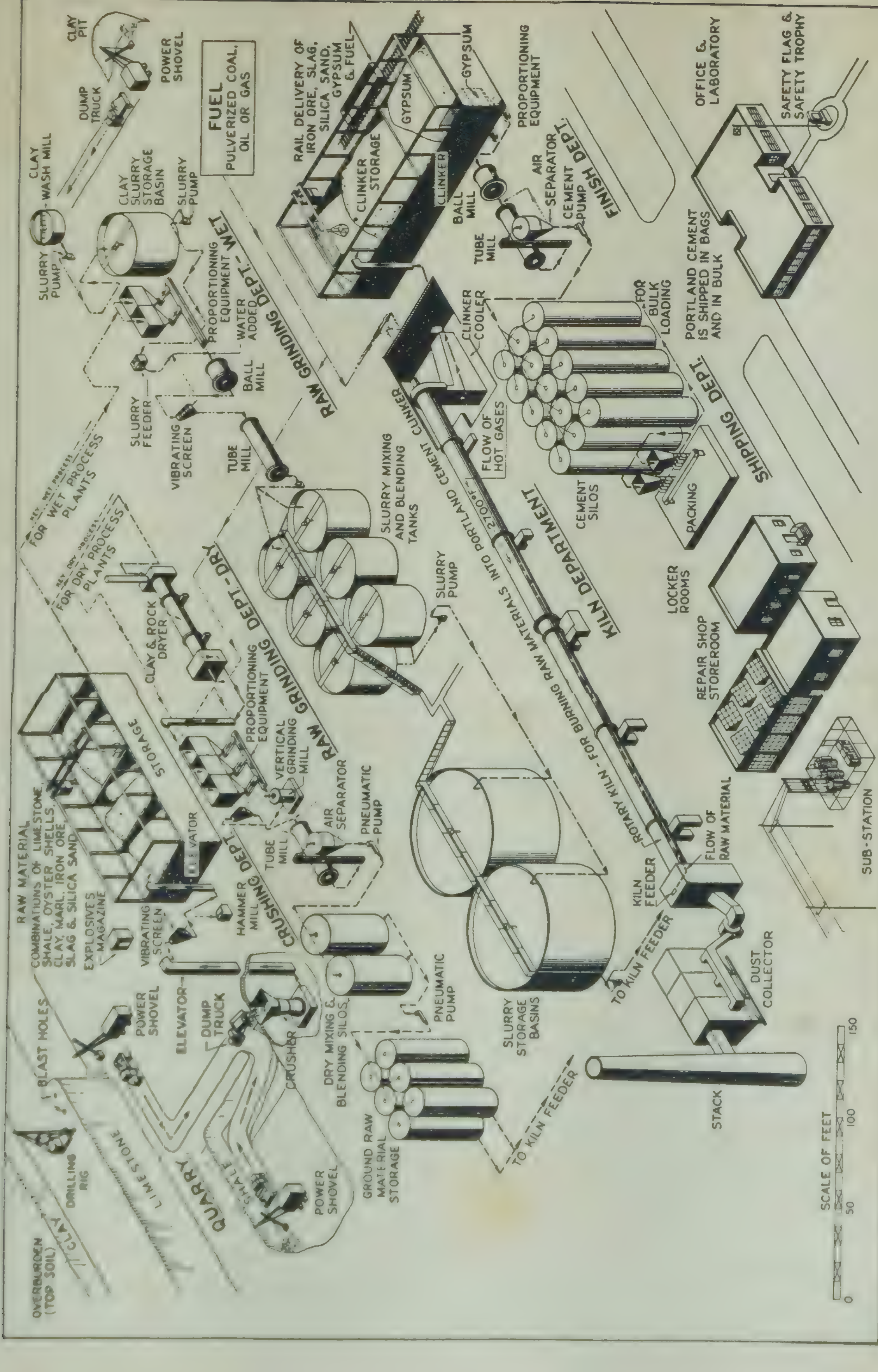


FIG. 1-2. Flow sheet of a plant for the manufacture of portland cement. (Courtesy of the *Portland Cement Association.*)

chemical reaction. Manufacturers of cement were faced with the problem of meeting the requirements of a score or more diverse specifications. Industrial endeavor toward standardization of behavior of cement suffered a temporary setback. Some industrial distress was caused by the increasing number and individual diversity of nonstandard specifications. Confusion was relieved in some degree, however, when in 1936 the Federal government adopted a specification for several types, and more so in 1940 when the ASTM adopted a specification for five types of portland cement.

Now during the 1950's, the Federal government has one, and the ASTM has two specifications for portland cement. These modern specifications provide not only for general-purpose cement but also for other types more suitable for specialized structures. One of the two ASTM specifications covers five different types[1] of cement having specifically designated applications as described later; whereas the other[2] covers three of the five types and provides, in addition, for the entrapment of a myriad of small bubbles of air within the concrete in which the cement is used. The contemporary Federal specification[3] provides for five types of cement having applications identical with those designated by ASTM as well as for five others of similar usage but having the property of *air entrainment*. An interesting commentary on the evolution of portland-cement specifications is contained in American Concrete Institute literature.[4] In addition to the foregoing specifications for true portland cements, both ASTM[5] and Federal[6] specifications are also available for portland-pozzolana types of cement. An ASTM specification[7] is also available for natural cement which, although it was a forerunner of portland, cannot be considered as a true portland cement, inasmuch as the material during manufacture never reaches sintering temperatures essential to the chemical reaction of lime with silica.

Other hydraulic cements of good quality and satisfactory performance which are not covered by nationally recognized specifications are available to the engineer. One of these that should be especially noted is Lumnite cement. It is an aluminosilicate cement that is sometimes used in concrete construction where unusually great strength or resistance to severe chemical attack may be required, or where temperatures as great as 1000°F may be encountered in service. Lumnite is not a portland

[1] Specification for Portland Cement, ASTM Designation: C 150.

[2] Specification for Air-entraining Portland Cement, ASTM Designation: C 175.

[3] Cements; Portland, Federal Specification SS-C-192.

[4] J. C. Pearson, Comments on Changes in Cement Specifications . . . , *J. ACI*, Vol. 19, No. 8, p. 705, April, 1948.

[5] Specification for Blast-furnace Slag Cement, ASTM Designation: C 205.

[6] Cement; Portland, Pozzolana, Federal Specification SS-C-208.

[7] Specification for Natural Cement, ASTM Designation: C 10.

cement, and concrete made with it requires somewhat different handling than does ordinary concrete. The experienced advice of its manufacturers should be solicited when its use is contemplated. *Ciment fondu* is a French equivalent of the aluminous type. Other European cements that might occasionally be used for concrete construction in this country are the iron-ore cements such as *erz* cement of Germany and *ferrari* cement made in France and Italy; the pozzolana-portlands such as *eisenportland* and *trass* cements of Germany, *gaize* and *metallurgique de fer* of France, and *silikatcement* of Sweden. True portlands from Belgium may sometimes be encountered.

Other cements such as white portland made by maintaining the iron content of raw mixtures at a very low value, waterproof portland made by incorporating organic soaps during grinding of clinker, oil-well portland made by special heat-treatment, and tinted or colored portlands made by intergrinding with limeproof pigments are all available for special uses. Such cements are generally used in monolithic concrete topping applied to ordinary concrete members. Expansive cements made by blending portland cement and metallic iron filings, sometimes accompanied by ammonium chloride, are sometimes used for grouting column bases and other parts of structures when expansion during setting is desirable.

An engineer can now select from among the several types of cement and can designate, by means of a standard specification, the most suitable cement for a special project or an unusual condition of service. The scope of his freedom of choice is best indicated by a brief description of the designated uses of each type of portland cement as set forth in Federal Specification SS-C-192:

Type I. For use in general concrete construction where the special properties specified for types II, III, IV, and V are not required.

Type II. For use in general concrete construction exposed to moderate sulphate action, or where moderate heat of hydration is required.

Type III. For use when high early strength is required.

Type IV. For use when low heat of hydration is required.

Type V. For use when high sulphate resistance is required.

Types I-A, II-A, III-A, IV-A, and *V-A.* For identical uses as the foregoing types of the same number where *air entrainment* is required.

The relative demand for each of these distinctive types of cement is indicated by our national average production[1] during the years 1945 to 1949. During this period the production of portland cement averaged 173,700,000 bbl; and the production of portland-pozzolana, masonry,

[1] Computed from yearly production given in "Minerals Yearbook," U.S. Bureau of Mines, 1949.

and natural cements combined averaged 2,700,000 bbl per year. The relative average production of distinctive types was as follows:

	Per cent
Types I and II combined	85.1
All air-entrainment types combined	8.9
Type III high early strength	3.4
Type IV low heat of hydration	0.1
Type V sulphate-resistant	0.05
All others (white, plastic, etc.)	2.5

Although the foregoing percentages are informative they fail to disclose a very significant fact: that production of air-entrainment types of cement increased fourfold during the 5-year interval. The noteworthy performance of concrete having entrained air has almost completely reassured us about the ultimate durability of well-proportioned and properly installed concrete exposed to freezing, the leaching action of fresh water, and the chemical attack of salt water. It is quite likely that the demand of progressive engineers for air-entrainment types of cement will continue to cause increased production for an indefinite time to come.

The manufacture of cement is widespread throughout the United States. It is produced in 150 mills located in 36 states and the island of Puerto Rico. The area of most significant production is the Lehigh Valley of Pennsylvania, and this is closely followed next in importance by California. The close proximity of argillaceous as well as pure limestone and coal is responsible for the advantageous position of the Lehigh Valley. However, because of technical progress in the use of clay, shale, chalk, and siliceous limestones accompanied by the availability of natural gas or petroleum, and because of improved plant-operating procedures, this one-time preeminent advantage is now greatly reduced. Increased use of a flotation process,[1] commonly used for ore dressing but first applied to the commercial production of cement at the Conshohocken plant of the Valley Forge Cement Co., may eliminate all factors other than cheap fuel as local advantages in the manufacture of cement. The schematic flow of material through a typical cement plant is shown in Fig. 1-2.

Cement is a conglomeration of many compounds in varying proportions. Its composition is dependent upon the impurities present in its raw materials, one of which is the fuel used in its manufacture, and upon the treatment given it during the operations of calcination and grinding. One of its most important constituents is isomeric in form, and others are suspected to be. The contribution of such dual-personality constituents to over-all quality is dependent upon their casual condition of

[1] C. H. Breerwood patent 1,931,921, Oct. 24, 1933.

existence. An idealized portland cement that might be assumed to exist only for the purpose of discussion is a chemical combination of lime and silica forming a mixture of tricalcium and beta dicalcium silicates. The manufacture of such a product would, because of temperature limitations, be commercially difficult and the product might be hard to manage in construction. Aluminous and iron compounds, normally present in cement raw mixtures, make it possible to approximate this idealized cement by means of practical production methods. Both the alumina and the iron compounds form effective fluxing agents, and they accelerate the reaction of lime and silica which exist largely as unmelted solids at kiln operating temperatures. These fluxing agents react at calcination temperatures with lime, to form tetracalcium aluminoferrite and tricalcium aluminate. They thus isolate some of the lime and make it unavailable for reaction with silica to form other compounds of great cementing value. Tricalcium aluminate is predominantly responsible for the setting and early hardening of cement.[1] Unfortunately, this constituent is also responsible for the deterioration of cement exposed to sulphates, inasmuch as its hydration product combines with the sulphate radical to form calcium sulphoaluminate and, in doing so, exhibits an increase of approximately 227 per cent in molecular volume.[2]

Other important constituents occurring in cement in lesser amounts are: uncombined calcium oxide, magnesium oxide, calcium sulphate, and poorly understood complex compounds of sodium and potassium. All these have a profound influence on the performance of cement in service. Free calcium oxide, usually trapped in the glassy structure of other major constituents, is the primary cause of unsoundness manifest by the warping or cracking of a pat of hardened cement paste exposed to low-pressure steam.[3] Crystalline magnesia occurring as a result of low calcination temperature, or slow cooling of clinker, is most responsible for autoclave unsoundness evident as abnormal expansion of hardened paste exposed to high-pressure steam.[4] Both the free lime and the magnesia are responsible for the long-time expansion of concrete continuously saturated with water. Calcium sulphate is deliberately added to clinker during finish grinding and is essential for retardation and establishment of a time of set that is practical for use in most concrete construction. An abnor-

[1] R. H. Bogue, The Nature of the Setting and Hardening Processes in Portland Cement, *PCA Fellowship Paper* 17, October, 1928; and R. H. Bogue and W. Lerch, Hydration of Portland Cement Compounds, *PCA Fellowship Paper* 27, August, 1934.

[2] R. H. Bogue, W. Lerch, and W. C. Taylor, Influence of Composition on Volume Constancy and Salt Resistance of Portland Cement Pastes, *PCA Fellowship Paper* 28, October, 1934.

[3] W. Lerch, *Concrete*, Vol. 35, No. 1, p. 109; Vol. 35, No. 2, p. 119, 1929.

[4] W. Lerch and W. C. Taylor, Some Effects of Heat Treatment of Portland Cement Clinker, *PCA Fellowship Paper* 33, July, 1937.

mally great amount of this constituent may indicate that the cement is of improper composition or was unsuitably calcined or cooled. Such cement, if prepared with a normal amount of retarder, might generate much heat and exhibit a *flash set* causing unwanted cracking of a structure. Sodium and potassium compounds are most certainly the cause of deterioration of concrete made with reactive aggregates, but the chemistry of the phenomenon is not yet well understood. These compounds are also believed to influence the formation of calcium silicates at kiln temperatures and to determine in some degree the amount and condition of the free lime remaining after calcination. The presence of other constituents such as compounds of manganese, phosphorus, titanium, and other elements in small amounts is of general occurrence. However, their significance in determining the quality or influencing the behavior of cement is uncertain.

The chemical processes of hydrolysis and hydration occur simultaneously during the reaction of cement and water. Hydrolysis is, briefly, the change of a compound into others as a result of the chemical action of water. Hydration is the combination of a material with water. Some of the constituents of cement disintegrate in the presence of water and, in doing so, they form other compounds which combine with water. The following outline of the reactions of cement with water should be considered as only generalities, since the reactions are not at all so simple and conclusive as they are described. Because of the ponderous names of the constituents and the unwieldiness of their common chemical symbols, they and their reaction products are usually referred to in cement technology in an abbreviated symbolic form. For example: tetracalcium aluminoferrite is expressed in conventional symbols as $4CaO.Al_2O_3.$-Fe_2O_3; whereas, in the abbreviated form used by cement chemists, it is written as C_4AF. In the same manner, tricalcium silicate is noted as C_3S, while hydrated tricalcium aluminate would be recorded as C_3AH_6. The abbreviated form of notation is used in the following description whenever its meaning is clear and its use is convenient.

Briefly, the C_3S disintegrates slowly by hydrolysis and the products of its dissociation then hydrate; the beta form of C_2S reacts directly with water but the reaction takes place slowly and requires many years for its completion. Gamma C_2S, produced by the inversion of the beta form as a result of the slow cooling of clinker, hydrates very slowly and its hydration product develops insignificant strength even after several years. Both the C_4AF and the C_3A combine actively with water and form hydrated products which contribute moderate strength to concrete within the first few hours or days of their existence. The hydration of C_3A contributes most to the initial set and the early hardening of concrete and, unless an adequate amount of retardant in the form of a sul-

phate is present, the reaction may be vigorous and cause rapid set. It may seem anomalous that sulphates are used to control the set and that they are also destructive to hardened concrete. However, the reaction of C_3A with retarders takes place largely while the mixture is plastic or, at worst, while the concrete is weak and capable of readjustment to the expansive forces of formation of calcium sulphoaluminate. The relatively energetic and prolonged reaction of C_3S, and the slow reaction of beta C_2S with water to form C_2SH_x is responsible for the progressive increase in strength of portland-cement concrete.[1]

The constitution, the chemistry of formation and utilization, and what might be called the "metallurgy" or phase relationships of cement are now fairly well defined. Phenomenal gains in our knowledge of cement have been made since 1920 by research chemists and physicists. Although many persons of most diversified national background have accomplished this and each deserves part of the credit, nevertheless most of the references given here are related to research performed in the United States. Sincere apologies are offered to the scientists of foreign lands who have contributed to our present state of knowledge but whose contributions have not been noted because of limited space.

Because of the chemical complexity of the material, it is suggested that the designing engineer should apply generally accepted standard specifications for cement and should leave improvements and departures from standard practice to the specialized fields of the cement technologist and materials engineer. Designers should also realize that, although special types of cement behave during mixing and placing in a manner similar to ordinary cement, they differ greatly and may be substantially slower in their rate of gaining strength. Special types should not be used without a thorough understanding of their properties.

1-4. Aggregate. Aggregate, either fine or coarse, is inert filler material added to cement paste to increase its bulk. Aggregates do have other functions and may impart beneficial properties to concrete, but a proper appreciation of their primary function as a filler makes the proportioning of concrete mixtures more easily understood. Fillers may be of either natural or artificial origin. Because of widespread distribution, natural sand and gravel and mechanically crushed rock are the most commonly used aggregates. Unusual materials such as blast-furnace slag, pumice, calcined clay, diatomaceous silica, asbestos, sawdust, vegetable fiber such as seaweed, and others are sometimes used as concrete aggregates. Materials deliberately made to have a cellular structure, such as Haydite, Lelite, Perlite, Waylite, and others, as well as natural materials such as vermiculite, are used for making acoustical, ther-

[1] R. H. Bogue and W. Lerch, Hydration of Portland Cement Compounds, *PCA Fellowship Paper* 27, August, 1934.

mal-insulating, and lightweight concretes. Even the small air voids entrapped in concrete when air-entraining cement is used should properly be thought of as part of the aggregate. It is customary to consider any sound filler material that will pass through a sieve having $\frac{1}{4}$-in.-square openings as fine aggregate. It follows that particles larger than $\frac{1}{4}$ in. in size are classed as coarse aggregate.

It is of major importance that aggregate be nonreactive with cement and water and that it be structurally sound, strong, and durable. Hardness and toughness are also desirable properties, especially in highway construction, where resistance to abrasion and impact are of functional importance. In general, igneous and metamorphic rocks as well as most siliceous sands and gravel are usually of excellent quality for use as aggregates. However, siliceous materials having constituents of amorphous or cryptocrystalline structure should always be suspected of being potentially reactive with high-alkali cement. Sedimentary rocks in general should be considered with suspicion unless thorough and complete tests or extensive experience in their use have proved their worthiness. Natural materials having pronounced planes of weakness or cleavage such as slate, shale, and micaceous materials are usually undesirable; whereas others having uniform shearing strength in all directions are ideal for use as aggregates.

Aggregates should be clean, since particles coated with clay, silt, organic matter, or crusher dust will not bond with the surrounding cement paste. The interface between an aggregate particle and its cementing medium is the most critical plane through which tensile or shearing failures are likely to occur. This interfacial area, if poorly bonded to the cement paste, may also provide channels for capillary percolation of water into the concrete mass. Natural sands are particularly prone to surface coating, especially when obtained from near the surface of a deposit. Material obtained from subaqueous sources is often found to be coated with algae and contaminated with marine animals and vegetation. Substances of organic origin are usually harmful to freshly mixed concrete. Fine, and sometimes coarse, material from sources of supply containing clay, silt, and natural overburden of disintegrated organic matter should be thoroughly washed before use. Even after washing the material should be tested, preferably in comparison with a material of known dependability such as *Ottawa* sand[1]. The contamination of aggregate with topsoil, humus, or earthy material containing products of organic decay even in small amounts is practically certain to cause early disintegration or complete collapse of a structure.

Specifications for concrete aggregates should require that the material

[1] Method of Test for Measuring Mortar-making Properties of Fine Aggregate, ASTM Designation: C 87.

be "clean, hard, strong, durable, and sound material free from harmful amounts of soft, friable, thin, elongated, or laminated pieces." It is customary to indicate the maximum amounts of harmful substances such as clay,[1] silt,[2] shale,[3] coal,[4] and organic matter[5] that will be acceptable in an aggregate. In some instances, particularly when a material has had little background in use, it may also be desirable to require that it meet empirical performance tests giving some measure of hardness,[6] toughness,[7] or resistance to repeated freezing.[8]

Aggregate should be inert to cement and water. It has of late been emphasized that some materials heretofore used with confidence as aggregates have, when used with certain cements, caused great expansion and cracking of structures.[9] Aggregate reactivity has been most often observed in Pacific Coast and Rocky Mountain states as well as in parts of the Missouri River Basin. Much more rarely have other cases been evident in New York State and in the Southern Appalachian Highlands. The mechanism of failure was first most reasonably hypothesized by Hansen,[10] and it is now factually accepted that sodium and potassium constituents of cement react with amorphous, pseudocrystalline, or microcrystalline siliceous mineral components of certain aggregates. Opal and chalcedony are serious offenders, but other siliceous minerals, and even natural and man-made glasses, may cause trouble.[11] Destruction is caused by forces resulting from the formation of hygroscopic silica-gel within the concrete which, in company with hydrated cement paste acting as a semipermeable membrane, creates osmotic pressure tending to burst the concrete. Where alkali-aggregate reactivity may be encountered it is imperative that definition of types and designation

[1] Method of Test for Clay Lumps in Aggregates, ASTM Designation: C 142.

[2] Method of Test for Amount of Material Finer than No. 200 Sieve in Aggregates, ASTM Designation: C 117.

[3] Method of Test for Soft Particles in Coarse Aggregate, ASTM Designation: C 235.

[4] Method of Test for Coal and Lignite in Sand, ASTM Designation: C 123.

[5] Method of Test for Organic Impurities in Sands for Concrete, ASTM Designation: C 40.

[6] Method of Test for Abrasion of Graded Coarse Aggregate by Use of the Deval Machine, ASTM Designation: D 289.

[7] Method of Test for Abrasion of Coarse Aggregate by Use of the Los Angeles Machine, ASTM Designation: C 131.

[8] Method of Test for Soundness of Aggregate by Use of Sodium Sulfate or Magnesium Sulfate, ASTM Designation: C 88.

[9] T. E. Stanton, *Eng. News-Record*, Vol. 124, p. 171, 1940.

[10] W. C. Hansen, Studies Relating to the Mechanism by Which the Alkali-aggregate Reaction Produces Expansion in Concrete, *J. ACI*, Vol. 15, No. 3, p. 213, January, 1944.

[11] T. M. Kelly, L. Schuman, and F. B. Hornibrook, A Study of Alkali-aggregate Reactivity by Means of Mortar Bar Expansions, *J. ACI*, Vol. 20, No. 1, p. 57, September, 1948.

of acceptable amounts of reactive minerals be stated. In some situations the danger must be circumvented by rigorously selecting low-alkali cement when none other than hazardous aggregates can be had. Tests for aggregate reactivity are now in the process of standardization.[1] A comprehensive discussion of the ways of evaluating aggregate reactivity is available in ASTM literature.[2]

Water is essential to alkali-aggregate reactivity, and repeated wetting and drying aggravates the rate and emphasizes the degree of failure. Water trapped in discrete crevices or small pores within aggregate particles is also troublesome in that it exerts disruptive forces upon freezing. For these reasons the use of impermeable aggregate, of poor ability to transmit or store water, is desirable. The density and absorption of aggregate[3] is a fair indication of its worthiness for use in concrete. In general, materials of greater density are most suitable for concrete exposed to weathering. This generality should, however, be applied with good judgment since some materials of dense structure contain discontinuous internal cells not likely to serve as water reservoirs, and particles of such dense materials may exhibit low *apparent* density. Porous aggregate of large pore size, even though such pores may be interconnected, is less susceptible to damage by freezing because large pores are less tenacious in their retention of water than are small pores.

Several other characteristics are also of great importance in determining the behavior of aggregates, but their consideration is often neglected in urban communities where relative utility of available materials has, in most instances, already been established by the time-consuming and sometimes expensive method of trial and error. However, in the building of structures in undeveloped areas where new and untried sources of aggregate must be used, these characteristics may be definitive in evaluating promising materials. One of these characteristics is surface texture. A material of rugged surface is—by reason of its greater likelihood of mechanically adhering to cement paste—more desirable than another of vitreous, conchoidal, or smoothly fractured surface. Materials of rougher texture are also less likely to develop continuous crevice areas beneath horizontally oriented particles from which cement paste may have been unfortunately washed by bleeding of water or sedimentation of cement.

Another significant characteristic is the coefficient of thermal expan-

[1] Method of Test for Potential Alkali Reactivity of Cement-aggregate Combinations, ASTM Designation: C 227.

[2] Symposium on Methods and Procedures Used in Identifying Reactive Materials in Concrete, *ASTM, Proc.*, Vol. 48, p. 1055, 1948.

[3] Method of Test for Specific Gravity and Absorption of Coarse Aggregate, ASTM Designation: C 127; also . . . Fine Aggregate, C 128.

sion of the material with respect to the same property of the cement paste or, in the case of coarse aggregate, the mortar. Many rocks have lower coefficients than paste or mortar; and, if the difference is great, the aggregate will contract less on cooling and cause tensile fractures of interfacial bond, thus providing channels for the percolation of water and subsequent deterioration by repeated freezing. Other influential characteristics are the specific heat and the thermal conductivity of aggregate. These determine to a great degree the temperature gradient within a concrete mass having temperature differences on opposite faces of a section. Where the gradient is sufficiently abrupt the concrete may be subjected to destructive internal strains.

Size and shape, as well as the relative number of particles of different size, are important in determining the suitability of a material for use as concrete aggregate. Remembering that a major function of aggregate is to act as the bulky filler in an expensive cement-water paste, and considering also that workable mixtures are necessary for ease in placement with minimum effort, it should then be obvious that particles offering least resistance to rearrangement among their kind are most desirable. Spherical particles meet this criterion of best shape. They roll against each other and their relative position in a group is easily changed. Furthermore, spheres have least surface area for a specific bulk volume and, as a consequence, less cement is required to coat their surface.

Cubes offer more resistance to rearrangement because of interferences of their edges and corners. For equal bulk volumes of material, cubes require 25 per cent more cement to coat their surface than do spheres. Flat, elongated, and prismoidal particles interlock with each other even more than do cubes. Their relative mobility in plastic mixtures is poor, and they behave much like a log jam. They impart harshness and encourage oversanding of mixtures in which they are used. Compared with spheres and cubes of equal bulk volume, they require much more cement to coat their surface. In attaining a prescribed consistency of concrete a greater bulk volume of spheres, as compared with cubes or prisms, can be added to a paste of specific fluidity because the spheres are more easily rearranged to provide plasticity and they use less of the paste to coat their surface. For this reason, natural gravel is usually more economical than is crushed stone in massive construction.

Size also influences the worth of a material used as an aggregate. For example, a single cube of 1-in. size has 6 in.^{2} of surface. If this cube is divided by planes through the center of each face, the volume of the material is still 1 in.^{3}, but the number of individual particles has become eight cubes—each of $\frac{1}{2}$-in. size. The total surface of the eight cubes is 12 in.^{2} area. Further subdivision of these particles produces no increase of their spatial volume, or what might be termed their bulking value, but

the number of particles and the sum of their individual surface areas soon reach extremely large values. During the mixing operation each aggregate particle must be wet by, and become intimately associated with, a companion film of paste. The sum of these coatings is immobilized, thus leaving less of the total paste available for the separation and flotation of particles, which condition is essential for plasticity. Because of their lesser surface area, a specific volume of large as compared with the same volume of small particles has less stiffening effect on a prescribed amount of paste. Consequently, a greater volume of large particles can be used as filler for a particular consistency of mixture, and this produces a greater volumetric yield of concrete. For this reason greater maximum sizes of aggregate produce more economical mixtures.

The relative frequency of occurrence of particles of different size is another factor influencing the utility of aggregate. For example, consider a cubical box of 1 ft^3 capacity. If this is filled with 1-in.-diameter spheres, each tangent to others, it will contain 12 layers each comprising 144 spheres. It might at first be inferred that the box is completely filled, but upon further consideration, it is evident that there are air spaces between adjacent spheres. These 1,728 particles, each of 1 in. diameter, constitute 0.52 ft^3 of bulk volume. The interstitial space amounts to 0.48 ft^3, or 48 per cent of the apparent volume of the material. The spheres could be more closely arranged in a system of hexagonal rather than cubical packing, and in this circumstance, the space between particles would be 44 per cent of their apparent volume. This space between aggregate particles is commonly termed *voids*. These uniformly sized particles, in their most compact arrangement, would require 0.44 ft^3 of paste to fill the voids and cement them into a solid mass.

If smaller particles were placed in each interstice, less paste would then be required to fill the voids. By using both fine and coarse aggregate having particles well distributed from small to large in size it is often possible to limit the interstitial space to less than 25 per cent of the apparent volume of the aggregate.[1] To produce plastic concrete of good workability, however, something more than a filling of the voids with paste is necessary. A surplus of paste must be provided to separate the particles at points of tangency or planes of contact and thus produce plasticity. Since well-graded aggregate possesses fewer voids, it requires less paste to produce a mixture of prescribed plasticity. For the same reason, a paste of specific dilution will tolerate the incorporation of more well-graded as compared with poorly graded aggregate, in preparing mixtures of suitable workability.

In almost all circumstances a statement of maximum particle size, and

[1] Method of Test for Voids for Aggregate in Concrete, ASTM Designation: C 30.

acceptable ranges of particle-size distribution, is necessary for procurement of suitable aggregate. When the gradation of aggregate is uncontrolled the consistency of the concrete is almost always erratic, and the quality of the concrete is usually variable. Furthermore, if aggregate of too large a size is used, the narrow clearances between steel and the face of a form may cause segregation of mixture components, or closely spaced reinforcement may behave like a sieve and separate parts of the mixture. Aggregate having particles well graded from fine to coarse in size facilitates placement of concrete in narrow spaces and in heavily reinforced structures.

Classification of aggregate with regard to size is accomplished by separating a representative sample of the material, using a standardized series of testing sieves.[1] Until about 1936, much confusion was experienced among aggregate producers, because some consumers expressed their requirements in terms of square openings while others preferred to use round openings.[2] However, cooperative action of manufacturer's associations, individual producers, public and private consumers, and various engineering societies, under the guidance of the National Bureau of Standards, has done much to improve the situation particularly with regard to coarse aggregate. Now, square-mesh sieves are almost universally used, and acceptable equivalents of square as compared with round openings have been established.[3] A system of square-mesh sieves logically related one to another was first produced in 1910[4] and is described in the 1913 ASTM Proceedings. A more modern system[5] providing closer separation of sizes is one so arranged that successive sieve openings are related in size as $1:\sqrt[4]{2}$. Only for the testing of coarse aggregate, however, are the intermediate sieves of this more closely spaced series generally used.

An exceedingly useful application of the Tyler series of sieves, which are related as $1:\sqrt{2}$, was suggested in 1918,[6] when it was proposed that concrete aggregate be classified in terms of an abstract number called the *fineness modulus*. This number, commonly abbreviated "F.M.," is the sum of the percentages of material coarser than the following sieves: 3 in., 1½ in., ¾ in., ⅜ in., Nos. 4, 8, 16, 30, 50, and 100. Its value is an index of the average surface area of the aggregate. Aggregates of different

[1] Method of Test for Sieve Analysis of Fine and Coarse Aggregates, ASTM Designation: C 136.

[2] Edmund Shaw, Sieve Testing of Aggregates, *Rock Products*, May 9, 1931.

[3] U.S. Department of Commerce, Simplified Practice Recommendation R 163-48.

[4] W. S. Tyler Company, Cleveland, Ohio.

[5] Specification for Sieves for Testing Purposes, ASTM Designation: E 11.

[6] Duff A. Abrams, Design of Concrete Mixtures, Structural Materials Research Laboratory, Lewis Institute, *Bull.* 1, 1918.

particle-size distribution may have similar fineness moduli; and furthermore, for aggregates of a specific type it is reasonably certain that those of similar fineness moduli will, if used in similar amounts, produce concrete of similar plasticity. For this reason, the fineness modulus of an aggregate is a worth-while adjunct to inspection and readily discloses a change in concrete-making quality of material from a particular source.

The highway departments of some states, as well as some other major consumers of aggregates, still adhere to the use of independently prepared specifications which are believed to be peculiarly suitable to their local problems. However, many progressive engineers, and most of the engineering societies by means of joint committee action in the promulgation of building-code requirements,[1] prefer to use nationally recognized standards,[2] which are in turn based to a great extent upon widely accepted simplified-practice agreements. The student is cautioned that, in some places, aggregate of best quality may be unavailable or obtainable only at great effort and expense. Local sources should be investigated if enforceable specifications are to be prepared and aggregate is to be obtained at reasonable cost. In any case, specifications should take into consideration local conditions as well as what is to be accomplished, and restrictions should be commensurate with the importance of the work.

1-5. Admixtures. An admixture is an extra component sometimes added to a concrete mixture for the purpose of creating a special property or for neutralizing a normal characteristic of the concrete; or to correct some deficiency of the mixture. The number of admixtures and the variety of advantages ascribed to them by their proponents are great. There are differences of opinion among engineers, with reference to some of them, as to their reliability and worth. Some are manufactured products of consistent behavior. Others are industrial waste products of little sales value, often representing unwanted expense or a troublesome disposal problem to their owners. The latter are seldom produced purposely, and sometimes their performance as concrete ingredients is inconsistent. These occasional components of concrete can, by their behavior, be arranged in three groups. (1) Some act mechanically during the plastic life of the concrete. (2) Others react chemically with one or more of the constituents of portland cement. (3) The most significant, however, are initially mechanical in their action, although later they participate in the reaction of cement with water.

Bentonite and other clays, silty sand, talc, and other chemically inert pulverized stones, all of subsieve particle size, are of the first category.

[1] Building Code Requirements for Reinforced Concrete, ACI 318-51, *J. ACI*, Vol. 22, No. 8, p. 589, April, 1951.

[2] Specifications for Concrete Aggregates, ASTM Designation: C 33: or, Specifications for Lightweight Aggregates for Concrete, ASTM Designation: C 130.

Their main effect is provision of cohesion and plasticity to poorly workable mixtures, and they act in two ways. Least significantly, they are fillers of small voids. Most importantly, they are spacers which mechanically separate aggregate particles; and as such, they are analogous to ball bearings in a raceway. They separate aggregate particles and thus reduce internal friction, and they provide room for more paste. They present great surface areas to be wet by cement paste, and because of their small size they stiffen and add skeletal structure to the paste. In this manner they reduce bleeding of water and sedimentation of cement. Furthermore, they alleviate harshness during placing, unreasonable shrinkage during the setting process, and abnormal permeability of the hardened concrete.[1] They are often effective for correcting deficiencies of poorly graded aggregate, but they usually require that *more water* be used for a prescribed consistency of concrete.

Air-entraining agents are peculiarly different admixtures of the mechanical category which usually permit a reduction of mixing water. As organic chemicals they are detrimental to strength, but this incidental behavior must be tolerated if their mechanical advantage is to be used. Microscopic bubbles of air are whipped into concrete during the mixing operation and, if they are stabilized so that they cannot collapse or escape, they perform all the mechanical functions of an inert fine solid. Moreover, they comprise innumerable discontinuous voids filled with elastic gas which cushions the strain of freezing and the expansive forces of chemical changes of the cement when instigated by harmful substances. It is reemphasized that air is the effective admixture, that it can be got into the concrete only by mechanical mixing, and that it can be kept there in an effective amount only by the great surface tension created by the foam-stabilizing organic agent. The remarkable discovery of aeration[2] has immeasurably improved our expectations of concrete durability and has been one of the most important developments of cement technology.

Chemical admixtures are added to concrete for the purpose of modifying the normal plastic life of the mixture, or for influencing its rate of gaining hardness and strength. A disadvantage of most chemical admixtures is that small changes in their amount cause great changes in their action. Furthermore, some may retard one cement and accelerate another. In this connection it should be remembered that cements, even though they may be of successive production lots from the same mill, are not always similar in constitution. Moreover, since chemical agents modify the normal reaction of one or more of the cement constituents, and since they are

[1] T. C. Powers, The Use of Admixtures for the Correction of Aggregate Gradation, *J. ACI*, Vol. 22, No. 1, p. 36, September, 1950.

[2] O. L. Moore, Pavement Scaling Successfully Checked, *Eng. News-Record*, Oct. 10, 1940.

effective in extremely small amounts, their action is critically influenced by changes in proportioning of mixtures.

Calcium sulphate in small amounts is a commonly used retarder and yet, with increased amounts, its behavior changes and it becomes a powerful accelerator. Organic materials such as gelatin, glue, sugar, other carbohydrates, and ligneous constituents of spent sulphite liquor used in papermaking, even if they are used in extremely small amounts, strongly affect the reaction of cement and water. Some of these are used as form coatings to retard surface hardening and thus make easy the production of textured effects when forms are stripped. Salts of metallic lead inhibit cement reaction and paints containing lead pigments may cause minor troubles at work sites.

Calcium chloride is a chemical admixture often used for the acceleration of repairs, as well as in winter concrete construction where its use is exceedingly helpful to the accomplishment of such work. However, its variability of action with different cements makes questionable the uniformity of behavior of the concrete at later ages. Despite the unfortunate probability that cement may vary in constitution, this frequently used admixture can be obtained of uniform quality.[1] When used in amounts not exceeding 3 per cent by weight of the cement, it shortens the plastic life of the mixture and in this manner is helpful in reducing bleeding. If it is used during warm weather or in heated concrete, the set may be accelerated to such an extent as to impair finishing, and heat of hydration may be increased to a degree that causes cracking. It may aggravate the dry shrinkage of hardened concrete and it usually increases expansion caused by alkali-aggregate reactivity. Nevertheless, it definitely improves the early strength of concrete.[2]

Proprietary brands of other chemical admixtures are numerous, but fairness to all producers of such materials precludes discussion of individuals. Some of them are harmless while others are hazardous, and any of them should be used only with discrimination. They are sold as surface hardeners, densifiers, plasticizers, water repellents, workability promoters, mixing-water conservationists, and dispersing agents. However, the last of these reputedly beneficial actions is illogical and is not supported by scientific principles.[3] Powdered aluminum deserves mention as a chemical admixture that is added to concrete to cause evolution of hydrogen gas, and this in turn creates porosity very much as does an air-entraining agent. However, the gaseous bubbles are unstabilized and they readily

[1] Specifications for Calcium Chloride, ASTM Designation: D 98.

[2] J. J. Shideler, Calcium Chloride in Concrete, *J. ACI*, Vol. 23, No. 7, p. 537, March, 1952.

[3] T. C. Powers, Should Portland Cement Be Dispersed? *J. ACI*, Vol. 17, No. 2, p. 117, November, 1945.

escape from the concrete. Much gas is necessary to accomplish the primary purpose—production of lightweight concrete—and variations of ambient temperature as well as cement alkalinity make the process difficult to control and the end result problematical.

Admixtures of the third category exhibit characteristics of both the foregoing types. They act not only as void fillers and as thrust bearings between large particles; they later associate with and participate in the reaction of cement and water. They differ from the truly chemical admixtures in that they are usable in large amounts and their reactivity is indirect. They are all finely divided pozzolanic materials which become chemically active only after the cement has combined with water. Their essential component is amorphous silica,[1] which reacts at normal temperatures in the presence of water with calcium hydroxide created by dissolution of the C_3S constituent of cement. The combination forms a strength-producing and densifying component of the hardened cement paste.

Natural materials such as pumicite or volcanic ash, diatomaceous silica, pulverized opaline cherts and shales; as well as artificial materials like pulverized aluminous and siliceous slags and calcined clays, and fly ash[2] —recovered from the stacks of coal-burning steam boiler plants—are all of pozzolanic nature. Some of these may be substituted for as much as 30 per cent of the cement with little loss in ultimate strength of the concrete and, furthermore, with beneficial effects on the heat of hydration, durability, and impermeability. All these are somewhat helpful in minimizing the harmful effects of alkali-aggregate reactivity.[3] These pozzolanic materials, during the plastic life of the concrete, also exhibit all the helpful mechanical attributes and harmful demands for *more water* as the inert materials discussed above. Natural cement, mixed with or substituted for part of the portland cement, is within this category.

The successful use of an admixture requires the solution of three serious problems. First, because most of them are variable among their kind and different in action with cement, they deserve precautionary study with the associated cement before they are used. Second, their adequacy of performance is difficult to measure at a construction site during the progress of the work. Consistency of action is not visually evident, and—without technical control—abnormality of behavior is not imme-

[1] R. E. Davis, Use of Pozzolans in Concrete, *J. ACI*, Vol. 21, No. 5, p. 377, January, 1950.

[2] R. F. Blanks, Fly Ash as a Pozzolan, *J. ACI*, Vol. 21, No. 9, p. 701, May, 1950.

[3] W. T. Moran, Use of Admixtures to Correct Alkali-aggregate Reaction, *J. ACI*, Vol. 22, No. 1, p. 43, September, 1950; and C. H. Scholer and G. M. Smith, Use of Chicago Fly Ash in Reducing Cement-aggregate Reaction, *J. ACI*, Vol. 23, No. 6, p. 457, February, 1952.

diately disclosed and may not become known until some years after the structure is built. Finally, their use is accompanied by problems of materials handling and batching. On large projects, where installation of special batching equipment, employment of capable technicians, and cost of testing devices are justified, these problems are easily solved. However, on ordinary projects where rigid control may unreasonably increase supervisory cost their solution is almost impossible.

A good example of the first problem is fly ash, which may be as variable in its performance as are the smokestacks from which it is collected, and yet it is admittedly a worthy beneficial admixture. Offhand it would seem that material from a single source could be trusted, but a change of fuel or operating conditions at its place of origin, or a new shipment of cement, might cause differences of reaction. Chemical agents in general, and air-entraining agents added on the work, are convenient examples of the second control problem. Such additives are effective in very small amounts, and sometimes they are disastrous if used beyond certain limits. For example, the effective amount of air-entraining agent in a cubic yard of paving concrete weighing about 2 tons approximates 2 avoirdupois ounces, whereas 4 oz may cause irreparable reduction of strength. The effect of 2 as compared with 4 oz in 2 tons of concrete is not visually evident and can be determined only by test.[1] Most workmen are contemptuous of such small components of a batch and—if unsupervised—they may give some batches a double dose, leave it out of others, and generally measure the admixture to suit their own convenience. Finally, the use of admixtures requires special storage and measuring devices seldom available at ordinary batcher plants. Moreover, the repeatedly similar batching of five as compared with four ingredients is more difficult to accomplish, not as 5 is greater than 4, which is 20 per cent, but instead as 32 is greater than 16, or 100 per cent.[2]

There are several dispassionate reasons why some engineers are skeptical of admixtures in general. Chemical admixtures are usually critical in the amount that can safely be used. By-products are often uncertain in their behavior. Successful use of these requires watchful observation and vigorous control, and this can be achieved only by expert technicians. Admixtures obscure the incongruous proportioning of basic ingredients, and without conscientious testing, poorly compounded mixtures are often

[1] Symposium on Measurement of Entrained Air in Concrete, *ASTM. Proc.*, vol. 47, p. 832, 1947.

[2] Consider the measurement of five different things, any one of which may with equal probability and without regard to the others be measured either right or wrong. There are 32 possible results of such an operation and only one of the results is desirable—that all be measured right and none wrong.

undetected. When they are used with preeminently satisfactory aggregates, many admixtures do nothing more than increase the cost of the concrete; whereas an equivalent monetary increase of cement is usually beneficial and introduces no extra problem of control. Nevertheless, in situations where undesirable aggregates are all that can be had, or when a specific resistance to some special condition of service must be built into the concrete, admixtures are indispensable.

1-6. Compressive strength of concrete. Concrete is an artificial stone and its excellent resistance to compression resembles the principal asset of natural stone. It is a pseudo fluid for a convenient part of its early life; hence it can be easily installed to form large monolithic parts of a structure. Furthermore, its strength and other properties can be regulated to some extent during its manufacture and, as a consequence, its cost need be no greater than is necessary for the service it is required to perform. For these reasons, concrete is an ideal material for foundations and for other large or intricate parts of structures that must be rigid and resistant to compressive forces.

The quality of concrete is usually specified or discussed in terms of unit compressive strength at the age of 28 days; when no age is mentioned this interval after mixing is usually implied. Other ages of reference such as 3 or 7 days are also used, especially with respect to high-early-strength concrete, and in such instances the age of reference should always be defined. The compressive strength of concrete, as it is most often determined by testing molded cylinders,[1] may range from approximately 1,000 to more than 8,000 psi. Compressive strength is also measured, in some instances, by testing cores of concrete drilled from the structure or by testing remnants of beams broken in flexure.[2] It has been demonstrated by means of elaborate and costly testing procedures that—when it is simultaneously subjected to lateral as well as axial pressures—concrete of moderate quality may support axial unit loads in excess of 70,000 lb when the lateral restraint is about one-third of this value. However, the triaxial-stress relationships of concrete have received very little study, and their effect upon local areas of concrete supporting concentrated loads is seldom considered in the design of reinforced-concrete structures.

Compressive strength, because it is a most important as well as an easily measured attribute, is often used as an index of the quality of other physical properties. It is generally assumed that—for concrete containing similar ingredients—a greater compressive strength is accom-

[1] Method of Test for Compressive Strength of Molded Concrete Cylinders, ASTM Designation: C 39.

[2] Method of Test for Compressive Strength of Concrete Using Portions of Beams Broken in Flexure, ASTM Designation: C 116.

panied by greater tensile strength, greater flexural strength, greater modulus of elasticity, greater density, less permeability, and greater durability. These assumptions are substantially true of any large family of observations; yet, because of somewhat poor correlation of compressive strength with other properties, and in consideration of inherent variation of individual measurements of compressive strength, the relationships should be considered little more than indications of probable trends when only a few observations of compressive strength are involved.

Even in most favorable circumstances, when the batching of ingredients is conscientiously performed with greatest accuracy and when specimens are prepared and tested by thoroughly competent technicians, the individual results of a series of tests are scattered around their collective average value in accordance with the principles of mathematical probability. It cannot be too strongly emphasized that—even when using the most precise equipment with the greatest of personal care—the results of a series of similar observations are quite different among themselves; many are similar and some may even be identical, and yet others may be so incredibly different as to seem not to be members of the same family. Such is the quality of concrete; most of it approximates what we wish it to be, much we would rather have nearer an average, and even with the best of control, some is inexplicably poor or superlative.

A most useful measure of the scatter or dispersion of a number of observations is their standard deviation; *i.e.*, the square root of the average of the squares of all individual deviations from the common average. This root-mean-square measure of dispersion is very helpful for estimating the trustworthiness of a few observations, estimating the over-all range of quality indicated by available observations, judging the normality among others of a seemingly incredible observation, and establishing limits beyond which needs for control are indicated.[1] The standard deviation is sometimes expressed as a percentage of the mean value of the observations with which it is associated, in which case it is called the *coefficient of variation.* This percentage measure of the dispersion of compressive tests—with respect to concrete having a reasonable degree of conscientious control—is usually somewhat more than 10 and less than 15 per cent. Allowance for this normal variation in quality is reflected in the long-established practice of proportioning controlled concrete mixtures in such a manner as to produce concrete having an average strength 15 per cent greater than is required by plans or specifications.

The ultimate strength of concrete is influenced primarily by the rich-

[1] "ASTM Manual on Quality Control of Materials," *ASTM Special Tech. Pub.* 15-C, January, 1951.

ness in cement of the water-cement paste used to glue the aggregate into a solid mass. Other factors such as aggregate type and quality, temperature and duration of water-cement reaction, workmanship during mixing and placing, treatment during the early hardening period, and age are of secondary but truly significant influence. These secondary factors approach primary importance as the quantity of cement in a mixture is reduced. In other words, a potentially strong mixture—containing a large amount of cement per unit of water—will tolerate a greater number and degree of unfavorable factors of secondary influence with less evident harm than will a mixture of lean cement content.

Attainment of a desired compressive strength of concrete was, until the year 1918, largely dependent upon auspicious circumstance; empirical proportions of cement and aggregate were hopefully expected to produce certain strengths. For example, volumetric proportions of 1:2½:5 were expected to produce concrete having a compressive strength of 1,500, and others of 1:2:4 were confidently presumed to attain 2,000 psi at the age of 28 days. What few supervisory tests were then performed were often disquieting.

Vastly significant progress in concrete technology—the result of research sponsored by the Portland Cement Association and performed at Lewis Institute—was made known during 1918 by Duff A. Abrams, then professor in charge of the Structural Materials Research Laboratory. The major result of these cooperative studies of cement and concrete was the disclosure of a means by which the strength of concrete could be more assuredly controlled. The fundamental concept of Abrams's water-cement-ratio theory of proportioning concrete was that, "With given concrete materials and conditions of test the quantity of mixing water used determines the strength of concrete, so long as the mix is *of a workable plasticity*."[1] This precept has since revolutionized the design and control of concrete mixtures, and its truth has been fully confirmed in practice.

This early research was performed when concrete ingredients were measured by volume. As a consequence, Abrams's empirical equations expressed the strength of concrete as a function of the volumes of water and cement comprising the paste. This water-cement ratio by volume, as it was once so extensively used, represents a somewhat illogical association of an actual volume of water with an apparent volume of cement. This peculiarity of the volumetric ratio should be recollected if reference is made to the literature of some 20 years following 1918. Use of the

[1] PCA Minutes of Annual Meeting, December, 1918. Italics have been added, inasmuch as the specific limitation of the principle is often overlooked. The rule does not apply to damp, sandy, crumbly mixtures used for precast units such as pipe or block; nor does it apply to harsh and greatly undersanded concrete.

ratio of cement to water *by weight* was initiated by Inge Lyse in 1932, as a result of his discovery that the relationship of strength to C/W could be manipulated as an empirical equation of the first degree. This ratio, C/W by weight, is of peculiar utility for the study of experimental mixtures. It was used in much of the technical literature of the decade following 1932, and yet, because of the inveterate custom of relating all the ingredients of mixtures to cement rather than to water, it is now seldom encountered and the inverse ratio W/C by weight is of most frequent current use.

There are indications that the strength attained at 28 days by concrete made with modern cement is approximately double that reported by Abrams in his tests using material of 1914 to 1918 production.[1] So also, but to a much lesser degree, have other physical properties of concrete been benefited by industrial improvements in the composition and processing of cement. These improvements, some of great and others of little significance, have curtailed the once common practice of proportioning mixtures for optimum compressive strength with minimum cement content. They have also made obsolete much of the mixture design data of the period 1920 to 1940. Now more emphasis is placed on the attainment of durability by limitation of excessive water, and rightly so, since a saving of cement is absurd if such economy is detrimental to the life of a structure.

When average materials are used and no preliminary tests of the materials are made, the extensively approved Code[2] of the American Concrete Institute limits the water in a mixture—including that contained in the aggregates—and designates the assumed compressive strength of the concrete as shown in Table 1-1. However, when artificial aggregates or

TABLE 1-1. Assumed Strength of Concrete Mixtures

Water content, U.S. gal per 94-lb sack of cement	Assumed compressive strength at 28 days, psi
$7\frac{1}{2}$	2,000
$6\frac{3}{4}$	2,500
6	3,000
5	3,750

admixtures are to be used, the Code requires, and when the use of controlled concrete is contemplated, it permits that other water contents determined in accordance with designated ASTM testing procedures shall

[1] H. F. Gonnerman and W. Lerch, Changes in Characteristics of Portland Cement as Exhibited by Laboratory Tests over the Period 1904 to 1950, *ASTM Special Tech. Pub.* 127, p. 22, June, 1951.

[2] Standard Building Code Requirements for Reinforced Concrete (ACI 318-51). Separate copies can be obtained at nominal cost by addressing the American Concrete Institute in Detroit, Mich.

be used. The method of determining maximum water for controlled concrete requires the establishment of a curve showing the relationship of water content to compressive strength. The ACI Code prescribes that this curve be based on a minimum of four observations made at each of three different water contents. In contrast, the New York City Building Code requires that such a curve for controlled concrete shall be based on four similar observations, but at each of four water contents. Both these codes require that the water content used on the work be that indicated by the curve as producing a strength 15 per cent greater than is called for on the plans or in the specifications.

The foregoing regulations give only minimum requirements; in many cases a greater number of observations may be desirable. For example, when only three observations of a relationship are available—and the nature of the relationship is unknown—such data are always most accurately described by a circular arc or by a straight line that might be envisioned as a special case of an arc of infinite radius. As a matter of fact, four observations are also inadequate for a clear indication of the true nature of the relationship, or for the choice of an empirical equation most suitable for describing the data. Furthermore, in so few batches, if the relative amount of fine and coarse aggregate in any single batch was poorly balanced this might produce freakish results and a misleading curve. For these reasons, in any research where secondary factors may influence the results, a spread of observations is essential. Though it is not invariably true, nevertheless it is generally assumed that an exponential equation is a most suitable approximation of the strength vs. water ratio relationship.

Hypothetical data, suitable for demonstrating the relationship of strength to water content, are shown in Table 1-2. These data are illustrative of some of the considerations involved in establishing a curve. They give a result that is similar to contemporary information having

TABLE 1-2. Compressive Strength, Psi, at 28 Days, as Influenced by Various Ratios of Water to Cement by Weight

w/c 0.37	w/c 0.46	w/c 0.55	w/c 0.64	w/c 0.73
6,390	5,230	4,000	3,450	2,720
5,910	4,390	3,450	2,700	2,560
6,280	4,500	3,650	3,200	2,300
5,220	4,680	4,350	3,010	2,650
5,950	4,700	3,860	3,090	2,560

widespread usage;[1] such curves are, of necessity, conservative in their indications. More reliable information is to be had by making tests with the materials actually to be used. These data simulate a situation wherein five batches of concrete of suitable consistency might have been prepared, and four specimens were molded from each batch. As is customary, unit strengths are rounded off to the nearest 10 lb. It should be noted that no single observation is of any worth-while significance alone;

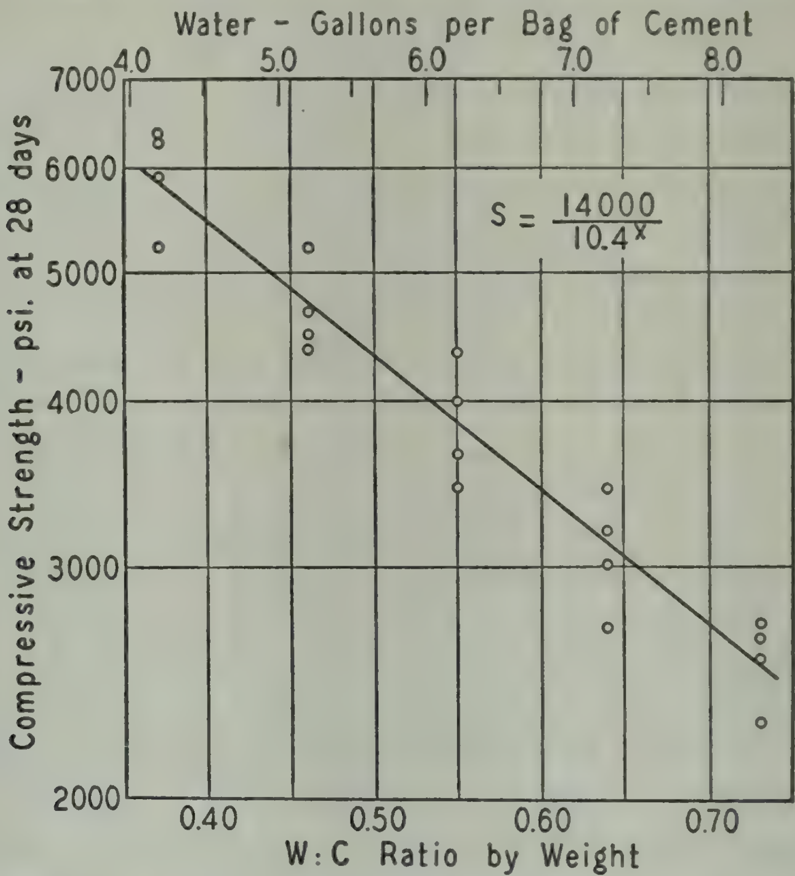


FIG. 1-3. Relation of compressive strength to water content.

each must be considered with regard to all the data. Furthermore, it is unequivocally stated that—with given aggregates—paste of greater fluidity requires more sand; otherwise the concrete might be cohesionless, harsh, or susceptible to segregation during placing, and permeable to water after hardening. In contrast, an overabundance of sand for a specific fluidity of paste is detrimental to strength, requires more cement for a prescribed consistency of concrete, aggravates settlement shrinkage, and may adversely influence durability. We shall assume that proper adjustment of fine and coarse aggregate—suitable for the desired consistency of concrete and for the different dilutions of paste—was made in these batches during the mixing operation.

[1] "Design and Control of Concrete Mixtures," 9th ed., Portland Cement Association, 1948.

Data from the table are shown plotted, using semilogarithmic coordinates, in Fig. 1-3. One of several methods might have been used to locate the curve, the least accurate being to plot the average values and then draw a straight line best fitting all of them. However, with a table of logarithms and a calculating machine, the constants of the curve can be computed from the average values in about the same time. Occasionally, when observations are meager in some areas of interest and prolific in others, or when they cannot be conveniently grouped to provide average values of equal weight, computation of the curve using individual values is the only reliable resort. The mechanics of this method, using average values from the table, are shown hereafter.

In principle, the general exponential equation

$$S = ab^x$$

is converted to the more easily managed linear form

$$\log S = \log a + x \log b$$

Test results are substituted there, forming separate equations for each associated pair of observed values. These are then solved simultaneously for the two unknown constant terms. In practice, by using tabular methods shown below, the equations need not even be formed. Moreover, the individual entries shown in columns 4 and 5 above need not be made when a calculating machine is used, since they can be accumulated in the machine; only their summation need actually be noted. The general form of the least-squares method of solving several simultaneous equations having two unknowns is

TABLE 1-3. Illustrative Computations

	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5
	W/C	Compressive strength	log S		
	x	S	y	xy	x^2
	0.37	5,950	3.7745	1.396565	0.1369
	0.46	4,700	3.6721	1.689166	0.2116
	0.55	3,860	3.5866	1.972630	0.3025
	0.64	3,090	3.4900	2.233600	0.4096
	0.73	2,560	3.4082	2.487986	0.5329
$n = \sum^{5}$	2.75	.....	17.9314	9.779947	1.5935
Mean (M)	0.55	.....	3.58628		

$$b = \frac{\Sigma xy - n(M_x M_y)}{\Sigma x^2 - n(M_x)^2}$$

and

$$a = M_y - b(M_x)$$

Substituting the tabular values in the above,

$$\log b = \frac{9.779947 - 5(0.55 \times 3.58628)}{1.5935 - 5(0.55 \times 0.55)} = -1.0163 \cdots$$

$$\log a = 3.58628 - (-1.0163 \times 0.55) = 4.1453$$

and

$$S = \frac{13{,}970}{10.38^x}$$

1-7. Modulus of elasticity of concrete. The modulus of elasticity of concrete in compression, E_c, is the ratio of the unit stress expressed as pounds per square inch to the deformation in inches per inch of gage length. A typical stress-strain diagram is illustrated in Fig. 1-4. It should be noted that the locus is a curve and that E_c is of variable value depending upon the magnitude of the stress. Many contradictory values may be assigned to the modulus of elasticity as is shown in the illustration. Of these, the secant modulus at the value of the unit stress used for purposes of design is the proper one to be used for E_c in the design of reinforced-concrete structures.

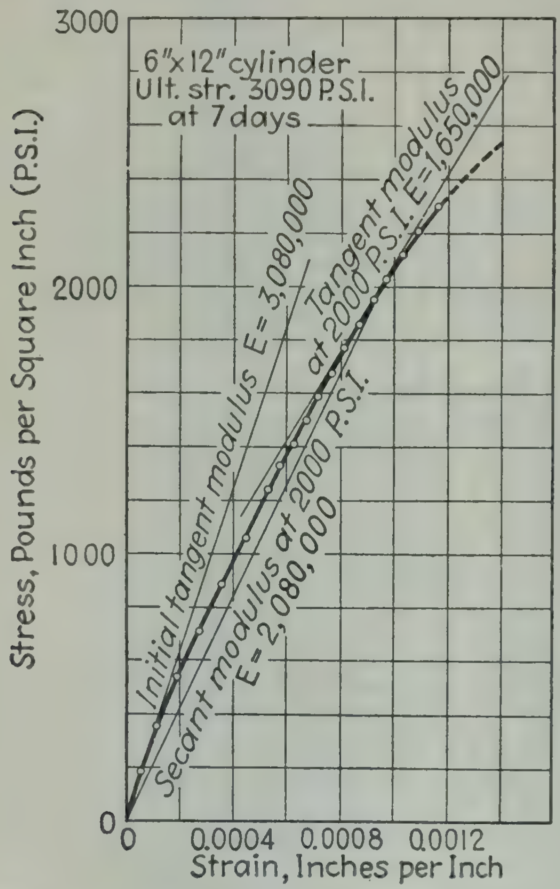


FIG. 1-4. Modulus of elasticity of concrete.

The ACI Code assumes that

$$E_c = 1{,}000 f'_c$$

where f'_c is the compressive strength of the concrete at the age of 28 days, unless otherwise specified. This relationship is an approximation that, in most circumstances, seems to be consistent. Various factors influence the elastic behavior of concrete, the primary one being the relative amounts of cement and water used to bind the aggregates together. The age and condition of curing affect E_c in that both factors are of direct influence on the completeness of combination of cement and water. The

density and degree of saturation of the hardened concrete are also influential factors.

In general, greater cement content, greater compressive strength, better curing conditions, greater age, and greater density of concrete tend to increase the value of the secant modulus E_c, which approaches the value of the inital tangent modulus as a limit. E_c varies slightly with the degree of water saturation of the hardened concrete mass, completely saturated concrete having a slightly higher modulus than the same concrete in a partially saturated condition. Another factor having a pronounced influence on the value of E_c is the type and quality of aggregate contained in the mass. As a general rule, the value of E_c is greater for concrete made with crushed stone aggregate consisting of prismoidal-shaped particles as compared with concrete made with gravel largely composed of rounded particles.

The secant modulus of elasticity is inconvenient to determine as a routine procedure, inasmuch as the operation is time-consuming and requires facile use of optical, mechanical, or electrical strain gages of great delicacy and precision. In contrast, with suitable instrumentation, the tangent modulus can be easily and quickly determined by sonic or vibrational methods.[1] The elastic properties of concrete are fairly well correlated with both the flexural and the compressive strengths of the material, and the elastic modulus is a useful index of the state of these other properties. It can be used to trace a progressive change in the quality of concrete with regard to time, and this can be done repeatedly and in a nondestructive manner, using only a few specimens. Observations of elastic characteristics are especially useful in studies of durability, because the deterioration of specimens can be noted as changes in elastic properties long before failure is visible or otherwise evident. Furthermore, in actual structures the elastic modulus influences the rate of conduction of vibrations through the concrete. Deterioration of structures can sometimes be disclosed as discontinuities or differences in the rate of propagation of wave fronts deliberately created within the concrete.[2]

In the design of reinforced-concrete structures by the straight-line method, it is assumed that concrete is an elastic material. For concrete of good quality the assumption is safe and practical, but it is fundamentally incorrect. In actual fact, concrete must be recognized as an extremely rigid plastic which is subject to progressive and cumulative deformation under load. This property, termed "plastic flow," is neg-

[1] Method of Test for Fundamental Transverse Frequency of Concrete Specimens for Calculating Young's Modulus of Elasticity (Sonic Method), ASTM Designation: C 215.

[2] Johannes Andersen and Paul Nerenst, Wave Velocity in Concrete, *J. ACI*, Vol. 23, No. 8, April, 1952.

ligible in most instances, but its existence should be recognized and considered in the design of unusual structures. Figure 1-5 shows the results of tests made by The Port of New York Authority to determine the magnitude of the permanent set or plastic flow under sustained load in connection with a study of the design of a reinforced-concrete arch crossing Riverside Drive and connecting with the George Washington Bridge.

The tests illustrated in Fig. 1-5 were made on concrete columns 10 by 10 by 48 in. containing 1 per cent reinforcement. Curves B, C, and D

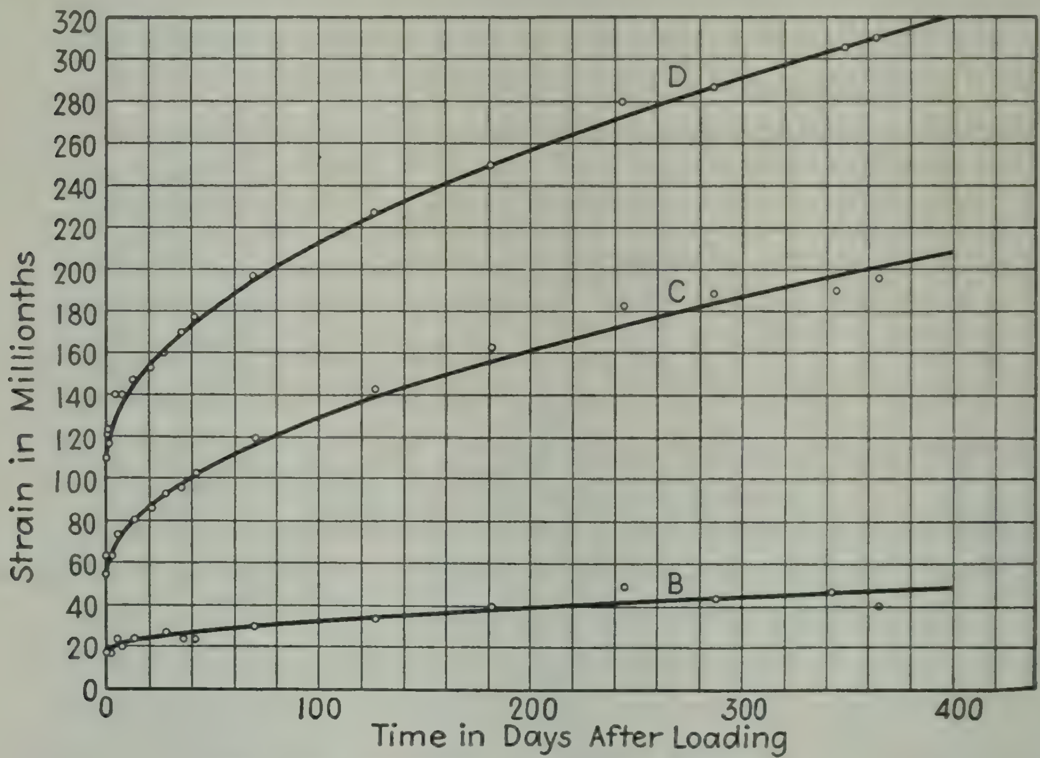


FIG. 1-5. Plastic flow due to sustained load.

indicate the permanent deformation over a period of 1 year of specimens loaded with sustained loads of 100, 300, and 500 psi, respectively. Permanent deformation under normal working load is not great, but it is measureable and is proportional to the amount and duration of loading.

1-8. Tensile strength of concrete. Concrete is poorly resistant to tensile loads; consequently, its tensile strength is assumed to be of negligible value in the design of reinforced concrete structures. Tensile strength varies from about 7 to slightly more than 10 per cent of the associated compressive-strength value; it is, proportionally, least for rich mixtures and for concrete of greater age.

1-9. Shearing strength of concrete. The shearing strength of concrete is influenced by the ingredients and the proportions of the mixture. It can be reasonably assumed that the strength in shear approximates one-fifth, and seldom exceeds one-fourth, of the strength of the concrete in compression. Pure, or "punching," shear is the sliding of adjacent

planes, one upon the other. This type of failure is resisted not only by the inherent cohesion of a material but also by internal reactions similar to friction and incidental to the granularity of the material. Concrete is of heterogeneous internal structure, and the interlocking and bridging action of strong aggregate particles distributes much of any shearing load as local concentrations of tension and compression within the mass. Shearing strength is improved by stronger paste, by greater integrity of the bond of paste to aggregate, and by stronger and more angular aggregate. This subject is discussed at greater length in Chap. 4.

1-10. Miscellaneous properties. The weight of ordinary concrete ranges between 140 and 160 pcf and is dependent upon the apparent densities and relative proportions of its ingredients, the entrained-air content, the degree of consolidation attained during its plastic life, and the amount of water physically absorbed by the hardened material. It has been customary for many years to assume the average weight of plain concrete to be 145 and that of reinforced concrete to be 150 pcf. Even for concrete containing deliberately entrained air, these assumptions are suitable for estimating purposes. With specific exception to some mixtures containing little or no coarse aggregate which in respect to general custom should be called mortar, concrete weighing as little as 75 pcf can be made with special aggregates.

Concrete mixtures made with normal portland cement and lightweight aggregates are often disconcertingly harsh and difficult to place using ordinary construction equipment and technique. Such mixtures usually weigh between 90 and 115 pcf. A phenomenal gain in plasticity and workability, accompanied by reduction of unit weight to a range approximating 75 to 100 lb, can be imparted to lightweight concrete mixtures by using either normal or air-entraining cement with an extra increment of air-entraining agent. About twice the amount of air-entraining agent normally used in ordinary concrete is essential for good workability and minimum weight, but this is not conducive to producing good compressive strength. Nevertheless, the best of lightweight aggregates are usually intractable in ordinary construction unless they are so plasticized, and furthermore, all of them make concrete that is relatively weak in comparison with ordinary concrete. This inherent disadvantage must be provided for in design.

An unfortunate practice of some proponents of lightweight concrete and of many vendors of lightweight aggregates is to discuss the unit weight of their sponsored product in terms of "oven-dry" weight, and to compare this with the water-saturated weight of ordinary concrete. Such prejudicial comparison is analogous to making measurements with an elastic yardstick. It should be remembered that concrete in a structure is almost never free of mechanically absorbed water, and that such moisture

comprises approximately 10 to 20 per cent of the weight of the concrete. The propensity to absorb water is a natural attribute of hardened cement paste, and conditions of either complete or partial saturation are common to both types of concrete. Therefore, in any weight comparison of light and ordinary concrete the datum of reference should at least be the same; both might be compared saturated, both might be compared in equilibrium with atmospheric moisture, or both should be converted to the artificial condition of "oven-dried to constant weight." Inapt comparisons have caused misunderstandings among engineers, contractors, and materials producers and may have hampered progress in the art of lightweight construction. Nevertheless, the potential worthiness of such construction is unquestionable.

Concrete shrinks when stored in air and expands in water. The amount of volume change is a function of the chemical composition of the cement, the water-cement ratio of the paste, the richness of the mixture, the reactivity of the aggregates, and the chemical environment of the concrete. The amount and condition of the magnesia, as well as the amounts of uncombined lime, calcium sulphate, tricalcium aluminate, tetracalcium aluminoferrite, and the so-called free alkali constituents of the cement are predominant factors influencing the permanent volume change. However, participation of some of these in producing expansion is dependent upon external circumstances such as exposure of the concrete to water-borne chlorides or sulphates, or a casual presence in the aggregates of critical amounts of reactive silica. Temporary and reversible changes in volume are caused by temperature changes and also by colloidal swelling and contraction of cementaceous calcium silicates actuated by changes in the degree of water saturation of the concrete. The total volumetric variation of concrete is the algebraic sum of permanent and temporary change. It is exceedingly difficult, if not impossible, to predict or control its full amount.

Seasonal and daily variation of ambient temperature of concrete is the most troublesome of the temporary changes which must be provided for in the design of large structures. The thermal coefficient of expansion of concrete is approximately 0.000006 in. per in. per °F. With respect to a not uncommon seasonal change of 100°F., this represents ¾ in. change in length per 100 ft of structure. Even though the thermal conductivity of concrete is fairly good, sudden daily or hourly variations of temperature may challenge the capability of the concrete to accommodate the change. One example of this, among many others, is the case of a pavement slab having its upper surface suddenly warmed by the sun while its lower surface is maintained cool by a damp or possibly frozen subgrade. If thermal accommodation of the concrete is too slow, the slab will warp; and if it is prevented from warping by mechanical restraint, tensile fractures are

likely to occur in its coolest surface. Less troublesome, but still serious, problems of temporary change are created by the slow shrinkage of concrete stored in air and by the slow expansion of concrete that is constantly wet by water. The shrinkage of concrete caused by evaporation of absorbed water ranges between 0.0002 and 0.0005 in. per in. A convenient average figure to remember is $\frac{3}{8}$ in. per 100 ft of structure. Expansion of concrete in water may be as great as 0.0002 in. per in. per year, and this approximates $\frac{1}{4}$ in. per 100 ft of structure. These volumetric changes must be considered in the design of long members such as pavement slabs, retaining walls, and dams; and provisions must be made for sufficient and properly located expansion and contraction joints. Means of caring for such deformations will be discussed later in greater detail.

1-11. Proportioning of concrete. Aside from the busy and engaging field of research, the proportioning of concrete begins with some person who must define, for the understanding of others, the minimum quality and other characteristics of the material with which a structure is to be built. To decide about details, this person might consider the recommendations of some authoritative guide such as the ACI Building Code, or he could call upon his own experience or possibly make some exploratory tests of available materials. He might even be directed by local regulations as to the details of his definition. The end result would be a specification. He should forthrightly ensure that his specification properly describes concrete capable of performing without failure the work required of it and, furthermore, that the concrete shall perform its assigned task for the contemplated life of the structure. Since concrete can be gravely harmed by the acts of irresponsible or unscrupulous persons, the engineer should also give directions regarding its manufacture, conveyance, installation, and early care.

In the preparation of specifications, the attainment of adequate strength was once of primary concern; whatever durability was perhaps attained was accepted as the best that could be had of the particular concrete. Because cement has been remarkably improved with respect to strength, and because of our greater technical knowledge of factors influencing durability as well as the strength of concrete, it is now the habit to consider durability as being of primary importance. Adequate strength can easily and assuredly be attained with properly adjusted mixtures. Recommendations of a joint committee of several engineering societies,[1] with regard to suitable water contents for various conditions of exposure, were once widely used for ensurance of durability. Now, since air entrainment is conclusively effective and special-purpose cements are available, this former means of assurance is less necessary. The only such

[1] "Report of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete," ASCE, AREA, AIA, ASTM, ACI, PCA, June, 1940.

restriction of the current ACI Code is to the effect that the water content of concrete exposed to freezing weather shall not exceed 6 gal per bag of cement. We can now be assured that a structure will endure, first by choosing a proper cement; and second, by requiring that, in relationship to water content, enough of it be used; and furthermore, wherever it might also be effective, we can demand that a specific amount of air be incorporated in the concrete. As contributive ensurance of durability we can insist that plastic cohesive mixtures of suitable consistency be installed, by methods which preclude adulteration, segregation, or other harmful mistreatment of the concrete.

One of several methods can be used to define and guarantee the functional quality of the concrete. The least dependable of these is the designation of arbitrary amounts, such as 1:2:3, or 1:2:4, or sometimes even 1:3:5 parts by volume,[1] of the solid ingredients comprising the mixture. Volumetric measurement of such proportional increments, and the quality of the resulting concrete, is extremely erratic because the loosely compacted volume of damp aggregate is amazingly variable at different moisture contents. Fine aggregate is particularly susceptible to what is called *bulking.*[2] If this seeming increase of the apparent volume of aggregates is uncompensated during measurement, batches may be deficient by as much as 25 per cent of their solid components; they are usually deficient in some components and they are always variable in quality, and they are not proportioned as specified. The student is warned that, so far as the hardened product is affected, such arbitrarily selected mixtures, prepared as either arithmetic or geometric series relationships, contribute no remarkable properties to the concrete.

Weighing of arbitrarily selected increments is somewhat more desirable than is volumetric measurement for small projects or for work of minor importance. More accurate compensation can be made for water carried into the batch by the aggregates. Arbitrary proportions are completely satisfactory, however, when provision is made to permit some shifting of aggregates from one part to the other if their gradation should change, and

[1] Fuller's rule for estimating the quantities of materials for concrete from known or assumed proportions given by volume can be useful on small projects of minor importance. It is as follows:

$$C = \frac{11}{c + s + g} \qquad S = C \times s \times \frac{3.8}{27} \qquad G = C \times g \times \frac{3.8}{27}$$

where c:s:g represents the ratios of ingredients by volume, such as 1:2:4, and C = barrels of cement per cubic yard of concrete, S = cubic yards of fine aggregate per cubic yard of concrete, and G = cubic yards of coarse aggregate per cubic yards of concrete. See Edward E. Bauer, "Plain Concrete," 3d ed., McGraw-Hill Book Company, Inc., New York, 1949.

[2] A. T. Goldbeck, National Crushed Stone Association, *Bull.* 1, 1927.

when the materials are weighed and some limitation is placed upon water. Another widely used method of ensuring adequate quality is to require that a minimum cement content—such as 5, 6, or 7 bags in each cubic yard of concrete—be used. This is not entirely effective unless the maximum allowable water content or the consistency of the concrete is also defined. A third way of establishing a quality level is to require that the concrete conform with a specific water-cement ratio; this, if it is to be effective, should also be accompanied by a definition of the consistency of the mixture.

The ailments and deteriorative tendencies of aged concrete are influenced greatly by the consistency at which the concrete was placed. The cohesion, consistency, plasticity, and workability of pseudo-fluid concrete mixtures are elusive properties. They are explainable, but such discussion is too involved to be presented here. These properties are crudely but effectively measured by what is called the *slump* of the concrete.[1] The slump of freshly prepared concrete is influenced by the fluidity of the water-cement paste, by the relative amounts of paste and aggregates, and by the textural characteristics of the aggregates used to give the paste skeletal structure.

Concrete mixtures are suspensions of solids in water. Excessive water, producing more fluid consistencies of greater slump, encourages segregation and sedimentation of the solid components of the semifluid mixture. When concrete mixtures are placed, at first the larger particles of coarse aggregate segregate and settle faster, leaving concentrations of both water and mortar at the surface of the pour. This is especially undesirable in pavement-slab construction. Later, while the concrete is quiescent but before it has begun to harden, the larger particles bear against each other and interlock; the smaller particles continue to settle and fall away from them, leaving voids filled with water. These water-filled voids are reservoirs susceptible to freezing. These voids can also form continuous or interrupted channels for the capillary percolation of water. Excessive manipulation of the concrete during placement aggravates these troubles. Therefore, it is desirable that the slump of concrete be the least that can be placed, with due regard to the nature of the structure, the size of the forms, and the clearances of reinforcement.

For pavements, or other construction where concrete can be discharged into the form at its final resting place, mixtures of preferably less than 2-in. and certainly less than 4-in. slump are desirable. For more intricately shaped structures or for deeper members such as walls, piers, or columns, concrete of less than 6-in. slump should suffice in all but the

[1] Slump Test for Consistency of Portland-cement Concrete, ASTM Designation: C 143. The slump is the subsidence, caused by gravity, of a plastic concrete specimen having an original height of 12 in.

most difficult of circumstances. Such slumps are good for the potential durability of the structure, but they are not always liked by contractors because they are not conducive to effortless placement of the concrete.

The formulation and adjustment of actual mixtures almost always involves awareness of, and the deliberate choice of, the least harmful of several conflicting interests. We must neutralize as best we can the inconvenient characteristics of cement paste;[1] and for this purpose we must, in most instances, use whatever natural materials happen to be handy in large amounts needing little processing. Casual deposits of the finer sizes of such materials are seldom as clean or as well graded as we might wish that nature had made them. Whatever fine aggregate we may by chance have must be pitted against coarse, and both of these against a semifluid paste of water and cement, to arrive at a mixture of suitable characteristics for placement; and, while doing this, we must keep in mind the effect of each of these components upon the integrity of the hardened concrete.

The first valid system of proportioning mixtures, based upon many experiments and free from the necromancy of favored numbers, was that proposed by Abrams,[2] in which he established the fundamental principle that both the water and the cement are the major factors determining the strength of concrete and, of almost equal importance, in which provisions were made for necessary adjustments incidental to the size, texture, and gradation of the aggregates. Another logical proposal of great significance was the announcement in 1921, by Talbot and Richart, of their voids–cement-ratio theory of proportioning mixtures.[3] Though this later theory resembled Abrams's theory in some respects, it differed specifically in its priority of consideration and emphasis upon the spatial characteristics of mixture components. This method was never widely used because of the soon realized obsolescence of volumetric batching of aggregates. Nevertheless, it had profound influence upon the direction of later research toward consideration of the absolute, rather than the apparent, proportions of mixtures.

A discovery of great practical significance was made in 1932 by Inge Lyse, then professor of engineering at Lehigh University, as a result of his study of mixtures with respect to their absolute volume relationships. Lyse found that the volume of water in a unit volume of concrete was

[1] T. C. Powers and T. L. Brownyard, Studies of the Physical Properties of Hardened Portland Cement Paste, *J. ACI*, October, 1946, to April, 1947.

[2] Duff A. Abrams, Design of Concrete Mixtures, Structural Materials Research Laboratory, Lewis Institute, *Bull.* 1, 1918.

[3] Albert N. Talbot and Frank E. Richart, The Strength of Concrete in Relation to the Cement, Aggregates and Water, *Univ. Illinois Eng Expt. Sta. Bull.* 137, October, 1923.

substantially constant in mixtures of different water-cement ratio, when such mixtures were prepared with similar ingredients and were of similar workability. In other words, with little regard to cement content, the quantity of water in 1 cu yd of concrete of prescribed consistency is for most practical purposes a constant. This is not invariably true, inasmuch as very rich and very lean mixtures require somewhat more water than do those of ordinary cement content; but, when the fluidity of the pastes for the strengths involved are not too different, the rule is useful for making moderate shifts from one strength to another. Another discovery of practical utility made by Swaze and Gruenwald[1] was that, if cement is included in the computation of fineness modulus of the granular ingredients, a substantially constant value is indicated for all mixtures from lean to rich. Direct use of this principle is somewhat involved, but it can be approximated in practice with another generality to the effect that the absolute volume of the coarse aggregate, in mixtures of similar consistency made with similar ingredients but of different water-cement ratios, is substantially constant.

Three proportions of materials for concrete are given in Table 1-4, for

TABLE 1-4. Examples of Proportioning of Concrete

Slump, in.	Parts by weight				Cement bags per cu yd	Ratio $C:f+c$	Ratio $f:f+c$
	Cement (C)	Fine aggregate (f)	Coarse aggregate (c)	Water (w)			
2	1.00	1.71	3.21	0.41	6.83	1:4.92	0.348:1
4	1.00	1.62	2.97	0.41	7.16	1:4.59	0.353:1
6	1.00	1.54	2.70	0.41	7.58	1:4.24	0.363:1

the specific purpose of illustrating several elementary principles of proportioning. It should be noted that, even though these mixtures are of different slumps, they are nevertheless of similar water-cement ratio. They should, as is indicated by Abrams's theory, be of similar strength. It is evident that their cement contents are greater with greater slumps; this is so because the amount of aggregate serving to stiffen the paste is respectively less at greater slumps, as is indicated by the ratio $C:f + c$. So also do the so-called wetter or more workable mixtures have greater fine-aggregate components, as is indicated by the ratio $f:f + c$.

Foregoing principles and approximations of fact are inadequate,

[1] Myron A. Swaze and Ernst Gruenwald, Concrete Mix Design—A Modification of the Fineness Modulus Method, *J. ACI*, Vol. 18, No. 7, March, 1947.

within themselves alone, for arriving at dependable mixtures. Something should also be known of the ultimate influence upon quality of each component of a mixture. Following is a brief discussion of some of the more important considerations.

Cement paste is, with respect to durability, the weak link in the chain of ingredients. Aggregates have lasted and will probably continue to last for ages; but hydrous cement compounds are susceptible to leaching, to volume changes induced by temperature and absorbed water, and to chemical changes caused by external influences. Nevertheless, enough paste must be used in mixtures to coat all the aggregate, to fill interstitial voids, thus minimizing porosity, and to separate aggregate particles so that mixtures are of suitable workability for placement. Hardened cement paste, in comparison with aggregates, is relatively soft and poorly resistant to mechanical wear. The least viscous and more diluted pastes are most susceptible to weathering, mechanical wear, and other damage. Furthermore, excessive manipulation during placement and during finishing of mixtures rich in paste can be more harmful to durability than would poorly proportioned mixtures. Entrained air as well as some other admixtures, because they inhibit segregation and sedimentation of mixture components, are effective aids for ensuring durability.

The relative water content of cement paste is the most influential factor determining strength. However, the shape and surface texture and the inherent strength of the aggregates are secondary factors not to be ignored. Furthermore, the relative fullness of large interstices, with smaller particles or with paste, has an appreciable influence upon strength. Excessive amounts of fine aggregate are detrimental to strength as well as to durability; so also are excessive slumps, which may have been arrived at by adding water, rather than by withholding aggregates from the mixture. Entrained air is detrimental to strength, but if not excessive, its effect can be neutralized by reducing the fine-aggregate component and also the water.

The volume constancy of concrete is influenced almost entirely by the behavior of the cement paste. The paste, in comparison with aggregates, is much more reactive to thermal changes. Even the ailment of aggregate reactivity is partly attributive to the nature of the paste. Hardened cement paste is a gelatinlike colloidal material, and it is typical of these that they are avid breathers of moisture; they quickly occlude and readily release great amounts of water in an attempt to maintain equilibrium with external conditions. In doing this, they swell when they are wet and contract when dry. Pastes richest in cement are most active in these respects. These swellings and contractions of paste are resisted by what might be envisioned as the bulk inertia of individual aggregate particles. Easily worked mixtures of great slump having a

greater ratio of paste to aggregate, and especially those of great cement content, are most susceptible to expansion and contraction. Minimum film thickness of paste between aggregate particles, the paste being of lean cement content, is conducive to minimum volume change instigated by absorbed water. In contrast, the thermal expansion and contraction of concrete is influenced little by richness of paste or richness of mixture. However, thermal conductivity and rapidity of accommodation to sudden changes of temperature are improved by richer pastes and richer mixtures.

Concrete having one of its faces seemingly dry can actually be transferring great amounts of water through the paste within the mass. However, the water leaves the drier surface as vapor at a rate usually so rapid as to cause the surface to appear dry. This avid permeation of water throughout the paste cannot be avoided. It is not too alarming, inasmuch as this water is saturated with leachings during most of its travel and leaves its dissolved constituents within the concrete. Much more serious, however, is the capillary movement of water through discrete channels within the concrete, perhaps with only occasional delays required to permeate a few intervening bulkheads of hardened paste. This action can dissolve and remove from the concrete essential material necessary for continuing integrity. Such troubles are minimized by placing concrete of minimum slump made with paste rich in cement. Furthermore, the creation of percolation channels as a result of sedimentation of the finer components of mixtures is discouraged greatly by entrained air, and to some extent by other admixtures.

After good workmanship, cement is the most costly component of concrete. If a structure is to endure and perform its function for a life span commensurate with its cost, adequate amounts of this essential ingredient must be used. However, the leaner and less viscous mixtures of great slump are least costly to make and are easily placed. From a contractor's point of view, water is the cheapest component of concrete. It facilitates effortless placement; and furthermore, if 7½ gal more than is really needed can be added to a batch of concrete, it seems when it is in the structure to be another cubic foot of concrete. It is, in these circumstances, paid for by the owner at the prevailing unit price of concrete; and later, it may again be paid for by the owner in his correction of undesirable attributes of the structure. Again from the contractor's point of view, sand is the next most economical ingredient of concrete. It is of fine particle size and great surface area, and it holds a wet mixture together well; although it is not so effective as water, it facilitates the self-induced flow of mixtures and it makes less laborious the efforts and the number of workmen required. One hundred pounds of sand will increase the bulk of a mixture perhaps 20 per cent more than will 100

lb of cement, and pound for pound, it costs perhaps one-tenth as much. Unfortunately, when it is used in excessive amounts, it is not beneficial to the longevity of a structure.

It is obvious that water is the most critical and least beneficial ingredient of concrete, yet we cannot make concrete without it. Nevertheless, we can discourage the indiscriminate use of water, and we can search out and control sources of unavoidable supply. For example, the aggregates usually carry a considerable amount of water upon their surface. Between 20 and 30 per cent of the total water in a mixture is introduced in this manner; if ignored it can seriously modify the intended characteristics of the concrete. The amount should be determined[1] and a corresponding reduction should be made in the amount added to the batch at the mixer. Moreover, corrections should also be made in the measurement of what seems to be aggregate but is, instead, partly aggregate and partly water. This should be done often. The surface moisture of fine aggregate in particular can change appreciably during a day's work and, unless corrections are conscientiously made, the concrete produced during the day can be of unwanted variable quality.

Air is a desirable component of concrete. If it is to be effective in promoting durability, it must be present in certain minimum amounts and it must also be dispersed as globules of almost microscopic size. From 2 to 6 per cent air by volume is enough to provide adequate results; more adds little benefit and is inconveniently harmful to strength.[2] The amount of air entrained is obviously influenced most by the quantity of frothing agent used. However, for a specific amount of agent, lean mixtures entrain more air than do others of greater cement content, sandy mixtures more than those of less fine-to-coarse ratio, and finer sands more than those of coarser gradation. Higher temperatures of concrete and longer mixing time cause more entrainment of air. Concrete having an entrained-air component acquires a peculiar property. It possesses what is called a *thixotropic structure;* that is to say, it behaves like a fluid while it is agitated but when left undisturbed it acquires a false rigidity. Yet, even though it may have assumed an apparently rigid state, it resumes the fluid condition if it is remanipulated. This phenomenon of false set is the beneficial property that discourages segregation and sedimentation and thus improves durability. Such concrete can be struck

[1] Method of Test for Surface Moisture in Fine Aggregate, ASTM Designation: C 70. Surface moisture carried by coarse aggregate is sometimes estimated, but can be measured by drying a sample of known weight upon a hot plate, using care that the absorbed moisture is not also evaporated; in the hands of a careful operator, this method is also suitable for fine aggregate.

[2] Method of Test for Weight for Cubic Foot, Yield, and Air Content (Gravimetric) of Concrete, ASTM Designation: C 138. Also Symposium on Measurement of Entrained Air in Concrete, *ASTM*, *Proc.*, Vol. 47, 1947.

off, troweled, and otherwise finished sooner than can ordinary concrete. Wet shrinkage of the mass, segregation of mortar, and bleeding of water are minimized by its falsely rigid structure.

The probable effect of modifying any single component of a properly adjusted concrete mixture is indicated in Table 1-5. This should be

TABLE 1-5. Effect upon Quality Attributes of Concrete, Caused by an Increase of Any Single Component of the Mixture

Attribute	Cement	Fine aggregate	Coarse aggregate	Water	Air	Mixing	Age
Slump	−	−	+	+	+	+	
Cohesion	+	+	−	−	+	+	
Workability	+	+	−	+	+	+	
Segregation	−	−	+	+	−	−	
Sedimentation	−	+	−	+	−	−	
Wet consolidation	−	+	−	+	−	−	
Bleeding	−	−	+	+	−	−	
Entrained air	−	+	−	+		+	
Durability	+	−	+	−	+	+	−
Strength	+	−	+	−	−	+	+
Elastic modulus	+	−	+	−	−	+	+
Freeze resistance	+	−	+	−	+	+	−
Wear resistance	+	−	+	−	+	+	+
Chemical resistance	+	−	+	−	+	+	+
Permeability	−	+	−	+		−	−
Wet expansion	+	−	−	−	+	−	−
Dry shrinkage	+	−	−	−	+	−	−
Density	+	−	+	−	−	+	
Form finish	+	+	−	−+	+	+	

interpreted as indicating the probable trend that would be caused by a moderate excess of any single constituent, and we suggest that it should not be applied to changes of more than one component at a time. It is generally safe to assume that a deficiency would produce opposite effects. It is quite possible that in some cases gross rather than moderate changes might produce somewhat different effects, and this should be kept in mind. Nevertheless, if the table is used with discrimination it can be helpful as a guide to the formulation of satisfactory mixtures.

1-12. Computations. The computations incidental to concrete proportioning require the application of little more than accurate arithmetic. With the exceptions of those concerned with the statistical analysis of data, and the computation of constants of curves such as the water-cement vs. strength relationship, all can be performed with suitable accuracy using a slide rule. Volumetric batching devices are now seldom encountered, and they are notoriously inaccurate; weighing batchers are

seldom more accurate than about $\frac{3}{4}$ per cent of any value that they may indicate. Often, the novice computes batch quantities to a greater apparent accuracy than is justified by the trustworthiness of field measuring equipment. Aside from the problems of research, the use of more than three significant figures can seldom be justified.

Certain values frequently used in concrete computations are customarily assumed to be constants. For example, the weights of a bag and a barrel of cement are always assumed to be 94 and 376 lb, respectively, since these are requirements of cement specifications. The specific gravity of cement is so nearly similar among different lots that it can be assumed to be 3.15 in value. Water is generally assumed to weigh 62.3 pcf, and 8.33 lb per gal. One cubic foot of water is equivalent to 7.48 gal. These assumptions are sufficiently correct to be within the limit of accuracy of field measuring devices. Other values essential to the computation of proportions and other characteristics of mixtures are related to the particular aggregates and the specific mixture involved, and these must be determined by test. Among these are the specific gravities of the respective aggregates, the amount of water required to produce a prescribed slump, the entrained air content of the mixtures, and the so-called cement factor or the quantity of cement in 1 cu yd of concrete.

The volumetric yield of a mixture is of great interest to both the contractor and the engineer. It is easily determined by weighing a known volume of concrete—usually $\frac{1}{2}$ cu ft—in an accurately calibrated measure. The true volumetric yield of a batch is equal to the sum of the weights of all its solid and liquid components, divided by the actual weight of 1 cu ft of the mixture. Knowing the amount of cement incorporated in the batch, and the yield of the batch, we can then compute the cement factor; this is usually done in terms of bags, barrels, or pounds of cement per cubic yard of concrete. It should be noted that this method of determining yield and cement factor automatically takes into consideration the bulking effect of any entrained air, and the shrinkage incidental to the solution of any of the solids in water. The volume of entrained air is, for all practical purposes, the difference in the volume of the batch as compared with the sum of the volumes of its solid and liquid components.

Computation of the absolute volume, or what might also be called the equivalent solid volume, of the granular ingredients is a basic calculation that may at first be difficult to grasp. It should be recollected that the specific gravity of a material is, by definition, the relative weight of the material as compared with the weight of an equal volume of water. If, for example, an aggregate could be melted and poured without voids into a cubic foot measure, it would weigh perhaps 2.65 times as much as 1 cu ft of water. Therefore, the weight of a solid or absolute cubic foot of

any material is its specific gravity multiplied by 62.3 lb, the latter being the weight of 1 cu ft of water. The true displacement or absolute volume of a granular material is its actual weight divided by its so-called weight per solid cubic foot.

Concrete proportions are indicated in many ways. For example, volumetric proportions much used in the past were often referred to as a one-bag batch such as 1:2:4, in which case the one part, or one bag, of cement was assumed to have an apparent volume of 1 cu ft. Weight proportions are often given in similar form, and in such instances, the ratios refer to pounds of aggregate per pound of cement. Weight proportions are sometimes given as 94:216:412, and it should be obvious that these are weights of aggregate per bag of cement. Water, when it is mentioned, is most often given as gallons per bag of cement. However, a most convenient form of designation that is rapidly coming into widespread use is 565:1,160:2,150:256, where the materials are arranged in the conventional order and represent the weights approximating 1 cu yd of concrete, the last figure given being pounds of water per cubic yard.

1-13. Mixing concrete. Except under very unusual circumstances, concrete is mixed in a power-driven drum equipped with blades so arranged as to agitate, stir, and interfold the plastic mass. The size and shape of the drum, its speed of rotation, the arrangement and condition of the blades, and the duration of mixing all influence the quality of concrete. Of these factors it is customary for the engineer to control the rate and duration of mixing. From experience it has been determined that a concrete mixer should revolve at an approximate peripheral speed of 200 fpm. Speeds greater than about 225 and less than 100 fpm are usually found to be unsatisfactory. As a general rule, the peripheral speed for greatest mixing efficiency is inversely proportional to the diameter of the drum.

The quality of concrete is improved to a certain extent by longer and more thorough mixing, but, beyond an indefinite optimum time, further mixing may produce a reduction in quality. Experience indicates that in no instance should the duration of mechanical mixing be less than 1 min after all the materials including the water are in the drum. Current practice on projects requiring concrete of uniformly good quality is to require a minimum mixing time of 1½ min for a mixer of 1 cu yd capacity and a greater time for larger mixers. A safe rule which may generally be used to indicate the desirable duration of mixing is to require 1½ min for the first cubic yard of mixer capacity and an additional ½ min for each additional cubic yard thereafter.

It cannot be too strongly emphasized that the duration of mixing should be counted from the time when all the materials, including the water, have been introduced into the drum. A batch of materials mixed

with less than the designed amount of mixing water is prone to roll and ball up in the mixer and, except with continued mixing of abnormally long duration, will be nonuniform in composition when discharged from the drum. This condition is particularly evident in large portable mixers of the truck-mounted type.

Job mixers differ mainly in details of design and arrangement of their operating controls rather than in their type or principle of mixing. The main abuse to which they are subjected is that of overcharging. They cannot efficiently or satisfactorily function when they are charged with a volume of material much greater than one-half the gross volume of the mixer drum. They are designed by their manufacturers efficiently and properly to mix a specific volume of concrete, and their capacity is indicated by their catalogue numbers. Thus a No. 54S mixer is designed to mix 54 cu ft of concrete, whereas a No. 27E mixer is intended to mix only 27 cu ft.

The use of mixers mounted on truck bodies, for the delivery of batched materials or premixed concrete from a remote central batcher plant to the site of the work, is now commonly encountered. Sometimes these are merely agitators of concrete already mixed before loading aboard the truck; sometimes they are used to mix the measured materials while in transit; and at other times they are restricted to the conveyance of measured materials and their mixing under supervision at the work site. They range in capacity from 2 to 8 cu yd. Most truck mixers are properly designed to produce uniform mixtures. Best designs are such that the ratio of drum length to diameter is little more than 1:1, the drum being equipped with suitable mixing blades arranged to provide good interfolding and some end-to-end transfer of concrete within the drum. Tilted drums, permitting high discharge of the batch for longer reach when chuting into a form, is also a desirable feature. Most truck mixers are equipped with mechanical water pumps, because rapid transfer of water into the drum materially reduces the time required for satisfactory mixing and improves the uniformity of the concrete throughout the batch.

Portland cement is generally well proportioned and well calcined, but even now some underburned cement having an undesirable component of gamma C_2S may occasionally be produced. Material of this nature is likely to form greater than normal amounts of laitance upon continued mixing. Prolonged or erratic duration of mixing also makes difficult the control of entrained air. For these reasons, the mixing of concrete for variable periods, or for intervals greater than about 30 min, should be discouraged.

1-14. Handling and placing of concrete. A most thorough and careful design can be completely defeated by improper practices in the

handling of ingredients and the placing of concrete. It is obvious that cement deserves proper storage and protection from moisture, but it is less often observed that aggregates also require attention and care. Aggregates should be protected from contamination with earth, coal dust, plaster, scrap lumber, and similar materials encountered at work sites. They should not be intermingled in storage, because this makes impossible their proper batch measurement. Coarse aggregate is susceptible to size segregation if it is dropped from a crane bucket or other conveyor; it particularly should be stored in bins, or in stock piles having slopes less than the normal angle of repose of the material. Unrestrained dropping of coarse aggregate causes large particles to concentrate at the toe of a slope, and this should be discouraged.

Similar considerations should be observed in the handling and placing of plastic concrete, and this is especially so of less cohesive and overly wet mixtures. Unrestrained dropping, steep chuting, and horizontal flow of concrete are extremely harmful and should not be tolerated. When concrete must be deposited in a deep form or slid down a steep chute, its progress should be restrained by baffles so that it continues to be a cohesive mass of uniform composition, rather than a goblike shower of segregated components. Whenever possible, concrete should be placed in a form at its final resting place in a structure. Chuting of concrete into a form at an angle, and flow of concrete within a form, should be permitted only when no other method of placement is possible. Lateral flow of concrete causes the coarse aggregate and the mortar to come to rest at different places in a form, and this may result in porous, honeycombed, frost-sensitive, or other weak regions of unsuitable concrete.

In almost all situations, concrete should be deposited vertically and in horizontal layers of reasonable depth. Great lifts of a single pour encourage segregation of coarser and sedimentation of the finer constituents of mixtures and, moreover, may cause unwanted displacement of forms. The creation, during deposition of concrete, of sloping planes or surfaces should be avoided; first, because such slopes aggravate differential flow and segregation of mixture components and, second, because such pitched surfaces constitute weak planes of potential shearing of the structure.

The deposition of concrete in deep water is especially difficult with respect to the foregoing considerations. This problem is further complicated, inasmuch as turbulence of water at the plastic concrete interface is likely to wash away cement, may greatly aggravate segregation, and causes weak and sloping bearing planes for subsequent lifts. Placement of concrete in water is accomplished by means of vertical pipes called *tremies*, which must be maintained filled with concrete and must have their lower ends submerged in plastic concrete for the duration of a

pour. Flow of concrete down the tremie is assisted by increasing the hydraulic head of concrete in the tube, by pounding or vibrating the tremie, and by raising the tube in its entirety so that its lower end is

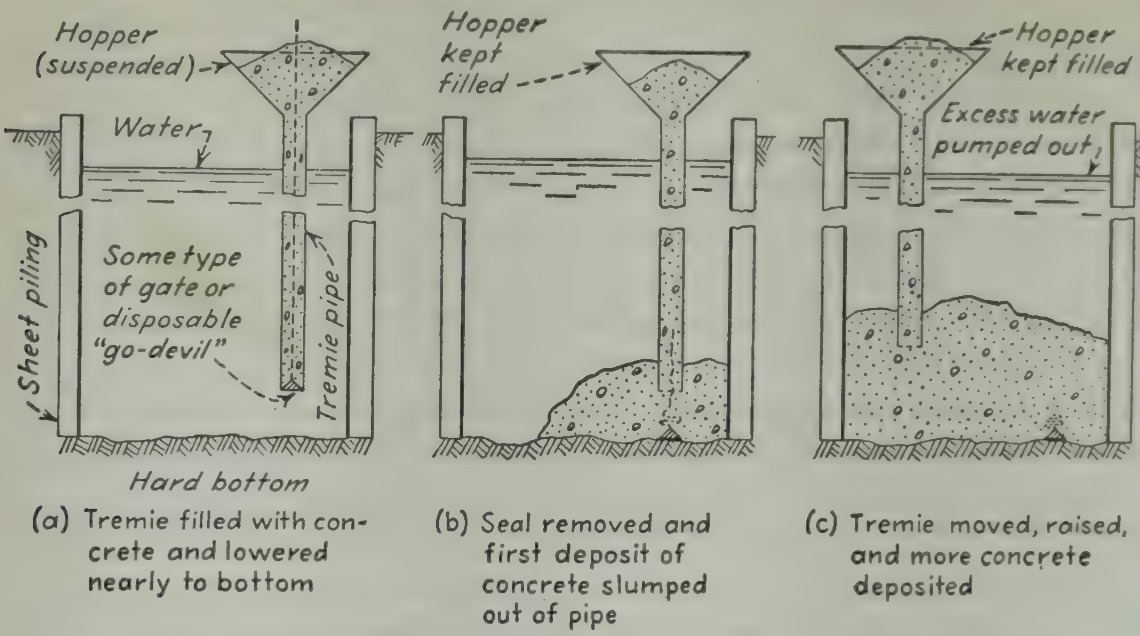


FIG. 1-6. Simplified illustration of depositing concrete by tremie.

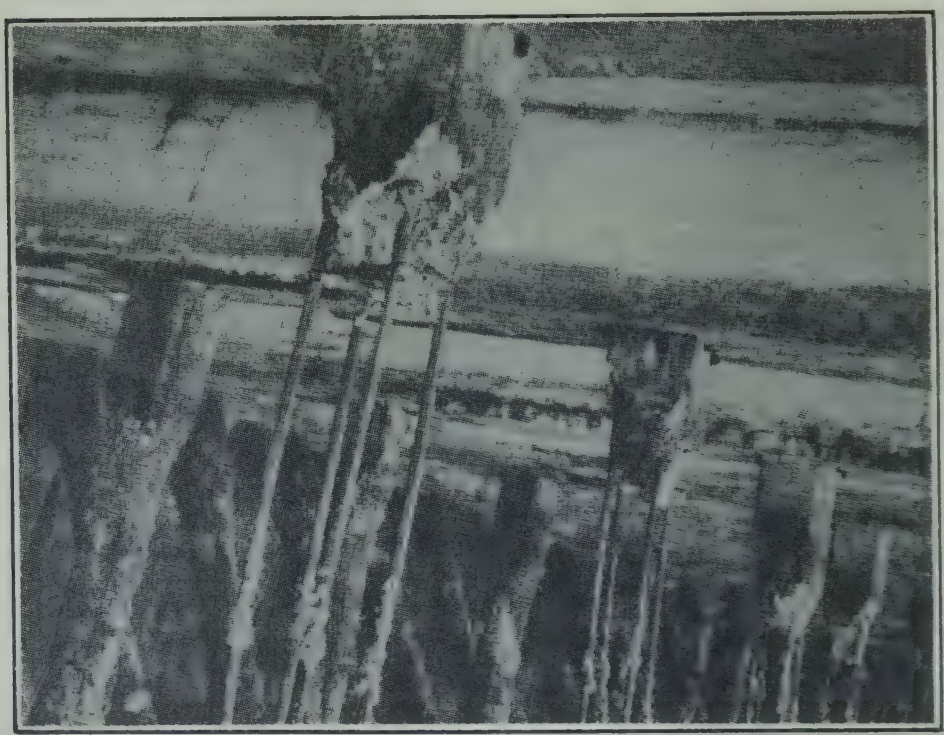


FIG. 1-7. A situation illustrating the need for good materials and the best workmanship for concrete construction that is exposed to sea water, especially in the tidal range.

nearly withdrawn from the submerged plastic mass. If the tremie is withdrawn so far as to lose its charge of concrete and become filled with water, then great difficulty is sometimes experienced in reestablishing placement of concrete without harming the partially completed pour. An illustration of this method of underwater placement is shown in Fig.

1-6. Rich mixtures are always necessary for underwater placement to compensate for possible losses of cement.

Many ingenious devices are used for the conveyance of concrete to its place in the structure. Wheelbarrows and two-wheeled buggies have been long used and are reliable conveyances for small projects. When they are equipped with pneumatic tires they are quite easily manhandled while heavily loaded. Cubical hinged-bottom buckets were once used extensively, but these have been superseded almost completely by cylindrical buckets provided with a rolling gatelike bottom discharge. At one time tall towers, provided with a fast elevator much like a mine skip, and equipped with long chutes supported by booms and guy wires, were often used for placing concrete. However, excessively long and too steep chutes caused great segregation of concrete, and such equipment is now in disfavor. Concrete has been successfully placed at great distances from the mixer by means of high-speed conveyor belts. Great quantities are placed by pumping from the mixer to the forms through several hundred feet of 6- or 8-in. pipe, with vertical lifts of as much as 200 ft. This is done with a ramlike pumping machine having the trade name *Pumpcrete*. Concrete is often placed pneumatically to create vertical walls of thin section and to provide encasement of irregular objects such as steel beams and columns, by means of the proprietary *Gunite* devices. These comprise mixing chambers, hose lines, and spray nozzles, operating on much the same principle as paint-spraying equipment. In tunnel construction, concrete is often conveyed to the form in side-dumping narrow-gage mine cars. Although it was once commonly done, it is now considered poor practice to modify the proportions of good concrete to suit the peculiarities and limitations of conveying and placing equipment. Now it is more often required that contractors use equipment suitable for the kind of concrete desired in the structure.

Concrete, except when it is placed underwater, should be consolidated by tamping, rodding, and spading. This is essential to the elimination of large casual voids, the complete encasement of reinforcement, and the proper contact of concrete with form faces and embedded fixtures. Mechanical vibration of the concrete is often used for this purpose, but this should be considered only as an auxiliary means of consolidation rather than as a substitute technique. Spading, in particular, is necessary for the production of good form finishes and smooth exposed surfaces of optimum durability. Spading should be done in such a manner as to work coarse aggregate *away* from the form and not toward it. This ensures that mortar is against the form where it is needed to minimize the possibility of honeycomb or air bells, at what might be thought of as the skin of the structure. Smoothly uniform surface textures enhance durability. Coarse aggregate worked away from a form during proper

spading tends to return to the face because of semifluid pressure of other coarse particles within the plastic mass.

The *proper* use of mechanical vibration is beneficial to the compressive strength and to the bond strength between concrete and steel. It facilitates the placement of more economical mixtures having lesser slump because of greater aggregate components. The *improper* use of vibrators, or their use on the wrong type of concrete, can cause greater harm than would placement of concrete without any effort toward consolidation. The consistency of concrete to be vibrated, as measured by its slump, should not exceed 3 in., and less workable mixtures are even more desirable. In addition, mixtures placed with vibrators should definitely be undersanded to the extent of being harsh. Workmen soon observe that concrete can be guided as a stream by leading the mass with a vibrator, and this requires less effort than does the use of a shovel. They tend to use them excessively and often move the concrete horizontally with the machine, thus causing unwanted segregation. Vibrators applied directly to the reinforcing rods can displace them from their proper locations and also tend to cause subsidence of the concrete from the undersurfaces of the steel.

Air entrainment is indescribably effective in neutralizing almost all the harmful aspects of poor workmanship, unsuitable equipment, bad construction practices, and willful abuse of concrete. Its use for ensuring durability despite all the pitfalls of construction is almost indispensable. Nevertheless, all concrete should be tamped, spaded, vibrated, and otherwise manipulated as little as is consonant with suitable consolidation and satisfactory finish. Pavement slabs in particular, as well as other pours of large area, and especially those which are made with overly workable mixtures, should be protected from excessive manipulation of concrete in the forms. The surface of such pours, when the concrete is overworked, accumulates a layer of mortar comprising fine sand, water, and less reactive components of the cement. This accumulation is especially weak and susceptible to spalling when frozen. Such accumulations should not be permitted to occur. Where they may have inadvertently done so, they should be removed by chipping, wire brushing, sandblasting, or other abrasive measures and the surface should be broomed with a slurry of rich mortar.

Brief mention should be made of a proprietary aid to construction called the *vacuum process*.[1] In this procedure domelike mats are applied to the fresh concrete immediately after strike-off, and special mats are sometimes even used as forms for a pour. These mats are evacuated with an air pump, and this helps to consolidate the concrete and removes

[1] K. P. Billner, Applications of Vacuum Concrete, *J. ACI*, Vol. 23, No. 7, March, 1952.

some of the batched water from the plastic mass. Leaner mixtures and concrete of somewhat undesirable water content are sometimes placed in this manner, and their residual water content is reduced after placement in the form. Specialized methods of finishing are required when pavement slabs are so constructed, and all the usual curing practices must also be applied. This construction procedure should not be confused with vacuum devices that are sometimes used for lifting and transporting hardened precast units.

1-15. Curing. Some aspects of the water-cement reaction are not yet clearly understood. For that reason, any outline of the process must be somewhat speculative. Nevertheless, the following brief discussion is offered as being helpful to an understanding of the need for curing, even though it is admitted that much of it may be conjectural. The reaction of cement and water, if it is to satisfy our objective of producing strong and durable concrete in a reasonable time, requires for its suitable progress not only time and favorable temperatures; it also requires the close and prolonged association of suitable amounts of the two reagents. The process is a progressive reaction of a liquid phase (water) with a relatively insoluble solid phase (cement). It appears, within our present knowledge, that the reaction continues certainly for many years and may require many decades if it ever is completed. The presence of a great amount of absorbed water within the concrete is essential to its worthwhile progress.

Water can react only at the surface of a cement particle and, at the instant this occurs, the particle is covered with a layer of hydrous calcium silicate gel. The reaction progresses inwardly—in a radial manner—from the new boundary surface of the particle, and this can occur only at such a time as more water may reach the new boundary. Considering only the calcium silicates of cement, their gelatinous reaction products are peculiarly bulky substances of great porosity and surface area. After some of the available water has been chemically fixed as a nonfluid part of the silicate molecule, the gel then seizes and forcefully retains a concentrated, densified, and viscous film of closely packed water molecules upon its great surface. This film of *adsorbed* water is more intimately associated with the solid gel than it is with the remaining free-water phase, and it increases the apparent bulk of the gel. The distinctiveness of adsorption, as compared with absorption, should be apparent. The first is a surface-energy phenomenon that is common to almost all matter but is especially evident of materials of colloidal size, whereas the second is a capillary effect.

It appears that part of the batch water is solidified as a chemical appendage of the silicate molecule; some more of the batch water is arrested, viscously concentrated, and forcefully attached by forces inci-

dental to interfacial energy to the extensive surface of the hydrophilic gel; and the remainder of the batch water permeates the porous structure of the gel. The latter is retained mainly by forces of capillarity. It seems reasonable to assume that only the capillary water is favorably disposed toward ultimate reaction with more cement to form more gel.

The porosity of the hydrous silicate gel is such that the vapor pressure of water exceeds the force of capillarity, the latter being a function of pore size and surface tension. Much of the absorbed capillary water escapes from the concrete by evaporation. Loss from within the concrete of the uncombined and unassociated capillary water can be avoided only if the concrete is maintained in an atmosphere saturated with water vapor, or in contact with liquid water. Since unassociated capillary water is all that can probably react with the remaining nuclei of uncombined cement, it should be prevented from escaping from the pores of the gel. If it should escape, it must be replenished if cement is to continue to react. It is quite likely that some of the interfacially captured water may revert, under the influences of environmental changes such as temperature, to capillary water. It is also a possibility that adsorbed water may slowly react with cement. Nevertheless, the actual behavior of concrete provides a strong implication that the capillary pore water is the more effective reagent.

Curing of concrete is the maintanence of pore saturation or, in effect, establishment of the handy presence of water capable of ultimately forming more gel. It is clear that the smallest cement particles are the first that should be completely converted to gel. This is confirmed by the fact that the same cement clinker, when it is more finely ground, produces better early strength. Present indications are that medium and coarse cement particles are not completely converted to gel even after many years. This probably accounts for the so-called *autogenous* healing of small cracks in excessively strained concrete, the effect even being evident of new cracks in old concrete. Incompletely converted cement particles are fractured by the strain, thus giving them new facial access to water. However, water must be within the concrete and must be free to make the healing reaction. Conservation of pore water is especially necessary during early life, if enough gel is to be formed to produce adequate strength and impermeability.

Curing is also a matter of thermal conservation, inasmuch as rates of chemical reaction are greater at higher temperatures. In fact, the reaction of cement and water at 35°F is almost insignificant in amount, and concrete maintained at that or lesser temperature is dangerously weak and porous. However, if free water is available and temperature of the concrete is subsequently increased, the reaction proceeds at a satisfactory

rate. Since newly placed concrete, if it is cold, cannot create enough gel to give it worth-while strength, there is great danger of damage being done to the concrete by the disruptive forces incidental to the freezing of water within the mass. The hydroxide-laden capillary water freezes at about 28°F.

Exothermic reaction of cement and water provides heat, and the presence of an actual excess required to produce plasticity of the mixture provides water, and these along with the cement are the essentials for the creation of hydrous gel. These favorable conditions can be conserved by holding forms in place and by sealing off exposed surfaces likely to evaporate water. Early covering of the freshly hardened concrete with damp earth, wet burlap, or impervious paper sheets is a practical means of minimizing evaporation. Spray-applied coatings of bitumen and other water repellents are also used for this purpose and, although they are not so effective as the foregoing, still they are much better than complete inattention to reduction of evaporation.

During winter construction of flat slabs and other thin sections, of relatively large surface area, the radiation of heat from the concrete is greater than that supplied by chemical reaction. In such circumstances, the concrete ingredients must be warmed to provide additional reserves of heat. This preheating must not be carried to extremes. Water in particular should not be overly hot because this causes too rapid reaction of the cement and expansion of the mass which, upon cooling, is almost certain to fracture extensively because of its low tensile strength. Good practice aims for temperature of the fresh concrete between 50 and 90°F, and this without using water in excess of 120°F. It is also necessary during winter construction that additional heat be supplied for several days after installation of the concrete. This is accomplished by enclosing structures with tarpaulins or by other means, and heating the enclosure with fire pots called *salamanders*. Electrical resistance heating has also been of limited application.[1] Pavement slabs are maintained heated by covering them with salt hay in layers sufficiently thick to provide insulation.

Extensive discussions of the influence of good and poor curing on strength and other physical properties of concrete are available.[2] Strength is very seriously impaired by poor curing practices, and it is suggested that reference be made to the technical literature. Although prolonged and efficient curing is most desirable, it is obvious that some compromise with ideals must be accepted in practice. To facilitate

[1] Prof. Chuzo Itakura, Electric Heating of Concrete in Winter Construction, a paper presented at the annual meeting of the ACI, Feb. 27, 1952.

[2] Curing of Concrete, a round-table discussion by various persons, *J. ACI*, Vol. 23, No. 9, May, 1952.

speedy construction and economy in the building of forms, it is desirable that the engineer establish a reasonable schedule of curing that is adequate for the integrity of his structure. This should also be fair to the builders of structures, most of whom reason that, when the concrete has set, it is at least hard and are hopeful that it is also sufficiently strong for them to forget it and proceed with other obligations. For ordinary concrete, not of a high-early-strength nature, the general procedures shown in Table 1-6 are suggested. It should be realized that wetting of

TABLE 1-6. Curing and Removal of Forms

Item	Forms held in place after concrete has set, days		Concrete kept wet after setting, days	
	Cold weather	Warm weather	Cold weather	Warm weather
Self-supporting floors and beams	14	10	7	10
Thin walls, 8 in. and less	4	3	7	7
Thick walls, and massive piers	3	2	7	7
Floors, and pavements on soil	..	..	4	7

concrete for more prolonged periods is necessary during hot summer months, and at all times in arid climates. Because continuous wetting during dry spells is troublesome and expensive it is often done fitfully and is sometimes neglected. Such bad practices are particularly detrimental to pavements, since shrinkage cracking and unreasonable wear are thus encouraged. Apparent hardness of dry surfaces should not be mistaken for good concrete inasmuch as gel formation is retarded in such circumstances.

1-16. Forms. Forms are intended to define the contour and locate the position of individual members with reference to the structure as a whole. To limit satisfactorily the size, shape, and position of parts of the structure, it is necessary that forms be built to resist the forces imposed upon them. It should never be forgotten that concrete is a semifluid during its early life. It is usually assumed that it exerts a horizontal pressure equal to the hydrostatic head of a liquid weighing 145 pcf.

Concrete, when it is vibrated, acts as a fluid throughout its depth; consequently, the full hydrostatic head should be considered. In contrast, concrete placed without the aid of vibration exerts a fluid pressure for a depth depending upon the rate and temperature of placement. The

consolidation and interlocking of aggregate and the initial setting of cement tend to neutralize fluidity; therefore, the pressures are actually somewhat less than are estimated for the full hydrostatic head. From literature of the Universal Form Clamp Company[1] it is indicated that concrete having a temperature of 50°F and placed at a rate of 6 vertical ft per hr exerts a maximum horizontal thrust of 1,030 psf, at a 9-ft depth of head. At greater depths the horizontal pressure is reported to be less until, at a head of 12 ft, it is reported to be 870 psf. The foregoing maximum pressure is approximately 80 per cent of the theoretical pressure that might be expected at 9-ft depth of action. Conditions such as higher ambient temperature, and others that are favorable to earlier setting of concrete, result in lesser maximum form pressures and, furthermore, the effects are evident at lesser depths. For example, the literature referred to indicates that concrete placed at 70 rather than at 50°F exerts only 740 rather than 1,030 psf maximum pressure, and the maximum effect is evident at only 7- rather than at 9-ft depth of head.

The foregoing comments are indications of the problems involved in the design and fabrication of adequate forms. Discussion of other aspects of the problem of forms will be found in Chap. 11.

1-17. Reinforcing steel. Concrete cannot be relied upon to withstand much tensile stress. This deficiency is overcome by embedding steel rods in those parts of a section that are subjected to tension. The two materials act in conjunction with each other, each doing the work for which it is best suited. The combination acting together as a unit, concrete resisting compression and steel resisting tension, is called *reinforced concrete*. It can be made a strong, durable, and economical system of construction, and it has been proved satisfactory for a great variety of structures.

Much has been said about concrete, and little about steel. The latter is of equal interest and importance. However, because of greater technical knowledge of the material, and better control during manufacture, reinforcement is a standardized material of great dependability. It is manufactured in the form of plain, deformed, or twisted bars and rods of various cross-sectional areas, ranging from the area of a $\frac{1}{4}$-in. to nearly the area of $1\frac{1}{2}$-in. round stock. It is also regularly available as wire, and as wire mesh in various sizes and combinations of weave.

At present we are experiencing a transitional improvement of the surface texture and some modification of available sizes of reinforcement. This may cause temporary confusion between the obsolescent and the newer styles of deformed bars. The strength and other physical properties, excepting size and shape of surface deformations, are common to

[1] "Forms for Architectural Concrete," Portland Cement Association.

TABLE 1-7. ASTM Physical Requirements for Concrete Reinforcing Steel

Properties	Type	Plain bars			Deformed bars		
		Structural	Intermediate	Hard	Structural	Intermediate	Hard
Tensile strength, psi	Billet steel	55,000–75,000	70,000–90,000	80,000 min	55,000–75,000	70,000–90,000	80,000 min
	Axle steel	55,000–75,000	70,000–90,000	80,000 min	55,000–75,000	70,000–90,000	80,000 min
	Rail steel	....	....	80,000 min	....	....	80,000 min
	C-D wire	....	....	80,000 min			
Yield point, min, psi	Billet steel	33,000	40,000	50,000	33,000	40,000	50,000
	Axle steel	33,000	40,000	50,000	33,000	40,000	50,000
	Rail steel	....	....	50,000	....	....	50,000
	C-D wire	....	....	0.8 tensile strength			
Elongation in 8 in., min, per cent*	Billet steel	$\frac{1{,}400{,}000}{\text{tensile strength}}$	$\frac{1{,}300{,}000}{\text{tensile strength}}$	$\frac{1{,}100{,}000}{\text{tensile strength}}$	$\frac{1{,}200{,}000}{\text{tensile strength}}$	$\frac{1{,}100{,}000}{\text{tensile strength}}$	$\frac{1{,}000{,}000}{\text{tensile strength}}$
	Axle steel	$\frac{1{,}400{,}000}{\text{tensile strength}}$	$\frac{1{,}300{,}000}{\text{tensile strength}}$	$\frac{1{,}100{,}000}{\text{tensile strength}}$	$\frac{1{,}200{,}000}{\text{tensile strength}}$	$\frac{1{,}100{,}000}{\text{tensile strength}}$	$\frac{1{,}000{,}000}{\text{tensile strength}}$
	Rail steel	....	....	$\frac{1{,}100{,}000}{\text{tensile strength}}$	....	....	$\frac{1{,}000{,}000}{\text{tensile strength}}$
	C-D wire	Reduction of area 30 per cent min (greater than 100,000 tensile strength, 25 per cent min)					
Bend test:†							
Under $\frac{3}{4}$ in., and under No. 6	Billet steel	$180°(d = t)$	$180°(d = 2t)$	$180°(d = 4t)$	$180°(d = 2t)$	$180°(d = 6t)$	$90°(d = 6t)$
	Axle steel	$180°(d = t)$	$180°(d = 2t)$	$180°(d = 4t)$	$180°(d = 2t)$	$180°(d = 6t)$	$90°(d = 6t)$
	Rail steel	....	....	$180°(d = 4t)$	....	....	$90°(d = 6t)$
0.3 in. or under	C-D wire	....	....	$180°(d = t)$			
$\frac{3}{4}$ in. and over, No. 6 and over	Billet steel	$180°(d = t)$	$90°(d = 2t)$	$90°(d = 4t)$	$180°(d = 4t)$	$90°(d = 6t)$	$90°(d = 6t)$
	Axle steel	$180°(d = t)$	$90°(d = 2t)$	$90°(d = 4t)$	$180°(d = 4t)$	$90°(d = 6t)$	$90°(d = 6t)$
	Rail steel	....	....	$90°(d = 4t)$	....	....	$90°(d = 6t)$
Over 0.3 in.	C-D wire	....	....	$180°(d = 2t)$			

* Specific modifications depending on size.

† d = diameter of pin around which specimen is bent; t = thickness or diameter of specimen.

both the older and the newer styles of bars.[1] However, a new system of numbering rather than the formerly used designation of bars by their nominal size, along with specific requirements for surface texture intended to improve mechanical bond of the bar to the concrete, are rapidly coming into use.[2] The numbering system is such that, beginning with 3 and including 11, each number represents the eighths of an inch diameter of an equivalent round bar having the same weight per foot as the deformed bar.[3] Data pertaining to both the older and newer bar sizes are given in Tables 1, 2, and 3 of the Appendix. Allowable bond stresses for both styles are given in the following section in Table 1-8.

Reinforcing bars are hot-rolled from car axles, tee rails of standard section, and new steel billets. Because of greater dependability from the standpoint of bending it is desirable, in the case of important structures, to require the use of bars manufactured from new billets. Billet-steel and axle-steel bars are produced in what are termed structural, intermediate, and hard grades. Rail-steel bars are similar in properties to the billet and axle types of hard grade. The essential differences in properties of the structural-, intermediate-, and hard-grade materials are, respectively, increasingly greater ultimate strengths and higher yield-point values accompanied by progressively less ductility. Reinforcement of higher yield value is desirable because it permits the use of greater working stresses, but, since concrete is generally weak in tension, high stresses in the steel may cause prolific cracking of the concrete. The best balance between these properties is a matter of personal opinion, professional judgment, and the satisfaction of building-code requirements.

Wire and wire mesh are ordinarily available in sizes ranging from the smaller No. 14 to the larger No. 7-0 gages and are manufactured from cold-drawn steel wire.[4] This is fabricated as mesh by welding in a large variety and combinations of weave. The ultimate strength of cold-drawn reinforcing wire is approximately equal to that of hard grades of hot-rolled bars, but the yield point of this material is appreciably greater than that of rolled bar steel. Table 1-7 indicates the comparative physical properties of these various grades of reinforcement as specified by the ASTM.

The modulus of elasticity in tension of plain carbon steel varies between

[1] Billet-steel Bars for Concrete Reinforcement, ASTM Designation: A 15; Rail-steel Bars for Concrete Reinforcement, ASTM Designation: A 16; and Axle-steel Bars for Concrete Reinforcement, ASTM Designation: A 160.

[2] Minimum Requirements for the Deformations of Deformed Steel Bars for Concrete Reinforcement, ASTM Designation: A 305.

[3] Nos. 9 to 11 depart somewhat from this rule in that they are equivalent to former 1-in., $1\frac{1}{8}$-in., and $1\frac{1}{4}$-in. square sizes.

[4] Cold-drawn Steel Wire for Concrete Reinforcement, ASTM Designation: A 82; Fabricated Steel Bar or Rod Mats for Concrete Reinforcement, ASTM Designation: A 184; Welded Steel Wire Fabric for Concrete Reinforcement, ASTM Designation: A 185.

28,000,000 and 31,000,000 psi. It is generally assumed to be a constant having a value of 30,000,000 psi. At normal atmospheric temperatures, steel is considered to be a truly elastic material throughout a loading range below a certain value termed its *proportional*, or *elastic*, limit. The proportional limit is not coincident with the yield point as determined by ordinary acceptance tests but is sufficiently close for all practical purposes.

Steel should be well embedded within the concrete, to protect it from corrosion and from damage by fire. If the concrete is plastic during placement and is made neither so dry as to be porous or honeycombed nor so wet as to be highly permeable to water, it provides good protection against corrosion of the reinforcement. Concrete, because its absorbed water is saturated with calcium hydroxide, is highly alkaline as is indicated by a hydrogen-ion concentration of somewhat more than 9.0 pH. Such an environment makes almost impossible the oxidation of steel and, in most instances, the demolition of old structures has shown that the steel was well protected. Ordinarily, a cover of 1½ or 2 in. is desirable for steel behind large flat surfaces exposed to air; 2 to 2½ in. at corners in air; and 3 in. when concrete is exposed to fresh water or moist earth. Local regulations with regard to fireproofing may sometimes dictate greater coverings, but too much concrete beyond the outermost bars of steel may aggravate spalling of the concrete. Rusting of the reinforcement is invited when cover of steel is inadequately thin.

Much controversy exists regarding the proper cover that should be used over reinforcement in structures immersed in or wetted by sea water. At least 3 in. cover is desirable and 4 in. is probably better. Even though types II and V cements are resistant to the sulphate constituents of sea water, and concrete made with these cements in company with entrained air is unquestionably resistant to such an environment, this may not afford enough protection to steel. Some engineers are of the opinion that very rich mixtures having a cement factor of 7 to 8 bags per cu yd are necessary for suitable protection of steel. They believe that the extra cost of concrete of perhaps $2.50 per cubic yard is justified when interference with service or replacement would be costly. Such rich concrete is, however, most susceptible to thermal cracking and this permits direct access of sea water to steel. Bituminous coatings, of a coal-tar rather than an asphaltic base, applied to the concrete within the tidal and spray range, are helpful in providing protection in such situations, but the coatings must be renewed every few years. An unfortunate example of the results of improper workmanship or materials is shown in Fig. 1-7.

1-18. Allowable unit stresses and safety factor. The allowable unit stress and safety factor are interdependent and are based upon our experience with reinforced-concrete structures. The safety factor is the

proportional amount by which the ultimate strength of the concrete, or the elastic limit of the steel, exceeds the permissible unit stress used in the design. Values shown in Table 1-8 are the permissible unit stresses to be used in the design of structures as given in the American Concrete Institute Building Code Requirements for Reinforced Concrete (ACI 318-51). There are other local rules and regulations, or codes, guiding both the design and the construction of reinforced-concrete structures but the authoritative nature and reliability of the ACI Code are of nationally recognized significance.

Some students may reach the opinion that the ACI Code, so often referred to here, is extremely arbitrary. It is, instead, the condensed wisdom of experienced engineers who know the many facets of the problem of designing structures. By open discussion of their opinions and experiences, and by mutual compromises, they have agreed upon this guide that serves the useful purpose of ensuring their less experienced colleagues of satisfactory attainment of safe results. Nevertheless, good judgment is still needed in the design of structures, inasmuch as there may be inconsistencies within any code. If good judgment and conservatism make it seem desirable to do so, there is no reason why an engineer should not make a structure better than the minimum limits set by such specifications of safe practice.

The problems given in the text are for the purpose of demonstrating the design of reinforced concrete as it is done in practice. A wide range of unit stress is used in problems, mainly to illustrate and emphasize that the quality of the materials, the uncertainties of construction, and the nature of the work are factors that must be considered. For example, the unit stress that may permissibly be used in steel reinforcement is not the same in different building codes; the obtainable quality of concrete or of steel may vary; wartime restrictions or other necessities for economizing on steel tonnage or individual opinion or professional judgment of the engineer may influence a design. As a matter amenable to judgment the prevailing situation regarding older styles of deformed reinforcement might be considered. Table 1-8 implies that these obsolescent types should now be classified as plain bars. Nevertheless, the Joint Committee Report referred to on page 37 recommended for such bars a permissible bond unit stress of $u = 0.05f'_c$ but not to exceed 200 psi for beams and one-way footings; this was increased to $u = 0.056f'_c$ in two-way footings when the bars were hooked. Whenever the older style of bar is used, it is suggested that these former criteria of safe practice might also be used. The following are cited as other examples of variations in practice that are matters amenable to judgment with respect to specific conditions of service:

1. Culverts or tunnels under deep embankments, bins designed for

maximum filling, foundations supporting massive superstructures, and, in general, most structures carrying relatively large determinable loads compared with any possible future increase—these may take advantage of greater allowable unit stresses that might be used in their design, provided that the accompanying cracking of concrete in regions of tension is not objectionable.

2. Bridges, crane girders, some floors, and other structures or members that could at some time be subjected to great loads, impacts, earth tremors, and other things causing forces of unknown magnitude—these

TABLE 1-8. Recommended Permissible Unit Stresses

Concrete							
		Allowable unit stresses					
Description		For any strength of concrete $n = \frac{30,000}{f'_c}$	Max value, psi	For strength of concrete shown below			
				$f'_c =$ 2,000 psi $n = 15$	$f'_c =$ 2,500 psi $n = 12$	$f'_c =$ 3,000 psi $n = 10$	$f'_c =$ 3,750 psi $n = 8$
Flexure f_c:							
Extreme fiber stress in compression.	f_c	$0.45f'_c$	...	900	1,125	1,350	1,688
Extreme fiber stress in tension in plain concrete footings........	f_c	$0.03f'_c$	...	60	75	90	113
Shear v (as a measure of diagonal tension):							
Beams with no web reinforcement.	v_c	$0.03f'_c$	...	60	75	90	113
Beams with properly designed web reinforcement...........	v	$0.12f'_c$	...	240	300	360	450
Flat slabs at distance d from edge of column capital or drop panel.	v_c	$0.03f'_c$	...	60	75	90	113
Footings....................	v_c	$0.03f'_c$	75	60	75	75	75
Bond u:							
Deformed bars:							
Top bars*..................	u	$0.07f'_c$	245	140	175	210	245
In 2-way footings (except top bars)...................	u	$0.08f'_c$	280	160	200	240	280
All others..................	u	$0.10f'_c$	350	200	250	300	350
Plain bars (must be hooked):							
Top bars....................	u	$0.03f'_c$	105	60	75	90	105
In 2-way footings (except top bars)....................	u	$0.036f'_c$	126	72	90	108	126
All others..................	u	$0.045f'_c$	158	90	113	135	158
Bearing f_c:							
On full area..................	f_c	$0.25f'_c$	...	500	625	750	938
On one-third area or less†......	f_c	$0.375f'_c$	...	750	938	1,125	1,405

TABLE 1-8. Recommended Permissible Unit Stresses.—(Continued)

Reinforcement A305 bars	
Tension:	
Structural-grade bars	f_s = 18,000 psi
Structural shapes	f_s = 18,000 psi
Intermediate- and hard-grade bars	f_s = 20,000 psi
Cold-drawn steel wire	f_s = 20,000 psi
Web reinforcement (all grades)	f_s = 18,000 psi
Column reinforcement:	
Verticals:	
Intermediate grade	f_s = 16,000 psi
Hard grade	f_s = 20,000 psi
Spirals useful limit:	
Intermediate grade	f_s' = 40,000 psi
Hard grade	f_s' = 50,000 psi
Cold-drawn wire	f_s' = 60,000 psi

* Top bars are horizontal bars so placed that more than 12 in. of concrete is cast in the member below the bar.

† The allowable bearing stress on an area greater than one-third but less than the full area shall be interpolated between the values given.

should be designed conservatively. This should also be done in the case of important structures which, in the event of their failure, could cause great loss of life or of their economic value.

All should realize that the saving of steel resulting from an increase in the permissible unit stress used in a design is not proportional to that increase. In fact, in most large structures properly designed, the saving is likely to be in the neighborhood of 5 to 10 per cent of that relative increase. Obviously, this is a small part of the cost of the reinforcement which, in turn, is usually a rather small percentage of the cost of the job. However, the safety of the structure may really decrease as the permissible unit stress is raised, hence the need for good judgment in determining these matters.

These comments are generally applicable to the concrete also, although the strength of the concrete is not so likely to be critical and the percentage of saving in the cost due to increasing permissible unit stresses in the concrete is usually far less than in the case of the steel.

1-19. Importance of workmanship. The need for honest and intelligent workmanship in the field during the building of a concrete structure has been emphasized here. It should be emphasized again and again. The best of designs can be ruined if the intent of the plans is not carried out in the field. Proper reinforced-concrete construction depends upon men—men who understand the action of structures, men who know the characteristics and the limitations of the material that they are handling, and men who are conscientious and determined to conduct their work with honor to themselves and with credit to their profession.

2

BEAMS SUBJECTED TO BENDING

2-1. Introduction. In general, a beam is a member which carries loads that act transversely with respect to its longitudinal axis so as to cause the member to bend. The most simple reinforced-concrete beams are those whose cross sections are rectangular in shape. In fact, an ordinary floor slab, like that shown in Fig. 2-1, may be thought of as a series of such beams of unit width b (usually 12 in.), represented by the piece $ABCD$ having a depth equal to t and a span equal to L.

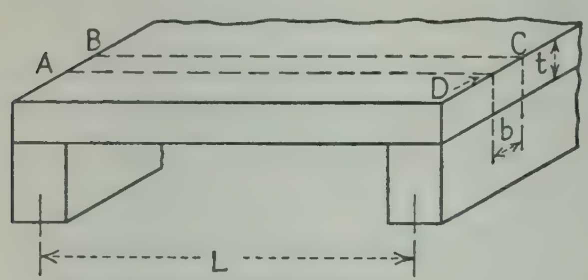


FIG. 2-1.

The effective span of a beam should be taken equal to the clear distance between supporting members plus some allowance for end bearing but not more than the distance between the centers of the supports. If a beam or slab is not poured integrally with its supports, the allowance for bearing need not exceed the depth of the beam or slab. Some engineers assume the span to be the clear distance plus about 4 in. for light members or plus 12 in. for heavy construction. When the end is supported by an edge beam or in a manner that restrains it only slightly, the end is assumed to be at the center of the support.

In dealing with such a beam there are two types of problem to consider. One is the analyzing or testing of existing and assumed beams of given dimensions to compute what forces they can withstand safely; the other is the designing or porportioning of beams to support certain given forces or loads. One of the fundamental theories by which these problems may be solved will be called the *straight-line* theory. The second will be called the *ultimate-load* theory.[1]

[1] Charles S. Whitney, Plastic Theory of Reinforced Concrete Design, *Proc. ASCE*, Vol. 66, December, 1940. V. P. Jensen, Ultimate Strength of Reinforced Concrete Beams as Related to the Plasticity Ratio of Concrete, *Univ. Illinois Eng. Expt. Sta. Bull.* 345. Articles by Corning, Anderson, Hognestad, Siess, Reese, and Lin in *J. ACI*, June, 1952.

At the present time, many engineers in Europe, and some in the United States, prefer the latter, but the former is still common practice in this country, and it has been used for many years. The student should be familiar with both systems. Therefore, to some extent, both will be presented together instead of being explained as utterly separate methods.

A good training in theory is essential for any designer. However, when one enters practical engineering work he will find that the design of reinforced-concrete structures is influenced to a large extent by general specifications, codes, and customary practices. Such codes may change—and they should do so as the art progresses and our knowledge increases. Each engineer should have a copy of the latest codes, such as the Building Code Requirements for Reinforced Concrete (ACI 318-51)) that has been prepared by the American Concrete Institute. This will be referred to here as the Code. Of course, there are many other specifications, building codes, and regulations in use: *e.g.*, the Building Code of the City of New York; the ASCE Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete; and United States Navy Department, Bureau of Yards and Docks, Specification for Concrete Construction, No. 13 Yd. The designer should be careful to comply with whatever ones govern his work, bearing in mind that they are general rules to be followed unless he has important reasons for making the construction even better and safer.

2-2. Table of symbols and their meanings. Most of the symbols which are used in the Code have been adopted for this text as far as it has been practicable to do so. An explanation of those which are used here is given generally wherever they are first encountered. However, those which are utilized in this chapter are grouped for convenience. They are fundamental symbols for use throughout the work.

The list is as follows:

a = depth of compression area assumed for stress block in ultimate-load design, in.

A_s = area of steel in tension, in.^{2}.

A_s' = area of steel in compression, in.^{2}.

b = width of rectangular beam or flange of T beam, in.

b' = width of stem of T beam, in.

c = lever arm between resultant tension and compression in ultimate-load design, in.

C, C_c = total force of compression in concrete, lb.

C', C_s = total force of compression in steel, lb.

d = depth from compression face of beam or slab to the center of gravity of the longitudinal tensile reinforcement, in. (called

effective depth). In some special problems with reinforcement distributed over a considerable depth of the member, d may be measured from the compression face to the row of bars that is farthest from it.

d' = depth from compression face of beam or slab to center of gravity of the longitudinal compressive reinforcement, in.

d_1 = distance between centers of gravity of tensile and compressive reinforcement in ultimate-load design, in.

D = total over-all depth of a beam, in.

E_c = modulus of elasticity of concrete in compression, psi.

E_s = modulus of elasticity of steel in tension or compression, psi.

ϵ = unit strain in concrete, in.

ϵ' = unit strain in steel, in.

f_c = compressive unit stress in extreme fiber of concrete, psi.

f'_c = ultimate compressive strength of concrete, usually at age of 28 days, psi.

f_s = tensile unit stress in longitudinal reinforcement, psi.

f'_s = compressive unit stress in longitudinal reinforcement, psi.

f_{yp} = yield point stress of tensile and compressive reinforcement, psi.

I_c = moment of inertia of transformed section in terms of concrete, in.^{4}.

j = ratio of distance between centroid of compression and center of gravity of tensile reinforcement or the extreme row of tensile reinforcement to the depth d.

k = ratio of distance between the compression face of the beam and the neutral axis to the depth d.

kip = 1,000 lb (sometimes abbreviated as "k").

L = span of beam or slab, usually in feet.

$m = f_{yp}/0.85f'_c$.

M = bending moment, or ultimate bending moment, ft-lb or in.-lb.

M_c = internal resisting moment in terms of the strength of the concrete, in.-lb.

M_s = internal resisting moment in terms of the strength of the steel, in.-lb.

n = ratio of modulus of elasticity of steel to that of concrete.

p = ratio of area of tensile reinforcement to the effective area of concrete in beams and slabs $= A_s/bd$.

p' = ratio A'_s/bd.

p_0 = critical percentage of reinforcement in tension in ultimate-load design.

S_c = section modulus of transformed section in terms of concrete, in.^{3}.

S_s = section modulus of transformed section in terms of steel, in.^{3}.

t = thickness of slab or flange of T beam, in.
T, T_s = total force of tension in steel, lb.
bd = effective area of a beam or slab, in.^{2}.

2-3. Distribution of stresses in a reinforced-concrete beam. In the ultimate-load theory, the resistance of a particular beam to pure flexure (bending) is determined either by the ultimate strength of the concrete f'_c or by the yield point stress of the steel f_{yp}. Then this resistance can be divided by a proper safety factor to determine what bending

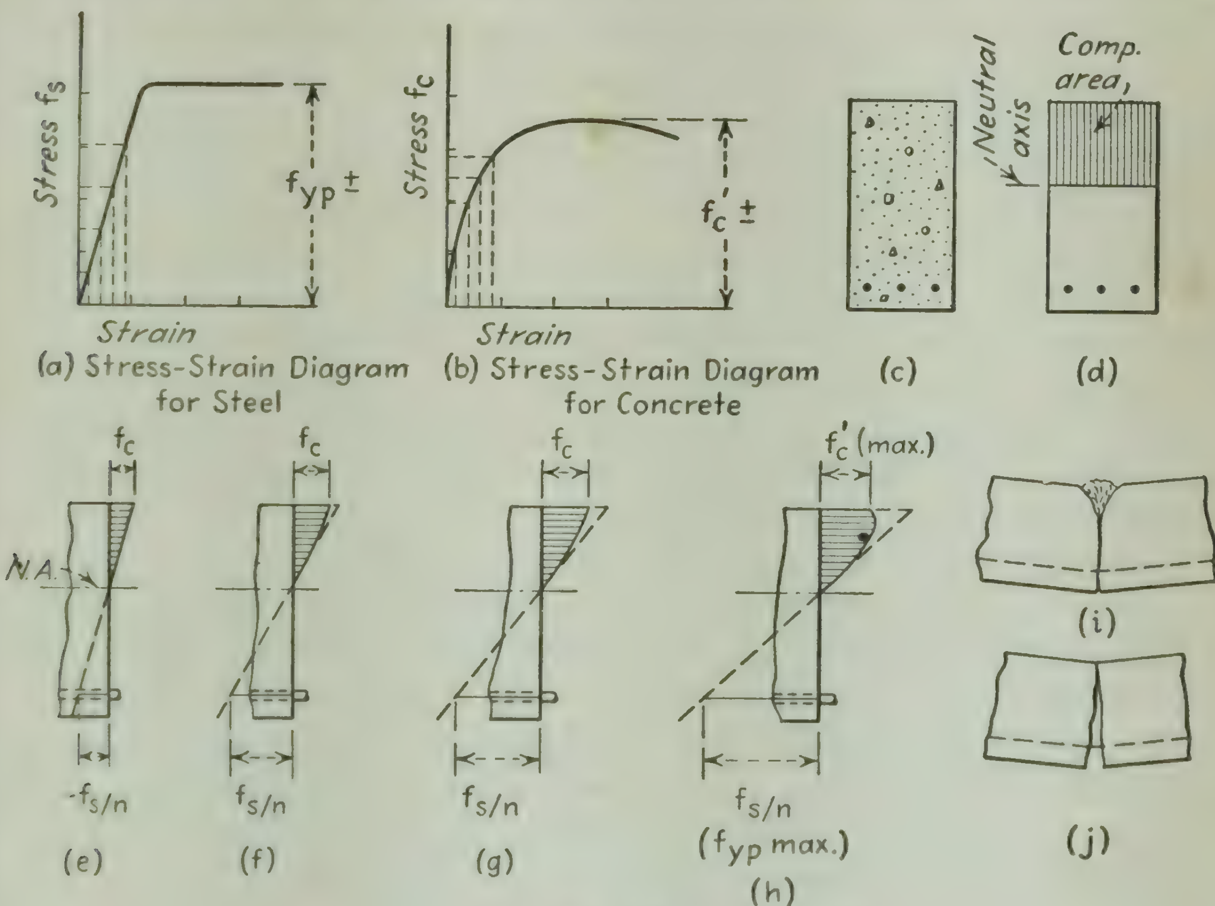


FIG. 2-2.

resistance is to be relied upon under working conditions. In the straight-line theory, f'_c and f_{yp} are divided by the same or different appropriate safety factors to determine the supposedly allowable unit stresses that may be permitted in the respective materials under working conditions.

First of all, examine Fig. 2-2 in which (*a*) represents the general character of the stress-strain curve of the reinforcement, and (*b*) pictures that of the concrete. In (*c*) is shown the cross section of a rectangular beam with reinforcing bars in the bottom. If this beam is bent so as to cause compression in the top and tension in the bottom, it will be assumed that the deformations or strains of the materials at any point will vary in proportion to their distances from a point of zero strain called the *neutral axis*. At very small loads, both the concrete in the bottom and the steel

will resist tension, but such loads are too small for practical consideration. However, as the loads and the bending increase, the concrete near the bottom will soon reach a tensile stress that causes it to crack. Then the bars must resist practically all the tension alone because the beneficial effect of any tension in a small area of uncracked concrete just below the neutral axis will be negligible. Now the member may be assumed to have an effective cross section like that pictured in Fig. 2-2(*d*), where tension in the concrete below the neutral axis is neglected entirely.

As the beam bends more, it would seem that the stresses in the concrete will vary from zero at the neutral axis to a maximum at the extreme top "fiber." However, Fig. 2-2(*b*) shows that the concrete is not truly elastic. Therefore, even though the strains increase, the stresses in the concrete will not do so proportionately. The assumed general character of the distribution of compressive stresses in the concrete as the bending increases is pictured in Sketches (*e*), (*f*), (*g*), and (*h*). As this increase occurs near the maximum, the top fibers yield plastically, or at least they continue to deform without offering proportionately increased resistance, causing the pressure diagram to be somewhat as shown in (*h*). Additional loading might then cause the beam to fail in compression somewhat as pictured in Fig. 2-2(*i*). Such failures are likely to be sudden and without warning; hence they are dangerous.

On the other hand, the bars resist tension in proportion to their strain as the loads increase until the elastic limit is reached. Beyond this point, such large plastic deformation of the steel would occur that the beam would "pop open" and fail in tension or by local crushing of the concrete at the top of the crack, as illustrated in Fig. 2-2(*j*). Such a failure is usually accompanied by such serious cracking and deflection that trouble can be detected in time to remove the excessive loads and to avoid collapse.

Experiments seem to indicate that concrete in flexural action in beams will resist a higher unit compressive stress than it will in 6 by 12 test cylinders. However, because of our present limited knowledge of this action, the cylinder strength will be used as the gage of the ultimate strength f'_c.

During the loading, after the concrete of the tensile area has cracked, the neutral axis may not remain stationary as indicated in Figs. 2-2(*e*) to (*h*), inclusive. Certainly it shifts upward as the tensile failure of Fig. 2-2(*j*) approaches, and it may shift downward as the compression failure of Sketch (*i*) occurs. Our knowledge of these matters is very limited.

The straight-line theory assumes that the stress distribution in the concrete at working stresses of approximately $\frac{1}{2}f'_c$ is like that of Fig. 2-2(*f*) except that any curvature of the compression diagram is neglected. At ordinary working loads, this assumption seems to be reasonably satis-

factory. It is probably as accurate as some of the other assumptions that have to be made in the design of reinforced concrete. It is probably impossible to predict accurately what the stresses in a reinforced-concrete beam will really be. However, such beams can be made to support loads safely.

In both theories, the stress in the steel is assumed to be that caused by the strain that would be in the concrete at the same distance from the neutral axis if the concrete could resist such a strain. That is why it is labeled f_s/n in Fig. 2-2, where $n = E_s/E_c$.

2-4. Resistance to bending, straight-line theory. A beam must curve if it is subjected to a bending moment because the parts that are in compression must shorten and those which are subjected to tension must elongate. The typical method that is used here to enable one to visualize these actions is shown in Fig. 2-14. The short irregular lines represent cracks. They are drawn in the regions where tension exists—the weak portions of the concrete which must be *reinforced* with steel. No cracks are shown in the portions that are subjected to compression because the concrete there will be effective by itself. Such pictures do not mean that concrete beams always crack so excessively; they are to indicate the condition that the beams may reach if they are loaded sufficiently. This visualization of excessively deformed members as almost a series of cracked portions that are tied together by the bars acting somewhat as a chain is helpful in understanding the action of reinforced concrete. If the bars are not parallel to the tensile force, it is satisfactory to assume that the effective area of a bar is its normal cross-sectional area times the cosine of the angle between the longitudinal axis of the bar and the direction of the force.

Figure 2-3(a) shows a small portion of a beam that is isolated as a free body. Neglecting the dead load and the shearing forces, let M represent the magnitude and the direction of the bending moment which is caused by the external loads. Since the beam is in equilibrium, this moment must be counteracted by a moment of equal magnitude and opposite direction. Obviously, this resisting moment must be provided by the strength of the materials composing the beam itself.

As previously explained, the tensile strength of the concrete is rather low and unreliable. Although its effect is considerable in many instances when the section is not cracked, it is advisable to assume that the tensile strength of the concrete is zero. Therefore, the tensile force T in Fig. 2-3(a) is assumed to be provided by the steel alone. It is sufficiently accurate to assume that a reinforcing bar has an intensity of stress f_s which is equal to the unit stress theoretically at its center. This value, multiplied by the area of the steel, gives the tensile force

$$T = A_s f_s \tag{2-1}$$

For multiple layers of bars, the total area of the steel is assumed to be concentrated at the center of gravity of the group of bars, and the tensile stress f_s is computed at that center of gravity. This is assumed to be an average stress on all the bars even though those farther from the neutral axis may be somewhat more highly stressed.

Figure 2-3(a) shows the assumed triangular distribution of stress in the concrete. Therefore, the maximum compression occurs at the top fibers, and the stress decreases to zero at the line O-O, the neutral axis. The height of this area which is in compression is called kd. Since this assumed pressure diagram of the compressive forces is a triangular wedge,

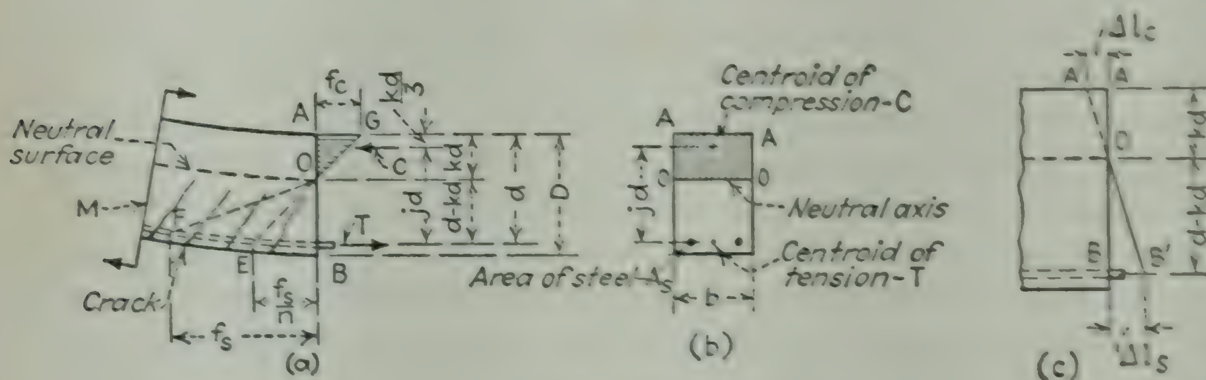


FIG. 2-3.

the resultant force C must equal the volume of the wedge, and it must be located at the center of gravity of this imaginary solid, which is at a distance $kd/3$ from the top fibers. It is therefore clear that

$$C = \tfrac{1}{2}f_c(kd)(b) = \tfrac{1}{2}f_ckbd \qquad (2\text{-}2)$$

It is also apparent that the magnitudes of these forces C and T must be equal in order to have equilibrium, for which $\Sigma H = 0$. Therefore,

$$\tfrac{1}{2}f_ckbd = A_sf_s \qquad (2\text{-}3)$$

From Fig. 2-3 it is easily seen that the lever arm of the internal resisting couple with forces C and T is the distance between the points of application of these forces, jd. Thus,

$$jd = d - \frac{kd}{3} \qquad \text{or} \qquad j = 1 - \frac{k}{3} \qquad (2\text{-}4)$$

Inasmuch as the moment of a couple equals the magnitude of either of the equal opposite forces times the lever arm between them,

$$M_c = \tfrac{1}{2}f_ckbd(jd) = \tfrac{1}{2}f_ckjbd^2 \qquad (2\text{-}5)$$

and

$$M_s = A_sf_sjd \qquad (2\text{-}6)$$

Formulas (2-5) and (2-6) are fundamental in the analysis and design of rectangular beams with tensile reinforcement only, and with no tension in the concrete. Both formulas are needed because reinforced concrete is not a homogeneous material. In practical design at ordinary allowable stresses the steel is usually the critical part of most reinforced-concrete beams. Varying assumptions of the distribution of the concrete stresses have a minor effect upon the lever arm of the steel, as the reader will realize after thinking the matter over carefully.

If the beam of Fig. 2-3 is progressively loaded to failure, the concrete may yield under compression before the steel gives way under tension. If so, the beam is said to be *overreinforced* because it has more than the necessary amount of steel. In case the reverse is true, and the steel fails first, the beam is *underreinforced*. From the standpoint of the efficient use of materials, the best design is one that results in a beam in which the maximum safe working strength of the concrete and that of the steel are reached simultaneously—a *balanced design*. However, cost and other practical matters affect one's designs. Sometimes it is wise to use more than the theoretical amount of reinforcement in order to simplify the details by making the rods in many different beams alike; sometimes it is advisable to use more concrete than needed for strength alone in order to have the thickness or general dimensions desired and to reduce cracking and deflection; often it is best to avoid an excessive variety of sizes which increases the formwork because the cost of a reinforced-concrete structure does not vary directly with the volume of concrete used in it.

The economy of any particular construction depends upon the relative costs of concrete and steel in place instead of upon whether or not it is a balanced design. Sometimes it is important to save steel even though more concrete is necessary; other times, especially in order to minimize the weight of the structure, thin sections may be the best and most economical even though they are heavily overreinforced.

By definition,

$$p = \frac{A_s}{bd} \qquad \text{or} \qquad A_s = pbd$$

Substituting this value of A_s in Eq. (2-6) gives

$$M_s = pf_sjbd^2 = Kbd^{2*} \tag{2-7}$$

Equations (2-5) and (2-7) may be restated as

$$bd^2 = \frac{2M_c}{f_ckj} = \frac{M_c}{K} \tag{2-8}$$

* K is computed as a coefficient which can be tabulated for balanced designs. For such data, see Tables 5 and 6 in the Appendix.

and

$$bd^2 = \frac{M_s}{pf_sj} = \frac{M_s}{K} \tag{2-9}$$

Another convenient form for Eq. (2-6) is

$$A_s = \frac{M_s}{f_sjd} \tag{2-10}$$

Equations (2-5) and (2-6) are in convenient form for analyzing beams, whereas Eqs. (2-8), (2-9), and (2-10) are handier for use in designing them on the assumption that they are to be balanced designs.

Let line AB [Fig. 2-3(c)] represent a plane cross section through a beam before the external loads are applied. If this section is considered to remain a plane after bending has taken place, it will move relatively to $A'B'$. The shortening due to compression at the top can be represented by Δl_c; the elongation of the rods caused by tension can be represented by Δl_s.

For any elastic material that is not stressed beyond its elastic limit, the modulus of elasticity equals the stress per square inch divided by the corresponding deformation in inches per inch of length, or $E = f/\delta l$. Therefore, although the magnitude of E_c may be somewhat uncertain, as shown in Fig. 1-4, assume that

$$E_c = \frac{f_c}{\delta l_c} \qquad \text{and} \qquad E_s = \frac{f_s}{\delta l_s}$$

where δl_c and δl_s represent the strains per unit of length. From Fig. 2-3(c), by the use of similar triangles, it is found that

$$\frac{\Delta l_c}{\Delta l_s} = \frac{kd}{d - kd} = \frac{k}{1 - k} \tag{2-11}$$

The ratio of the modulus of elasticity of steel to that of concrete is

$$n = \frac{E_s}{E_c} \tag{2-12}$$

$$n = \frac{f_s/\delta l_s}{f_c/\delta l_c} = \frac{f_s(\delta l_c)}{f_c(\delta l_s)} \tag{2-13}$$

Referring to Eq. (2-13), it is clear that, if the deformations of the steel and of the concrete are equal, as when side by side at the same distance from the neutral axis, then

$$n = \frac{f_s}{f_c} \qquad \text{or} \qquad f_s = nf_c \tag{2-14}$$

These equations neglect the effect of plastic flow of the concrete.

Equation (2-14) explains again why EB of Fig. 2-3(a) is labeled f_s/n. To some scale, AG and EB represent stresses in the concrete for a straight-line variation of stress and for truly elastic action. The stress in the steel would then have to be $n(EB)$, which is plotted as FB.

In order to assist in determining a value for the modular ratio n, the Code gives it the following approximate magnitude:

$$n = \frac{30{,}000}{f'_c}$$

Obviously, the qualities of the steel and the concrete should be known or decided upon before making the calculations.

Some other formulas that are useful in developing office standards and tables or diagrams for balanced design are shown in Tables 5 and 6 of the Appendix. Another formula that is useful for the analysis of beams is

$$k = \sqrt{2pn + (pn)^2} - pn \tag{2-15}$$

Notice that p is whatever the area of the tensile reinforcement and the effective area of the beam cause it to be whether it is a balanced design or not. Furthermore, this shows that k and the area of concrete in compression are larger for a heavily reinforced beam than for one that is lightly reinforced, and that k is also larger for weaker concrete having a larger modular ratio n.

The assumptions upon which these formulas are based should be studied carefully. It must be remembered that they are for beams with tensile reinforcement only, additional methods of calculation for beams with compressive reinforcement being derived in subsequent articles. They also assume that the concrete cracks so that it cannot withstand the tensile forces. This is equivalent to saying that, before the member will fail, these conditions will occur, and the beam will be safe in spite of them but that any resistance to tension that the concrete may provide will merely add to the safety of the structure. Then, finally, the value of n for concrete is considered to be the same for tension as it is for compression—an assumption that will be made use of later.

2-5. Problems in the analysis and design of beams, straight-line theory. Many of the problems to be illustrated in this text are solved by the use of the slide rule. Its accuracy will be sufficient for practical purposes, although differences may appear in the third significant figure. The student will find that, in many cases, the calculations are not carried out to unnecessary extremes but are limited in accordance with good judgment.

Generally, E_c is assumed to be equal to $1{,}000f'_c$, and this value is used in the determination of n. However, the allowable working stress in the concrete f_c is much less than the ultimate strength f'_c. The ratio of f_c to

f_c may be looked upon as the *safety factor* of the member in terms of the strength of the concrete, although this is seldom if ever strictly true. The ratio of the yield-point stress (or elastic limit) of the reinforcement to the allowable working stress f_s is fairly reliable as the measure of the safety factor in terms of the steel. For beams, the code gives it a magnitude of approximately 2.0. For purposes of illustration in problems, different allowable unit stresses f_c will be utilized for the concrete because there are many cases in which an engineer may wish to be more conservative than the maximum limit permitted by the Code. Such instances may occur when the future loading is uncertain, when shocks of unknown magnitude may be applied, when the difficulties of the field work may make it seem that greater conservatism is advisable, and when greater stiffness is desired.

Furthermore, an examination of Fig. 2-3(a) will remind one that a high working stress in the longitudinal reinforcement will be accompanied by correspondingly severe cracking of the concrete in the vicinity of the steel. Therefore, the designated values of f_s also are not constant but are varied in order to remind one that the specifications and good judgment may limit its magnitude to suit any special case.

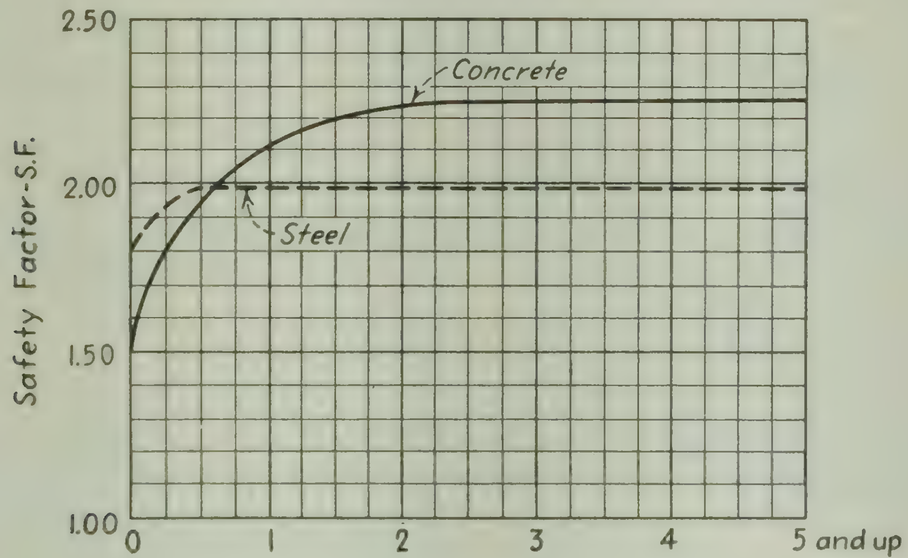
It is the practice of some engineers to allow higher computed unit stresses in reinforced concrete when the dead load is predominant than when the live load is so. This means that they consider the dead load to be static and determinable; hence a smaller safety factor is needed for it. The live load is likely to be variable, unknown in actuality, subject to excessive increases, and applied suddenly. Therefore, a larger safety factor is needed to allow for such uncertainties. This idea has considerable justification if used wisely.

Figure 2-4 has been prepared to show a suggested curve of permissible relationship between the L.L./D.L. ratio and the safety factor for the design of beams. The author has used 1.5 as a minimum safety factor because of the uncertainties of construction and design, even when there is only dead load to support. Furthermore, a smaller safety factor is likely to result in too much cracking and deflection of one's structures. The 2.25 figure is set as a maximum for concrete but, when great uncertainties and the likelihood of fatigue make it desirable, one can well be more conservative than that. Modern reinforcement is so reliable that a safety factor of 2 seems to be adequate for it in any case except when cracking and deflections are to be minimized.

It is suggested that Fig. 2-4 be used only as a starting point for further study. General specifications of this character will have to be discussed thoroughly before they are adopted. Perhaps different specifications should be used for school buildings with light live loads than for warehouses with heavy ones.

For use in the solution of problems, the weight of concrete will be taken at 150 pcf. The areas and perimeters of reinforcing bars of the ordinary sizes are given in Tables 1, 2, and 3 in the Appendix.

Analysis of Rectangular Beams. The term "analysis of a beam" is used to denote the testing of an existing or assumed beam by computations to see whether or not the beam can safely support the loads on it or to be applied in the future.



Ratio of live load: Dead load or live load + impact: Dead load – $\frac{LL}{DL}$ or $\frac{LL+I}{DL}$

S.F. for concrete = $\frac{\text{28-day cylinder strength}}{\text{allowable unit stress}} = \frac{f'_c}{f_c}$

S.F. for steel = $\frac{\text{yield point stress}}{\text{allowable unit stress}} = \frac{f_{yp}}{f_s}$

FIG. 2-4. Curves showing one suggested relation between allowable unit stresses and live-load: dead-load ratios for beams.

Example 2-1. If the beam shown in Fig. 2-5(a) is subjected to a bending moment of 300,000 in.-lb, and if $n = 15$, compute f_s and f_c.

Incidentally, the designer is not so much interested in the magnitudes of f_s and f_c for any given case for their own sakes as he is in comparing them with the allowable unit stresses to see whether or not the member is safe on the one hand and economical on the other.

$$A_s = 3 \times 0.44 = 1.32 \text{ in.}^2 \qquad \text{(see Table 3, Appendix)}$$

$$p = \frac{A_s}{bd} = 1.32 \div (10 \times 14) = 0.0094$$

$$k = \sqrt{2pn + (pn)^2} - pn = \sqrt{2 \times 0.0094 \times 15 + (0.0094 \times 15)^2} - 0.0094 \times 15 = 0.408$$

This value of k may be roughly checked by the use of Fig. 10 of the Appendix, or its magnitude may frequently be determined with sufficient accuracy directly from this diagram.

$$j = 1 - \frac{k}{3} = 1 - \frac{0.408}{3} = 0.864$$

$$f_c = \frac{2M}{kjbd^2} = \frac{2 \times 300{,}000}{0.408 \times 0.864 \times 10 \times 14^2} = 868 \text{ psi}$$

$$f_s = \frac{M}{A_s jd} = \frac{300{,}000}{1.32 \times 0.864 \times 14} = 18{,}800 \text{ psi}$$

Example 2-2. Assume E_s and E_c = 30,000,000 and 2,500,000 psi, respectively; also, the allowable f_s and f_c = 18,000 and 900 psi, respectively. Compute the safe resisting moment of the beam shown in Fig. 2-5(*b*).

$$n = \frac{E_s}{E_c} = \frac{30{,}000{,}000}{2{,}500{,}000} = 12$$

$$A_s = 4 \times 0.79 = 3.16 \text{ in.}^2$$

$$p = \frac{A_s}{bd} = 3.16 \div 15 \times 28 = 0.0075$$

$$k = \sqrt{2pn + (pn)^2} - pn = \sqrt{2 \times 0.0075 \times 12 + (0.0075 \times 12)^2} - 0.0075 \times 12 = 0.344$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.344}{3} = 0.885$$

$$M_s = A_s f_s jd = 3.16 \times 18{,}000 \times 0.885 \times 28 = 1{,}410{,}000 \text{ in.-lb}$$

$$M_c = \tfrac{1}{2} f_c kjbd^2 = \tfrac{1}{2} \times 900 \times 0.344 \times 0.885 \times 15 \times 28^2 = 1{,}610{,}000 \text{ in.-lb}$$

From the fact that the foregoing figures show the safe value of M_s to be less than M_c, it is apparent that this beam is somewhat underreinforced. Using the magnitude of M_s and solving for the simultaneous value of f_c for purposes of illustration gives

$$1{,}410{,}000 = \tfrac{1}{2} \times f_c \times 0.344 \times 0.885 \times 15 \times 28^2$$

$$f_c = 788 \text{ psi}$$

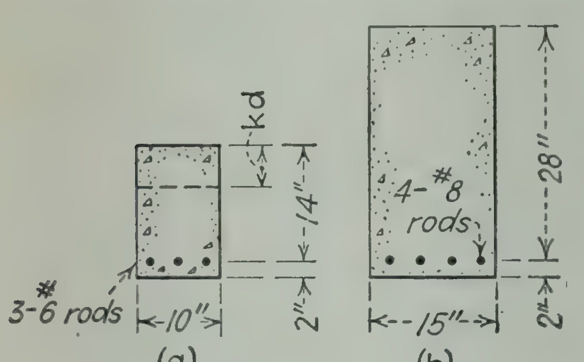


FIG. 2-5.

It can also be said that f_c varies as the magnitude of the resisting moment, or

$$f_c : 900 :: 1{,}410{,}000 : 1{,}610{,}000$$

Example 2-3. Assume a simply supported slab 9 in. thick with No. 5 round rods 5 in. c.c. (center to center) located 2 in. above the bottom as shown in Fig. 2-6(*a*). Determine the safe uniform live load for this beam if the span = 9 ft, n = 12, and the allowable f_s and f_c = 20,000 and 900 psi, respectively.

First, imagine a slice 12 in. wide to be cut out of the slab from one support to the other, parallel to the reinforcement. Each such piece will be a rectangular beam. It will contain an equivalent of $1\tfrac{2}{5}$ rods,[1] Thus

$$A_s = \frac{12 \times 0.31}{5} = 0.74 \text{ in.}^2 \qquad d = 7 \text{ in.} \qquad b = 12 \text{ in.}$$

$$p = \frac{A_s}{bd} = \frac{0.74}{12 \times 7} = 0.0088$$

$$k = \sqrt{2pn + (pn)^2} - pn = \sqrt{2 \times 0.0088 \times 12 + (0.0088 \times 12)^2} - 0.0088 \times 12 = 0.37$$

[1] See Table 2, Appendix.

$$j = 1 - \frac{k}{3} = 1 - \frac{0.37}{3} = 0.88$$

$$M_s = A_s f_s jd = 0.74 \times 20{,}000 \times 0.88 \times 7 = 91{,}200 \text{ in.-lb.}$$

$$M_c = \tfrac{1}{2} f_c kjbd^2 = \tfrac{1}{2} \times 900 \times 0.37 \times 0.88 \times 12 \times 7^2 = 86{,}200 \text{ in.-lb}$$

The strength of the concrete, in this case, limits the safe resisting moment, and the slab is slightly overreinforced. The simultaneous computed magnitude of f_s is

$$f_s = 20{,}000 \times \frac{86{,}200}{91{,}200} = 18{,}900 \text{ psi}$$

Considering that the weight of concrete is usually assumed to be 150 pcf, the dead load of the slab is $150 \times 9/12 = 112$ psf of horizontal area. Assuming that $M = wL^2/8$

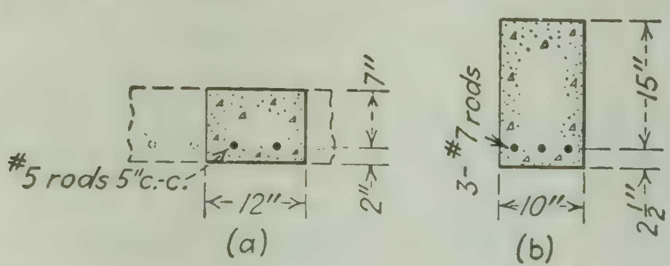


FIG. 2-6.

for a simply supported beam and that the safe live load is the reserve supporting capacity of the slab over and above that required for the dead load, it is apparent that

$$M = 86{,}200 = \frac{w \times 9^2 \times 12}{8} \qquad \text{or} \qquad w = 710 \text{ psf}$$

Therefore, the live load permissible in this case is $710 - 112 = 598$ psf.

Design of Rectangular Beams. The term "design of a beam" is used to denote the determination of the size and the materials that are required to constitute a beam that can safely support specified loads under certain definite conditions of span, stresses, etc. Ordinarily, there are many beams of varying proportions which can safely serve the same purpose. However, it is generally reasonable and economical to proportion a rectangular beam so that its depth equals about twice its width unless these dimensions are controlled by other conditions. Sufficient lateral stiffness, economy and efficiency in the use of materials, strength in shear as well as in bending, space for proper placing of rods—these are some of the practical reasons for using such proportions.

The Code specifies that the clear distance between lateral supports of a beam shall not exceed thirty-two times the least width of the compression flange. This is because of the possibility of failure by lateral buckling. Personally, the author prefers to be more conservative than this for important work, using a limit of $L/b = 20$.

Example 2-4. Design a beam to carry a bending moment of 400,000 in.-lb if $n = 10$ and the allowable f_s and f_c = 18,000 and 1,100 psi, respectively.

From Table 5 of the Appendix, with $f_s = 18{,}000$, $n = 10$, and f_c = a bit less than 1,125, find k = approximately 0.38 for a theoretically balanced design. Then

$$j = 1 - \frac{k}{3} = 1 - \frac{0.38}{3} = 0.873$$

Using Eq. (2-8)

$$bd^2 = \frac{2 \times M_c}{f_ckj} = \frac{2 \times 400{,}000}{1{,}100 \times 0.38 \times 0.873} = 2{,}190$$

It would be satisfactory to use $K = 189$ from Table 5 of the Appendix instead of $f_ckj/2$, as shown in Eq. (2-8).

The problem now resolves itself into a case of "cut and try." There are a multitude of possible values for the width and depth of the beam. However, one way is to assume d and test for b, changing the assumptions until proper and reasonable dimensions are found. Assuming $d = 18$ in. gives

$$b = \frac{2{,}190}{18^2} = 6.75 \text{ in.}$$

Experience will soon show that this value of b is so small that it will be difficult or impossible to place the reinforcing rods properly. However, assuming $d = 15$ in. gives

$$b = \frac{2{,}190}{15^2} = 9.75 \text{ in., or, say, 10 in.}$$

Taking this value of $d = 15$ in. and substituting it in Eq. (2-10) yields

$$A = \frac{M_s}{f_sjd} = \frac{400{,}000}{18{,}000 \times 0.873 \times 15} = 1.7 \text{ in.}^2$$

If three No. 7 rods are used, $A_s = 3 \times 0.6 = 1.8$ in.^{2}. Placing these rods in the assumed beam gives a section as pictured in Fig. 2-6(*b*). The 2½ in. of concrete below the steel provides slightly more than the minimum required cover of 2 in.

Ordinarily it would be unnecessary to test this beam further because the width of the member and the area of the steel used are slightly greater than the minimum required by the calculations. However, for illustration, the values of f_s and f_c will be computed by analyzing the beam as follows:

$$p = \frac{A_s}{bd} = 1.8 \div (10 \times 15) = 0.012$$

$$k = \sqrt{2pn + (pn)^2} - pn = \sqrt{2 \times 0.012 \times 10 + (0.012 \times 10)^2} - 0.012 \times 10 = 0.383$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.383}{3} = 0.872$$

$$f_s = \frac{M_s}{A_sjd} = \frac{400{,}000}{1.8 \times 0.872 \times 15} = 17{,}000 \text{ psi}$$

$$f_c = \frac{2M_c}{kjbd^2} = \frac{2 \times 400{,}000}{0.383 \times 0.872 \times 10 \times 15^2} = 1{,}066 \text{ psi}$$

The beam is slightly overreinforced, a fact which is shown by the relative magnitudes of f_s anf f_c given above.

In assuming depths of beams for designing ordinary structures, the following may be of some service as a general guide or starting point, d being in inches and the span L in feet:

1. For slabs for roofs and floors, assume $d = L/3$ to $L/2$ in. (if $L = 8$ ft, $d = 2.6$ to 4 in.). The larger depth is preferable for stiffness.

2. For light beams and heavy slabs, assume $d = 0.8L$ in.

3. For heavy beams, and headers or girders supporting crossbeams, assume $d = 1.0L$ to $1.25L$ in., depending upon the intensity of loads and lengths of spans.

4. For ordinary continuous beams and girders, assume d somewhat less than given above.

The planning of the arrangement and spacing of reinforcement is far more important than one might at first expect. The necessary cover over the rods should be secured; this means real cover of concrete beyond

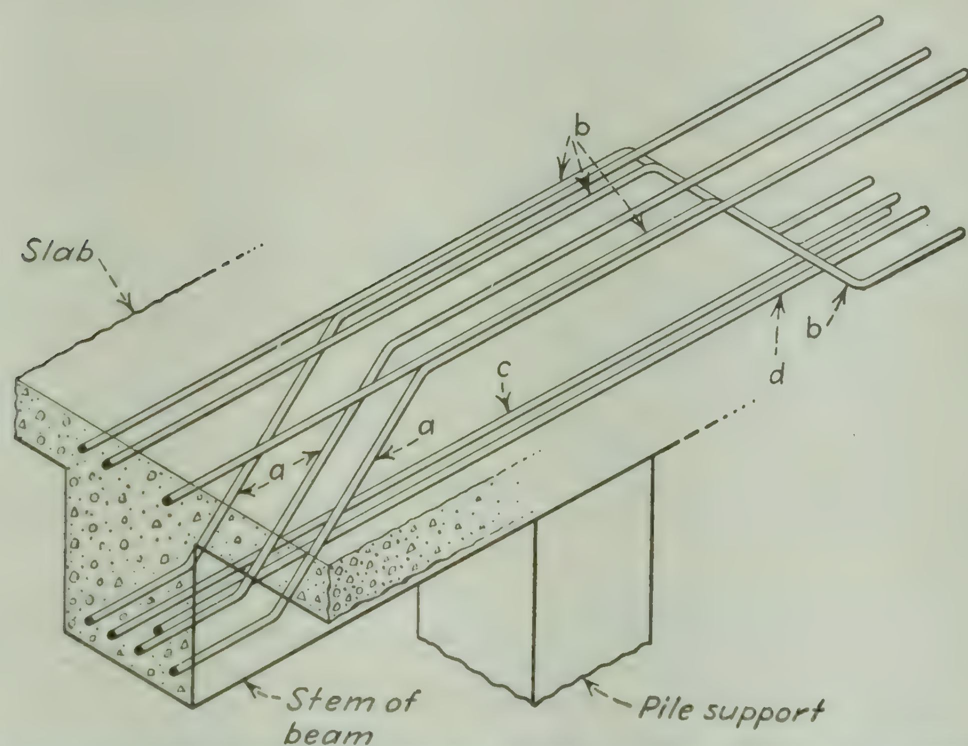


FIG. 2-7. Example showing how rods too close together may cause honeycombing of the concrete.

the surface of the steel, not a dimension to the center of a rod. Not only should the spacing of the main reinforcement be ample to permit the aggregate to pass between adjacent rods, as shown in Tables 8 and 8A of the Appendix, but this should also be adequate at splices, at overlaps, where bent rods adjoin others, and where main rods are beside dowels.

Here is one case as an example. Some continuous T beams in a wharf were 16 in. wide with No. 5 stirrups and four No. 11 main bars about $3\frac{1}{4}$ in. c.c. Near the supports three of these bars were bent up from each side and continued across the top as shown by bars a and b in Fig. 2-7 to resist tension in the top of the beam. The fourth bar from each side continued across and overlapped the bent ones, as shown by c and d. All were detailed to lie in the same plane at the top and bottom of the beam. Rods a and b formed a screen at the top where they passed each

other. Rods c and d did likewise where they lapped over a and b, which were already close together. After a few years it was discovered that the latter had caused honeycombing in the concrete cover under the rods. The sea water and salty air entered and caused rusting of the steel. This, with probable aid from freezing of water in the voids, produced longitudinal cracks in the bottoms of the beams, even loosening the concrete cover in several places. Extensive and costly repairing of the beams was necessary, whereas some deeper girders that were immersed more but were made properly showed no disintegration.

Incidentally, a bitumastic coating sprayed or brushed on such exposed beams may help a little in protecting the members.

Many times it is desirable to use approximate formulas when making a first try at the design of a beam in order to minimize time and labor. This tentative design can be checked later by more theoretically correct methods. A casual inspection of Tables 5 and 6 in the Appendix shows that the value of k generally lies somewhere between 0.3 and 0.45; hence j is somewhere from 0.9 to 0.85. Therefore, assume $k = 0.38$ and $j = 0.88$, as average values. Then Eq. (2-5) gives

$$M = \frac{1}{2} f_c \times 0.38 \times 0.88bd^2 = \frac{1}{6} f_c bd^2 \qquad \text{or} \qquad f_c = \frac{6M}{bd^2} \qquad (2\text{-}5a)$$

and Eq. (2-6) becomes

$$M = 0.88 A_s f_s d \qquad \text{or} \qquad A_s = \frac{M}{0.88 f_s d} \qquad (2\text{-}6a)$$

These formulas will be labeled as shown to denote that they are approximations of the original ones. They are easily remembered and are useful if one wishes to get a scale on sizes required, especially when he has no books to which he can refer.

The values of K shown in Tables 5 and 6 in the Appendix are also very useful in expediting the design of rectangular beams. To illustrate this, assume the following problem:

Design a beam along the edge of a large hatch in a floor. It is simply supported and has a span of 22 ft. It carries a uniformly distributed live load of 1,500 plf, as well as its own weight. An intersecting beam at its center delivers to it a reaction of 20,000 lb. Assume $n = 8$ and the allowable f_c and $f_s = 1{,}200$ and 20,000 psi, respectively.

This is a girder with a very heavy load. From the ratios of depths to spans previously given, assume $d = 1.2 \times 22 = 26.4$ in., or call $d = 27$ in., $D = 30$ in., and $b = 18$ in. The dead load of the beam is

$$w = \left(\frac{18 \times 30}{144}\right) 150 = 560 \text{ plf}$$

$$M = (1{,}500 + 560) \frac{22^2}{8} + 20{,}000 \times \frac{22}{4} = 235{,}000 \text{ ft-lb}$$

From Table 6, $K = 173$, then

$$bd^2 = \frac{M}{K} \text{ or, using the assumed } d,\ b = \frac{235{,}000 \times 12}{27^2 \times 173} = 22.4 \text{ in.}$$

This is greater than the assumed b of 18 in. It seems wise to deepen the beam; hence assume $d = 30$ in., $D = 34$ in., and $b = 20$ in. The new dead load is 720 plf, and $M = 244{,}000$ ft-lb. Testing for b again,

$$b = \frac{M}{d^2K} = \frac{244{,}000 \times 12}{30^2 \times 173} = 18.8 \text{ in.}$$

This shows that b might be 19 in., but the 20-in. dimension is conservative, satisfactory, and simple for use in formwork.

Technically, k and j should be computed for these conditions, but the magnitude of j would change but slightly, being a little more than the 0.892 given in Table 6. Therefore, since the loads are approximations to start with, it is conservative and close enough for practical purposes. Using $j = 0.892$,

$$A_s = \frac{M}{f_sjd} = \frac{244{,}000 \times 12}{20{,}000 \times 0.892 \times 30} = 5.47 \text{ in.}^2$$

Table 3 in the Appendix shows that nine No. 7, seven No. 8, six No. 9, five No. 10, or four No. 11 rods may be used. Next, considering Table 8 in the Appendix, and assuming ¾-in. aggregate, seven No. 8 rods with five in the bottom row and two in the second row 3 in. above it gives a satisfactory spacing and arrangement of the rods, with $A_s = 5.53$ in.^{2}.

Checking this beam, if considered necessary, and assuming that the rods are concentrated at their center of gravity (⅞ in. above the bottom row), the following are found:

The minimum depth $D = 30$ in. for d, + ⅞ in. for the distance to the centers of the bottom rods, + ½ in. for stirrups (to be discussed later), + say 2½ in. for cover = 33⅞ in. The assumed D of 34 in. is therefore satisfactory.

$$p = \frac{5.53}{20 \times 30} = 0.0092$$

$$k = \sqrt{2 \times 0.0092 \times 8 + (0.0092 \times 8)^2} - 0.0092 \times 8 = 0.309$$

(Check this with Fig. 10 of the Appendix.)

$$j = 1 - 0.103 = 0.897$$

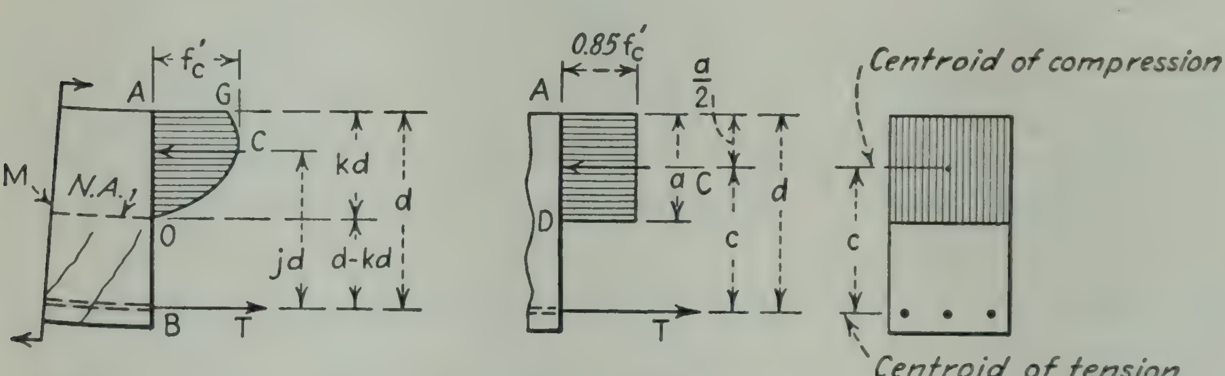
$$A_s = \frac{244{,}000 \times 12}{20{,}000 \times 0.897 \times 30} = 5.43 \text{ in.}^2 \text{ required}$$

$$f_c = \frac{2 \times 244{,}000 \times 12}{0.309 \times 0.897 \times 20 \times 30^2} = 1{,}170 \text{ psi}$$

2-6. Resistance to bending, ultimate-load theory. Assume a portion of a beam that is bent and cracked as pictured in Fig. 2-8(a). It is loaded to the point of imminent failure. If it is a balanced design under this condition, the concrete will be stressed to what will be called its ultimate compressive strength, and the steel will be stressed to its yield point in tension. If the beam is underreinforced, the steel will reach its yield point before the concrete is stressed to its ultimate value; if overreinforced, the concrete will reach its maximum resistance first.

For convenience, the symbols used are kept the same as those shown in explaining the straight-line theory wherever they are applicable.

Figure 2-8(a) shows the assumed shape of the pressure diagram for the concrete. Whitney has proposed the substitution of a rectangular "stress block" as shown in Sketch (b). This is chosen so as to result in the same total compressive force C, or sufficiently close thereto. This method simplifies the computations somewhat. It is not intended to mean that the actual compressive stresses are constant from the compression edge to the neutral axis, but it is a convenience in making computations. Actually any variation of stresses near the neutral axis will not cause much effect upon the resisting moment because of their short lever arms from the axis. The use of $0.85f'_c$, where f'_c is determined by test cylinders, is seemingly a conservative value to use as the maximum assumed compressive stress.



(a) (b) (c)

FIG. 2-8.

Other shapes and other values have been proposed for the compressive-stress block, but they do not change the general method of analysis. The slight changes in mathematical results may seem to be more accurate if the shape of Fig. 2-8(a) is used, but one may well doubt if the many uncertainties involved justify this. Whatever inaccuracy there may be in the assumptions is reduced in its effect when the computed ultimate bending resistance is divided by the safety factor to be used for working conditions. Whitney's method will be used here. Other methods may differ in detail, but Whitney's procedures seem to illustrate the basic principles very well.

The ultimate-load theory seems to produce results that agree fairly well with those of many tests. In fact, certain empirical values of factors have been proposed to make this so. However, most of the tests have been made upon relatively short deep members. One is reluctant to assume that the results found from such small specimens can be extrapolated to large-scale members used under the conditions that are customary in practice. Nevertheless, such tests provide very useful

evidence. Coefficients determined from their results can be modified in the future if this is found to be desirable.

The flexural resistance of a beam at its ultimate load will vary with the properties of the materials. Referring to Fig. 2-8(a) again, Whitney found that, for a strain $\epsilon = 0.003$ at the compression edge and $\epsilon' = 0.00133$ at the steel corresponding to $f_{yp} = 40{,}000$ psi, $k = \epsilon/(\epsilon + \epsilon') = 0.692$ and that the area of the pressure diagram equaled $0.803\epsilon f'_c$ per unit width. Its center of gravity was $0.571kd$ from the neutral axis. The total compressive force $C = 0.803f'_c kd = 0.556f'_c d$. The lever arm jd between the forces C and T is $jd = d - kd(1 - 0.571) = 0.703d$. Therefore, the resisting moment for a beam of unit width is

$$M = 0.556f'_c d \times 0.703d = 0.391f'_c d^2$$

Thus the location of the neutral axis, the magnitude of the compression force, the lever arm between the resultant compression and the tension, and the resisting moment are much different than they were for the straight-line theory. In other words, when the extreme compression edge begins to get stressed highly, plastic redistribution of stress occurs, but the total compressive resistance and the resisting moment continue to increase to some maximum so that the beam really can support more bending moment with the same safety factor than the straight-line theory would indicate.

From such investigations as the preceding one, Whitney concluded that the actual area of the probable compression diagram might be replaced by a fictious rectangular one as shown in Fig. 2-8(b), for which the center of gravity is made to correspond closely with that of the diagram of Sketch (a). The uniform ordinate is assumed to be $0.85f'_c$. The depth AD in Sketch (b) is called a to emphasize that point D is not the real neutral axis. The lever arm of the resisting couple is called c.

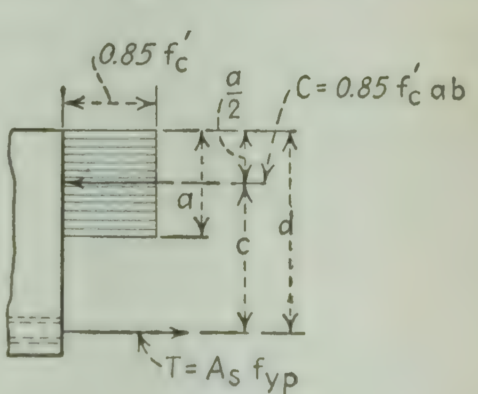


FIG. 2-9.

Figure 2-9 is used as a reference for a case where the flexural strength is controlled by the steel stress f_{yp}. Let $p = A_s/bd$ and $m = f_{yp}/0.85f'_c$. It should be remembered that the total compression C must equal the total tension T. Keeping $0.85f'_c$ constant for a given concrete,

$$a = \frac{C}{0.85f'_c b} = \frac{T}{0.85f'_c b} = \frac{A_s f_{yp}}{0.85f'_c b} = \frac{A_s m}{b} \tag{2-16}$$

$$\frac{a}{d} = \frac{A_s m}{bd} = pm \tag{2-17}$$

$$c = d - \frac{a}{2} = d - \frac{A_s m}{2b} \tag{2-18}$$

$$\frac{c}{d} = 1 - \frac{pm}{2} \tag{2-19}$$

$$M_s = cT = cA_s f_{yp} = A_s f_{yp}\left(d - \frac{A_s m}{2b}\right) \tag{2-20}$$

Substituting $A_s = pbd$ in Eq. (2-20), and dividing by bd^2, gives

$$\frac{M_s}{bd^2} = pf_{yp}\left(1 - \frac{pm}{2}\right) \tag{2-21}$$

Similarly, in terms of the concrete,

$$M_c = C\left(d - \frac{a}{2}\right) = 0.85f'_c ab\left(d - \frac{a}{2}\right) \tag{2-22}$$

From this,

$$\frac{M_c}{0.85f'_c bd^2} = \frac{a}{d} - \frac{a^2}{2d^2}$$

or

$$\frac{a}{d} = 1 - \sqrt{1 - \frac{2.35M_c}{f'_c bd^2}} \tag{2-23}$$

using the negative sign of the radical. Then, from $c = d - a/2$,

$$\frac{c}{d} = 1 - \frac{a}{2d} = 1 - \frac{1}{2}\left(1 - \sqrt{1 - \frac{2.35M_c}{f'_c bd^2}}\right)$$

or

$$\frac{c}{d} = \frac{1}{2}\left(1 + \sqrt{1 - \frac{2.35M_c}{f'_c bd^2}}\right) \tag{2-24}$$

Whitney's studies of and comparisons with the results of many tests indicated that, for a beam of balanced design, it is sufficient to assume that $a/d = 0.537$ and $c/d = 0.732$. Then, approximately,

$$\frac{M_c}{bd^2} = \frac{f'_c}{3} \quad \text{or} \quad M_c = \frac{f'_c bd^2}{3} \tag{2-25}$$

for concretes having a 28-day strength of 2,500 psi and over. Its use is conservative for weaker ones.

In general,

$$A_s f_s = \frac{M}{c} \quad \text{or} \quad A_s = \frac{M}{f_s(d - a/2)}$$

For critical conditions determined by the steel strength,

$$A_s = \frac{M}{f_{yp}[d - (a/2)]} \tag{2-26}$$

The critical percentage of steel needed to develop the full strength of the concrete is, from Eq. (2-17),

$$\frac{a}{d} = p_0 m = p_0 \frac{f_{yp}}{0.85 f'_c} = 0.537$$

or

$$p_0 = 0.456 \frac{f'_c}{f_{yp}} \tag{2-27}$$

However, this yields a much larger value than is customary with the straight-line theory. On the other hand, it is not always practicable to pack so much steel in a beam.

The use of more than the necessary amount of steel is wasteful except to minimize cracking and deflection. For analysis in such cases, Eq. (2-25) is good enough.

In using the ultimate-load method, the safety factor may be applied through what is called the *load factor*. Since the computations give the ultimate load for the beam, the working load must be smaller. Much judgment is required in determining the proper magnitude for this load factor.

Since the dead load is usually determinable with considerable accuracy, the load factor need only provide for the necessary reserve strength one wishes to have in the materials. Some persons specify a load factor of only 1.2 for dead loads. The author, however, suggests that more reserve be allowed for ordinary structures. When the dead load is equal to or greater than the live load, a load factor of 1.5 for dead load may be more advisable.

For live loads, a load factor of 2 is reasonable because the future use of a structure and the loads that may come upon it are so uncertain. When impact loads are probable, one should be even more conservative. If repetition of loads will be great, and if repeated reversal of stress is likely, there may be danger of fatigue. If so, the member may be considered to be about 60 to 70 per cent as strong as its computed ultimate strength indicates.

Remember that, with the ultimate-load method, one is not interested in the actual stresses in a beam at working loads. He is, however, very much interested in what reserve strength is available between the working conditions and failure of the beam.

As a comparison, rework Example 2-2 by means of the ultimate-strength theory. The following values are to be used: $b = 15$ in., $d = 28$ in., $A_s = 3.16$ in.^{2}, $p = 0.0075$, $f'_c = 2{,}500$ psi, $f_{yp} = 36{,}000$ psi, safety factor for concrete $= 2{,}500/900 = 2.75$, and safety factor for steel $= 36{,}000/18{,}000 = 2$.

From Eq. (2-27),

$$p_0 = 0.456 \times \frac{2{,}500}{36{,}000} = 0.032$$

This shows that, since $p = 0.0075$, the beam is underreinforced and the steel controls the design. Using

$$m = \frac{f_{yp}}{0.85f'_c} = \frac{36{,}000}{0.85 \times 2{,}500} = 17$$

in Eq. (2-20),

$$M_s = A_s f_{yp}\left(d - \frac{A_s m}{2b}\right) = 3.16 \times 36{,}000\left(28 - \frac{3.16 \times 17}{2 \times 15}\right)$$

$$M_s = 3{,}000{,}000 \text{ in.-lb (approx)}$$

Using the safety factor for the steel as 2, the safe working moment would be

$$3{,}000{,}000/2 = 1{,}500{,}000 \text{ in.-lb}$$

This compares with the 1,410,000 in.-lb found in Example 2-2.

Other computations also indicate that, when a beam is underreinforced, both the straight-line and ultimate-load theories give about the same results when the same safety factors or load factors are used. When the concrete controls, they may differ considerably, the members designed by ultimate-strength methods being permissibly smaller. However, lighter sections may not be desired because of deflections.

Example 2-5. Design a continuous slab to support a live load of 300 psf. Assume the following: $L = 12$ ft, $f'_c = 3{,}000$ psi, $f_{yp} = 40{,}000$ psi, load factor for dead load (D.L.) = 1.5, load factor for live load (L.L.) = 2, M at end $= wL^2/12$, M at center $= wL^2/16$, size of reinforcement = No. 5.

Assume a slab 6 in. deep with $d = 4.75$ in., D.L. = 75 psf. Use a 1-ft strip.

D.L.: $75 \times 1.5 = 113$

L.L.: $300 \times 2 = 600$

Design load $w = 713$ psf

$$\text{End } M = -\frac{713 \times 12^2 \times 12}{12} = -103{,}000 \text{ in.-lb}$$

$$\text{Center } M = +\frac{713 \times 12^2 \times 12}{16} = +77{,}000 \text{ in.-lb}$$

For balanced design at end, use Eq. (2-25), with $b = 12$ in.

$$M = f'_c bd^2/3 \qquad \text{or} \qquad 103{,}000 = \frac{3{,}000 \times 12d^2}{3}$$

$$d^2 = 8.6 \text{ in.} \qquad \text{or} \qquad d = 3 \text{ in. (approx)}$$

This is too thin anyway. A minimum depth for adequate stiffness would be

$$L/3 \text{ in.} = 4 \text{ in.}$$

Use the 6-in. slab.

Since the depth exceeds the minimum, the slab will be underreinforced. From Eq. (2-23),

$$\frac{a}{d} = 1 - \sqrt{1 - \frac{2.35M}{f'_c bd^2}}$$

$$\frac{a}{4.75} = 1 - \sqrt{1 - \frac{2.35 \times 103{,}000}{3{,}000 \times 12 \times (4.75)^2}} = 0.16$$

$$a = 4.75 \times 0.16 = 0.76 \text{ in.}$$

From Eq. (2-26),

$$A_s = \frac{M}{f_{yp}[d - (a/2)]} = \frac{103{,}000}{40{,}000[4.75 - (0.76/2)]} = 0.59 \text{ in.}^2$$

Use the No. 5 bars at 6 in. c.c. ($A_s = 0.6$) in the top at the end. For the center,

$$\frac{a}{d} = 1 - \sqrt{1 - \frac{2.35 \times 77{,}000}{3{,}000 \times 12 \times (4.75)^2}} = 0.12$$

$$a = 4.75 \times 0.12 = 0.57 \text{ in.}$$

$$A_s = \frac{77{,}000}{40{,}000[4.75 - (0.57/2)]} = 0.43 \text{ in.}^2$$

Use No. 5 bars at 8 in. c.c. in the bottom.

2-7. The transformed-section method. This is a method of analysis that is based upon elasticity of the materials and upon the straight-line theory.

The formulas of the preceding articles do not theoretically consider any reinforcement that is in the compression zone of the beam. Neither are they satisfactory for members of irregular shape or with rods distributed over the general cross section of the beam. Formulas might be developed to cover such special cases, but they are generally too cumbersome for satisfactory use. However, for much design work, it is sufficient to use the previously illustrated formulas to determine the area of tensile reinforcement and to test the strength of such reinforcement. By necessity, much use of judgment is involved in the application of the formulas.

Therefore, it is desirable to have a general, flexible, and standard procedure for use in design offices that can be used for all problems, and to have it so made that any man can check the computations and obtain the same results as any other man. Furthermore, in important work, the computations are to be a matter of record and open to inspection. In case of any difficulty in the future, they may even become evidence in court. On this account, each designer should be able to use this method of analysis when it is wise to do so even though much of his work in practice may be done by more easily performed methods.

The methods of analysis and design that have been explained previously have treated the concrete and steel as two separate materials which act together in the beam. It has also been stated that, if concrete and steel are deformed equally, the unit stress in the steel at working loads may be assumed to be practically n times as great as that in the concrete. This relationship can be utilized to good advantage because 1 in.^{2} of steel may be considered equivalent to n in.^{2} of concrete, as far as its resistance to deformation is concerned. Therefore, if all the rods in a beam are assumed to be replaced in the cross section by the equivalent square inches of concrete in the same location with regard to the neutral axis, an imaginary beam is obtained in which the steel is said to be "transformed" into concrete. Thus, for any beam of given shape, dimensions, and make-up, there is a definite substitute beam of homo-

geneous material which can be used in its stead for the purpose of calculation. Such a substitute beam will have a definite location for its neutral axis, which is at the center of gravity of the section; it will also have a definite moment of inertia. Therefore, these values can be substituted in the equation $M = sI/c$ in order to find the resisting moment of the beam.

Figure 2-10(a) shows the cross section of a reinforced-concrete beam. The substitute, or transformed, beam is pictured in Fig. 2-10(b). The neutral axis of this "transformed-concrete" beam is represented by the line O-O.

The location of this axis is found by utilizing the fact that, for a beam of homogeneous material but of any shape, the static moment of the area of the cross section about the neutral axis is zero. Then the area above the neutral axis times the lever arm to its own center of gravity must equal the area below the neutral axis times the lever arm to its particular center of gravity.

Furthermore, to find the moment of inertia of the transformed-concrete beam about this neutral axis—called I_c—it is merely necessary to make the usual calculation for $\Sigma x^2(\Delta A)$ about the line O-O. Then the section modulus of this transformed beam will naturally have two values, *viz.*, I_c/kd for the top and $I_c/(d - kd)$ for the bottom. With these values, the compressive stress in the concrete is simply

$$f_c = M \div \frac{I_c}{kd} \tag{2-28}$$

whereas the tensile stress in the steel is

$$f_s = n\left(M \div \frac{I_c}{d - kd}\right) \tag{2-29}$$

Although the method of finding f_c and f_s that is indicated above is satisfactory, it is sometimes hard to visualize what is going on and to avoid errors resulting from improper use of n or of the distances to the extreme fibers. On this account it is an advantage to have the section modulus for use in determining the compressive stress in the concrete differentiated from that utilized in finding the tensile stress in the steel. Therefore, call the former S_c and the latter S_s. Then

$$S_c = \frac{I_c}{kd} \tag{2-30}$$

and

$$S_s = \frac{I_c}{n(d - kd)} \tag{2-31}$$

Equation (2-31) is found from Eq. (2-29) as follows:

$$f_s = \frac{nM}{I_c/(d - kd)} = \frac{M}{I_c/[n(d - kd)]} = \frac{M}{S_s}$$

Therefore, by finding S_c and S_s immediately after calculating I_c, the designer can realize thereafter that the one with the subscript c goes with the concrete and the one having the subscript s is for the steel-stress calculations. However, Eqs. (2-28) and (2-29) will also be used in later problems. Curves for use in practical design are given in Figs. 4 and 5 of the Appendix. When working to any given specification as to f'_c, and therefore n, tables or curves giving section moduli can be prepared for a variety of beams so that these properties can be used in designing without repeated computations.

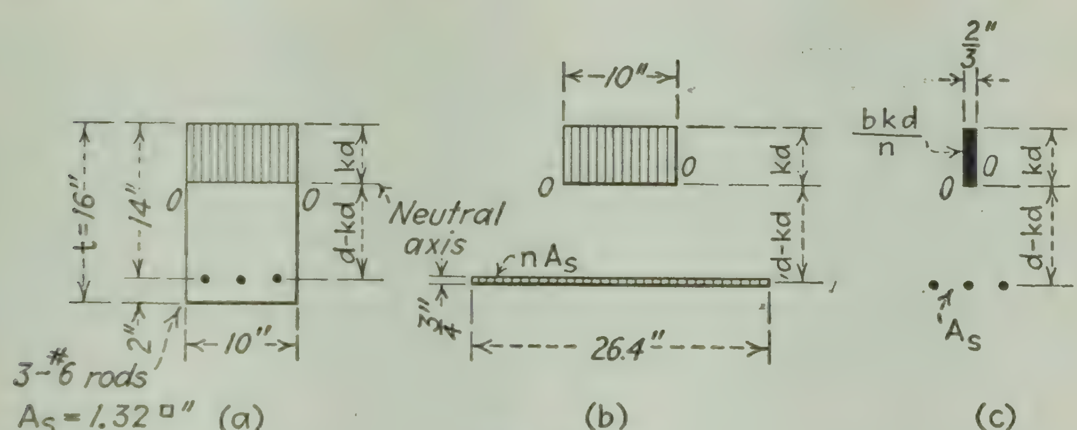


FIG. 2-10.

In general, the transformed-section method is simple and understandable. The general procedure is to assume a section for the beam and then to analyze it. The transformed-section method will be explained further by direct application to a problem.

Example 2-6. Let Fig. 2-10(a) represent the cross section of a beam with dimensions and make-up as shown. Assume that this beam carries a bending moment of 300,000 in.-lb and that the materials are such that n equals 15. Compute f_c and f_s.

The area of the rods is $3 \times 0.44 = 1.32$ in.². They carry a total tensile force of A_sf_s which, with the compressive forces, is needed to hold the beam in equilibrium. Since the steel is n times as effective as the same area of concrete, the rods are replaced by an area of concrete that will be $nA_s = 15 \times 1.32 = 19.8$ in.². If this area is arbitrarily assumed to have a depth equal to the diameter of the rods, its length will be $19.8 \div 0.75 = 26.4$ in., as shown in Fig. 2-10(b), which pictures the transformed section of the beam of Fig. 2-10(a) in terms of concrete.

On the other hand, replacing the concrete of the area in compression above the neutral axis O-O by steel will produce an equivalent area of steel in compression equal to $bkd \div n = 10 \times kd \div 15 = 0.667kd$. Since the real neutral axis O-O does not shift because of this juggling of figures, the distance kd must remain unchanged, and the width of the substituted steel area must become 0.667 in. as shown in Fig. 2-10(c). This last figure may be called a "transformed section" of the beam of Fig. 2-10(a) in

terms of steel. This procedure is not necessary but is given here merely to show the student that it is possible to substitute an equivalent steel beam instead of the concrete one. Hereafter, the substitute steel beam will be discarded in order to adhere to one standard system—the transformed section in terms of concrete.

The value of the unknown distance kd is found by taking moments of the equivalent areas of Fig. 2-10(b) about the neutral axis O-O, where the sum of these moments is zero.

Solving for kd gives

$$\frac{bkd(kd)}{2} = nA_s(d - kd)$$

$$\frac{10(kd)^2}{2} = 15 \times 1.32(14 - kd)$$

$$kd = 5.72 \text{ in.} \qquad \text{and} \qquad d - kd = 14 - 5.72 = 8.28 \text{ in.}$$

$$I_c = b\frac{(kd)^3}{3} + nA_s(d - kd)^2 = \frac{10(kd)^3}{3} + (15 \times 1.32)(14 - kd)^2 = 1{,}984 \text{ in.}^4$$

Notice that I of the transformed section of the rods about their own axes is neglected.

When there are two or more rows of reinforcement in tension or compression, it is theoretically desirable to consider each row separately in the computations, using the lever arms from the neutral axis to each row individually. The effective depth d should be measured from the compression edge of the beam to the row of bars that is farthest from that edge. However, for much ordinary construction, it is sufficiently accurate to assume that the tensile steel is concentrated at its center of gravity, and to measure d from the compression edge to that center of gravity. Furthermore, multiple rows of compressive reinforcement are seldom used.

From Eq. (2-30),

$$S_c = 1{,}984 \div 5.72 = 347 \text{ in.}^3$$

From the ordinary flexure formula, $M = sI/c$, with $s = f_c$, the value of f_c is found. Thus,

$$f_c = \frac{M}{S_c} = \frac{300{,}000}{347} = 865 \text{ psi}$$

The section modulus to use in finding the stress in the rods, using Eq. (2-31), is

$$S_s = \frac{I_c}{n(d - kd)} = \frac{1{,}984}{15 \times 8.28} = 15.9 \text{ in.}^3$$

$$f_s = \frac{M}{S_s} = \frac{300{,}000}{15.9} = 18{,}900 \text{ psi}$$

It is important to notice that, if a structure is designed upon the basis of a concrete with a certain strength—and hence with a certain value of n—and if a stronger concrete is used later on in the real building of the structure, the computed stresses in the concrete will exceed the previously calculated values of f_c. However, this excess will not be greater than the proportional increase in the ultimate strength of the concrete. On the other hand, the steel will have less than its calculated stress. No structural harm will result from the substitution. If the design is made upon the basis of high-strength concrete but poor workmanship or materials cause the concrete to be much weaker, the result is on the side of danger.

Now analyze a reinforced-concrete beam which supposedly does not crack. For instance, take the beam of Fig. 2-10, and analyze it on the basis that the concrete can withstand tension. Assume $n = 15$.

The principles of the transformed-section method can again be used in solving this last problem. The concrete above the neutral axis will be in compression as usual, but all the concrete below that axis—and the rods, too—will be in tension. Of course, the neutral axis will be in a new position, which is found as follows:

$$\frac{b(kd)^2}{2} = \frac{b(t - kd)^2}{2} + (n - 1)A_s(d - kd)$$

$$\frac{10(kd)^2}{2} = \frac{10(16 - kd)^2}{2} + 14 \times 1.32(14 - kd)$$

$$kd = 8.6 \text{ in.} \qquad \text{and} \qquad d + 2 - kd = 16 - kd = 7.4 \text{ in.}$$

Continuing the solution, using the same principles as those of the previous problems,

$$I_c = \frac{b(kd)^3}{3} + \frac{b(d + 2 - kd)^3}{3} + (n - 1)A_s(d - kd)^2$$

$$I_c = \frac{10 \times 8.6^3}{3} + \frac{10 \times 7.4^3}{3} + 14 \times 1.32 \times 5.4^2 = 4{,}010 \text{ in.}^4$$

$$S_c \text{ for top} = \frac{I_c}{kd} = \frac{4{,}010}{8.6} = 466 \text{ in.}^3$$

$$S_c' \text{ for bottom} = \frac{I_c}{d + 2 - kd} = \frac{4{,}010}{7.4} = 542 \text{ in.}^3$$

$$f_c \text{ at top} = \frac{M}{S_c} = \frac{300{,}000}{466} = 643 \text{ psi}$$

$$f_c \text{ at bottom} = \frac{M}{S_c'} = \frac{300{,}000}{542} = 553 \text{ psi}$$

Of course, if the steel has the same deformation as the concrete which is at the same distance from the neutral axis, its unit stress is n times that of the concrete. The stress in the latter at the location of the rods is

$$f_x : f_c \text{ at bottom} :: (d - kd) : (t - kd)$$

$$f_x : 553 :: 5.4 : 7.4, \text{ or } f_x = 404 \text{ psi}$$

$$f_s = nf_x = 15 \times 404 = 6{,}050 \text{ psi}$$

An analysis of the results of this last problem shows that, if the concrete does not crack, the compressive stress in it will be 643 psi instead of 865 psi, but the tensile stress in the concrete will be 553 psi. This tension is very high because, using a concrete with an ultimate compressive strength of 2,000 psi, the limit of its tensile strength will probably be about $0.10 \times f_c' = 0.10 \times 2{,}000 = 200$ psi. Therefore, the concrete must surely crack under a stress that is nearly three times this value. The action cannot be that of a homogeneous solid beam, but, before the member will fail, it will behave in the general manner that has been assumed for such beams; that is, it will crack; the concrete will resist the compression; and the steel will withstand all, or nearly all, of the tension.

The tensile strength of the concrete is too unreliable to permit the designer to depend upon it, even when the calculations may appear to warrant it. Furthermore, a construction joint or an unintentional delay

in placing adjacent concrete until after the initial set has occurred may result in a member that cannot withstand tension effectively. Therefore, because life and property are frequently at stake, reliance upon the tensile strength of the concrete should be avoided.

2-8. Beams with compressive reinforcement, straight-line theory. In the case of continuous rectangular beams and T beams, it is general practice to have some of the steel run through the bottom of the beam at the support where the lower portion of the beam is in compression. Sometimes it is necessary or advisable to place rods in a beam so

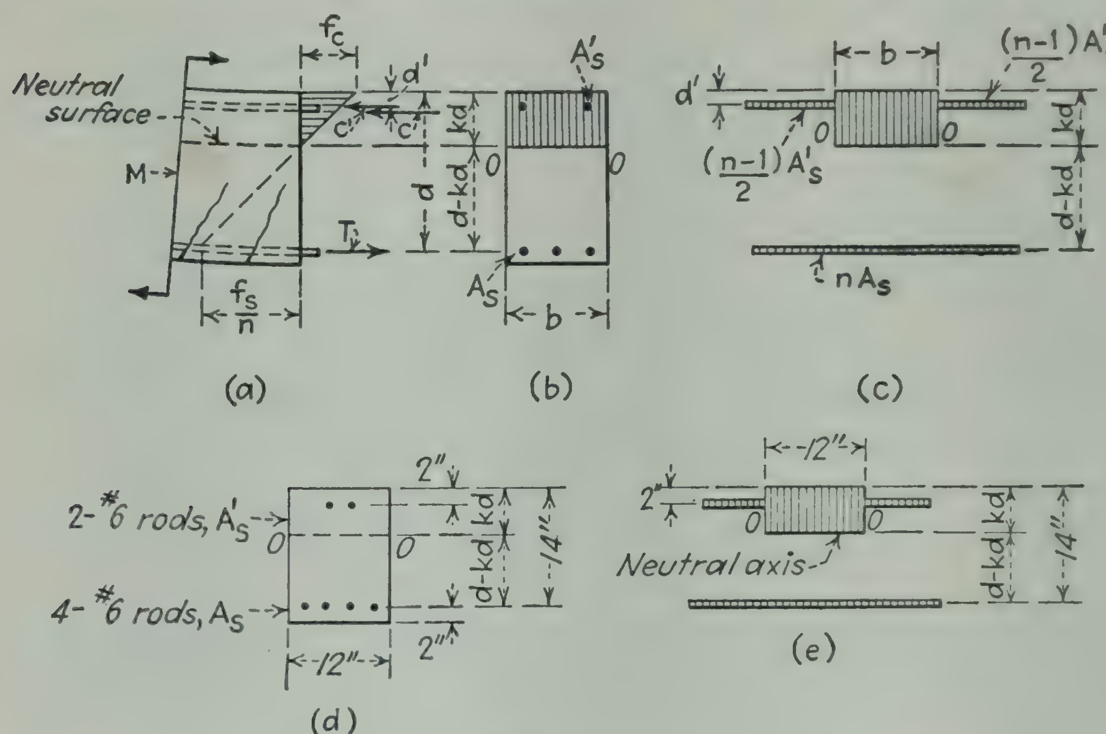


FIG. 2-11.

as to reinforce its compressive strength, as well as to have other rods which carry tension. For cases like these, it is necessary to realize that both the steel and the concrete will work together in withstanding the compression. The fundamental principles set forth in Arts. 2-3 to 2-6, inclusive, apply here, but the formulas do not consider the presence of the compressive steel. Therefore, the transformed-section method will be used in solving such problems when adhering to the straight-line theory. However, the use of the formulas for rectangular beams may be helpful in determining approximate sizes for more rigorous analysis.

Let Figs. 2-11(*a*) and (*b*) represent a piece of a beam that has compressive reinforcement and is subjected to bending. The length is assumed to be infinitesimal.

Let A'_s = the area of the compressive reinforcement in square inches and f'_s = its unit stress. It is apparent that the internal resisting couple is made up of the tension T and the resultant of the two compressive

forces C and C', with the proper lever arm. Figure 2-11(c) shows the imaginary transformed cross section of this beam with the steel replaced by its equivalent amount of concrete. The area of the concrete that is displaced by the rods should be deducted from the cross section of the concrete. This is equivalent to saying that the area that is substituted for the compressive steel outside the solid rectangle bkd should be $(n - 1)A'_s$.

The 1951 revision of the Code actually permits the use of twice the area of any compressive reinforcement in beams when computing the transformed section, but the value of this increased steel times the computed stress in it must not exceed the allowable tensile strength of the bars. In other words, the area of concrete substituted for the compressive steel can be taken as $2n - 1$. This is admittedly empirical. It is an introduction of a portion of the ultimate-load theory. Some engineers prefer to be more conservative and to use $(n - 1)A'_s$ as the imaginary concrete area to substitute for the compressive reinforcement. On this account, this has been adhered to in the preparation of Figs. 6 to 9, inclusive, in the Appendix. However, both methods will be illustrated.

The Code rightly requires that compressive reinforcement in beams or girders be held in by ties or stirrups not less than ¼ in. in diameter, spaced not farther apart than 16 bar diameters or 48 stirrup diameters.

The values of kd, I_c, S_c, and S_s can be found as before and as illustrated in the following problems. Figures 6 to 9 in the Appendix are also useful for design and checking purposes.

Example 2-7. Compute the safe resisting moment for the beam shown in Fig. 2-11(d), assuming that n equals 10 and that the allowable f_s and f_c equal 18,000 and 1,000 psi, respectively. At first, assume $(n - 1)A'_s$ as a substitute area for the bars that are in compression. Then recompute the problem using $(2n - 1)A'_s$ for this area.

A. First Solution.

$$A_s = 4 \times 0.44 = 1.76 \text{ in.}^2$$
$$A'_s = 2 \times 0.44 = 0.88 \text{ in.}^2$$
$$nA_s = 17.6 \qquad \text{and} \qquad (n - 1)A'_s = 7.92$$

Take moments about the neutral axis, and solve for kd.

$$\frac{12(kd)^2}{2} + 7.92(kd - 2) = 17.6(14 - kd)$$
$$kd = 4.8 \text{ in.} \qquad \text{and} \qquad (d - kd) = 9.2 \text{ in.}$$

Taking the moments of inertia about the neutral axis,

$$I_c = 12 \times \frac{4.8^3}{3} + 7.92(4.8 - 2)^2 + 17.6(14 - 4.8)^2 = 1{,}994 \text{ in.}^4$$
$$S_c = \frac{I_c}{kd} = \frac{1{,}994}{4.8} = 415 \text{ in.}^3$$
$$S_s = \frac{I_c}{n(d - kd)} = \frac{1{,}994}{10 \times 9.2} = 21.7 \text{ in.}^3$$

Since the moment equals the allowable unit stress times the section modulus,

$$M_c = 1{,}000 \times 415 = 415{,}000 \text{ in.-lb}$$
$$M_s = 18{,}000 \times 21.7 = 391{,}000 \text{ in.-lb}$$

Therefore, M_s controls the design. The compressive stress in the top rods is

$$f_s' = 18{,}000 \times \frac{2.8}{9.2} = 5{,}480 \text{ psi}$$

This last equation for f_s' is based upon the fundamental assumption that the stresses vary directly as their distances from the neutral axis. The distance of the compressive reinforcement above *O-O* is 2.8 in.

The magnitude of j for this beam is not quite equal to $1 - (k/3)$ because the stress in the top rods shifts the centroid of compression slightly away from the center of gravity of the triangular wedge that would apply for the concrete alone. However, it is safe to assume $j = 1 - (k/3)$ in most cases.

B. Second Solution. As before, $nA_s = 17.6$ in.² for the assumed tensile concrete. The assumed added area of concrete in compression on the basis of doubling the area of the compressive bars is found as follows: If the two bars in compression having an area of $2 \times 0.44 = 0.88$ in.² are assumed to be pulled out of the concrete, two holes will be left in the rectangular area bkd of Fig. 2-11(c). It requires an area of concrete A_s' to fill these up. Hence the excess area left over is

$$n \times 2A_s' - A_s' = (2n - 1)A_s' = (2 \times 10 - 1)0.88 = 16.72 \text{ in.}^2$$

Taking moments of the areas about the neutral axis,

$$\frac{12(kd)^2}{2} + 16.72(kd - 2) = 17.6(14 - kd)$$
$$kd = 4.54 \text{ in.} \qquad d - kd = 9.46 \text{ in.}$$

Taking the moments of inertia of the areas about the neutral axis,

$$I_c = 12 \times \frac{4.54^3}{3} + 16.72(4.54 - 2)^2 + 17.6(14 - 4.54)^2$$
$$I_c = 2{,}058 \text{ in.}^4$$

Then

$$S_c = \frac{I_c}{kd} = \frac{2{,}058}{4.54} = 453 \text{ in.}^3$$
$$S_s = \frac{I_c}{n(d - kd)} = \frac{2{,}058}{10 \times 9.46} = 21.8 \text{ in.}^3$$
$$M_c = 1{,}000 \times 453 = 453{,}000 \text{ in.-lb}$$
$$M_s = 18{,}000 \times 21.8 = 392{,}000 \text{ in.-lb}$$

A comparison with the previous solution of this problem shows that the computed safe bending resistance of the beam based upon the strength of the compression side is considerably larger, as should be expected. However, the difference in the computed resistance based upon the strength of the steel has changed little. This, too, should be expected unless the compressive reinforcement is very large and the beam is quite deep.

Figures 6 to 9, inclusive, in the Appendix are for use in checking one's calculations roughly. For example, refer to Example 2-7, and check it by means of these curves. They give values of kd and S_c on the basis of a

section of beam 1 in. wide. For kd, this value is the same for any width, but the actual section modulus for the whole beam in terms of concrete is the data from the diagram times the width b. The section modulus in terms of steel is S_c times $kd/n(d - kd)$.

The checking procedure is as follows:

1. $\dfrac{(n-1)A'_s}{b} = \dfrac{9 \times 2 \times 0.44}{12} = 0.66$

2. $\dfrac{nA_s}{b} = \dfrac{10 \times 4 \times 0.44}{12} = 1.47$

3. Using the left-hand portion of Fig. 6 as the one nearest to $(n - 1)A'_s/b = 0.67$, with $d = 14$ in., $kd = 4.75$ in. (approx). Then $d - kd = 9.25$ in.

4. Using the right-hand portion of Fig. 6, $S_c/b = 33$ (approx), and

$$S_c = 33 \times 12 = 396 \text{ in.}^3$$

Then

$$S_s = \frac{396 \times 4.75}{10 \times 9.25} = 20.3 \text{ in.}^3$$

5. With the preceding values,

$$M_c = 1{,}000 \times 396 = 396{,}000 \text{ in.-lb}$$
$$M_s = 18{,}000 \times 20.3 = 366{,}000 \text{ in.-lb}$$

These are near enough to serve as a rough check.

If the diagrams of Figs. 6 to 9, inclusive, of the Appendix are plotted to a sufficiently large scale, they will be very helpful in office work. They will also be easier to read, and the results will be more accurate.

The preceding problem shows the analysis of an existing or assumed beam. In practical design work it is important to be able to determine tentative sections by approximate methods which yield results that will serve as fairly good trial sections. Obviously, it is tedious to make a guess and then have the analysis of the beam show that the guess was not even approximate. Therefore, the following procedures are recommended, being based upon the fundamental action of a beam with compressive reinforcement:

A. For design

1. Assume $k = 0.38$ and $j = 0.88$. Also assume the trial dimensions for b, d, and d', or use values of b and d based upon the use of Eqs. (2-5) and (2-6).

2. Solve for A_s as in the case of beams with tensile steel only. Then find f_c for the same case.

3. The value of f_c just found minus the allowable value gives the approximate amount of overload of stress in the concrete. Since the diagram of compressive stress is triangular, one-half of this overload

times the area under compression bkd gives the total excess force that the steel may be assumed to withstand.

4. The unit stress in the compressive steel f_s' can be approximated by dividing the allowable f_s by $(d - kd)$ and multiplying by $(kd - d')$. The total excess force divided by this value of f_s' gives the trial value of A_s'. (Then add perhaps 25 per cent to it.)

B. For analysis

1. Assume $k = 0.38$ and $j = 0.88$.
2. Solve for $f_s = M \div 0.88\, A_s d$.
3. The total compressive force C = the tensile force $T = f_s A_s$.
4. The compressive force in the steel $C_s = A_s' f_s(kd - d') \div (d - kd)$.
5. $C - C_s$ = total compressive force in the concrete = approximately $b(kd/2)f_c$. Solve for f_c. If this magnitude of f_c is more than the allowable value, more strength is needed; if it is too far below the allowable f_c, there is more strength than required, and the trial section is not the most economical.

These trial procedures are based upon an assumed value of kd.

Another useful approximate formula for trial design and analysis of such beams may be derived by assuming $k = 0.36$, $j = 0.88$, and $n = 10$. Also assume that the compressive reinforcement is $\frac{2}{3}kd$ from the neutral axis. Then the total compressive force

$$C = \frac{M}{0.88d} = \frac{bkdf_c}{2} + (n - 1)A_s'\frac{2}{3}f_c$$

$$\frac{M}{0.88d} = f_c(0.18bd + 6A_s') \tag{2-32}$$

The first trial method will now be applied to a particular problem.

Example 2-8. Assume that a simply supported beam must be limited to a width of 12 in. and a total depth of 18½ in. It must resist a bending moment of 800,000 in.-lb. Assume also that n equals 10, the cover over the rods equals 2½ in., and the allowable f_s and f_c equal 20,000 and 1,350 psi, respectively. Design the beam.

Using the trial method for design given under A,

$$kd = 0.38(18.5 - 2.5) = 6.1 \text{ in.} \qquad \text{and} \qquad (d - kd) = (16 - 6.1) = 9.9 \text{ in.}$$

$$A_s = \frac{M}{f_s jd} = 800{,}000 \div (20{,}000 \times 0.88 \times 16) = 2.84 \text{ in.}^2$$

$$f_c = \frac{2M}{kjbd^2} = 2 \times 800{,}000 \div (0.38 \times 0.88 \times 12 \times 16^2) = 1{,}560 \text{ psi}$$

Then

$$\text{Excess force} = \tfrac{1}{2}(1{,}560 - 1{,}350) \times 12 \times 6.1 = 7{,}680 \text{ lb}$$

$$\text{Approx } f_s' = 20{,}000(6.1 - 2.5) \div 9.9 = 7{,}270 \text{ psi}$$

$$\text{Approx } A_s' = \frac{7{,}680}{7{,}270} = 1.06 \text{ in.}^2$$

Try two No. 5 rods in the top and three No. 9 in the bottom. Using these, the student can analyze the trial section, finding $kd = 6.4$ in., and $d - kd = 9.6$ in.,

$I_c = 3{,}980$ in.^{4}, $S_c = 622$ in.^{3}, $S_s = 41.5$ in.^{3}, $f_c = 1{,}290$ psi, $f_s = 19{,}300$ psi, and $f_s' = 7{,}840$ psi. These show that the assumed rods are sufficient for the purpose.

Compressive reinforcement is often used for practical reasons rather than for the purpose of adding compressive strength to beams. It is probably most useful over the supports of continuous beams. Figure 3-11 pictures the rods in such a beam. Note the presence of one pair of rods running full length through the top, whereas another pair does likewise in the bottom. Incidentally, if the lap of the bottom rods above the column were made long enough, four rods could be counted in the section as compressive reinforcement at the support.

2-9. Beams with compressive reinforcement, ultimate-load theory. Let Fig. 2-12 picture a beam with compressive reinforcement. The rectangular-stress block is assumed as before, and its resultant magnitude and lever arm are retained although the bars are present. The full area of the compressive reinforcement is also used, although this means that the concrete area occupied by the bars has been used once and is now being used again as steel area. The total compression in the top bars will be called $A_s' f_{yp}$, its lever arm from the tensile steel, d_1. Notice that the yield-point stress is used in these bars, and it is assumed to have the same value as for the tensile steel.

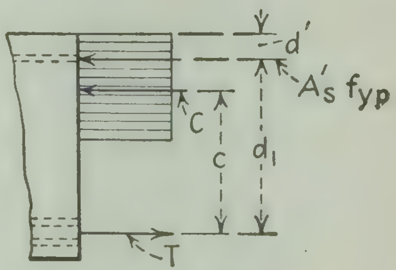


FIG. 2-12.

The resisting moment of the beam can be looked upon as the sum of two couples; *i.e.*, the concrete compression and the tensile steel as one couple, the compression bars and the tensile steel as the other. Obviously, the tensile force must equal the sum of the two compressive forces. Taking moments about the tensile steel, assuming the compressive resistance to be that of a balanced design,

$$M = 0.85abf_c'\left(d - \frac{a}{2}\right) + A_s' f_{yp} d_1 \tag{2-33}$$

Using Eq. (2-25) for the first term of this equation gives

$$M = \tfrac{1}{3}bd^2 f_c' + A_s' f_{yp} d_1 \tag{2-34}$$

For convenience one can assume that one part of the tensile steel is used to balance the stress in the compressive reinforcement whereas the other part is needed to balance the compressive stress in the concrete. Because of the relatively large strains in the concrete, the compressive steel A_s' can be assumed to be stressed to its yield point. The remainder of the compressive resistance must therefore be provided by the concrete. If all the strength of the concrete is not needed (the beam being

underreinforced), the compression in the concrete will be assumed to be as large as it has to be but no larger. Therefore, with c from Eq. (2-18) as the lever arm, Eq. (2-34) can be modified as follows: M = steel to balance concrete $\times\, c \times f_{yp}$ + compressive steel $\times\, d_1 \times f_{yp}$. Therefore,

$$M = (A_s - A'_s)\left[d - \frac{(A_s - A'_s)m}{2b}\right] f_{yp} + A'_s f_{yp} d_1 \qquad (2\text{-}35)$$

This is useful in analyzing real or assumed members.

If Eq. (2-35) is divided by bd^2, and if $p' = A'_s/bd$,

$$\frac{M}{bd^2} = (p - p')\left[1 - \frac{(p - p')m}{2}\right] f_{yp} \qquad (2\text{-}36)$$

This can be used if the percentages of reinforcement are known or determined in advance.

It seems that the simplest way to handle a problem of design is to assume a depth of beam d desired, then assume A'_s, and determine d_1. Deduct the moment $A'_s f_{yp}\, d_1$ of Eq. (2-35) from M. With the remaining moment M' and the chosen d, design the member as a beam without compressive reinforcement. To the A_s thus found, add A'_s to find the total tensile steel needed.

Example 2-9. Design a continuous rectangular beam to withstand a uniform D.L. of 1,200 plf and a L.L. of 3,000 plf plus a concentrated load of 10,000 lb at the center of the 25-ft span. Assume f'_c = 3,500 psi and f_{yp} = 40,000 psi. The beam is to be as shallow as it is feasible to make it. All bars are to be No. 9. Assume a load factor of 1.5 for D.L. and 2 for L.L.

From Fig. 1 in the Appendix, the maximum bending moment is found to be at the ends. This section is therefore critical.

$$\text{D.L.:}\quad M = -\frac{WL}{12} = -\frac{1{,}200 \times 25^2}{12} = -62{,}500 \text{ ft-lb}$$

$$\text{L.L.:}\quad M = -\frac{WL}{12} - \frac{W'L}{8} = -\frac{3{,}000 \times 25^2}{12} - \frac{10{,}000 \times 25}{8} = -187{,}200 \text{ ft-lb}$$

Moments for ultimate-load design:

$$\begin{array}{lrl} \text{D.L.:} & M = -62{,}500 \times 1.5 = & -93{,}800 \\ \text{L.L.:} & M = -187{,}200 \times 2 = & \underline{-374{,}400} \\ & \text{Total} = & -468{,}200 \text{ ft-lb or } 5{,}620{,}000 \text{ in.-lb} \end{array}$$

Try a depth d = 18 in. with d' = 2½ in. Assume two No. 9 bars in the bottom where the compression occurs, as shown in Fig. 2-13(a). Then A'_s = 2 in.^{2} and d_1 = 18 − 2.5 = 15.5 in. From the last term of Eq. (2-35), the moment value of the compressive reinforcement is

$$A'_s f_{yp} d_1 = 2 \times 40{,}000 \times 15.5 = 1{,}240{,}000 \text{ in.-lb}$$

The moment for the concrete is

$$M' = 5{,}620{,}000 - 1{,}240{,}000 = 4{,}380{,}000 \text{ in.-lb}$$

Using Eq. (2-25) for balanced design,

$$M' = \frac{f'_c bd^2}{3} = 4{,}380{,}000 \text{ in.-lb}$$

$$\frac{3{,}500b \times 18^2}{3} = 4{,}380{,}000 \qquad \text{and} \qquad b = 11.6 \text{ in.}$$

From Eq. (2-27), to balance the concrete,

$$p_0 = 0.456 \frac{f'_c}{f_{yp}} = \frac{0.456 \times 3{,}500}{40{,}000} = 0.04$$

Then A_s to balance the concrete is

$$A_s = p_0 bd = 0.04 \times 11.6 \times 18 = 8.36 \text{ in.}^2$$

Total tensile steel is

$$\Sigma A_s = A_s + A'_s = 8.36 + 2 = 10.36 \text{ in.}^2$$

This requires 11 No. 9 bars.

In Fig. 2-13(*b*), the width is increased to 12 in. because Table 8A of the Appendix shows that, even with ¾-in. aggregate, this width is desirable in order to place four

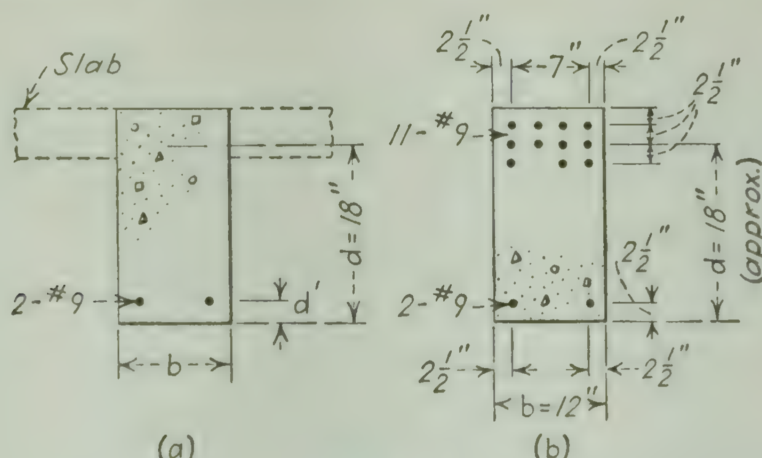


FIG. 2-13.

No. 9 bars in a row. The 11 bars might be arranged as shown in the drawing. Seven No. 11 bars could be used in two rows of three each and one in a third row under the middle bars. However, the author prefers to limit the reinforcement of such beams to two rows of steel if possible. This is in order to facilitate the placing of the reinforcement and the pouring of the concrete.

2-10. T beams. The use of simple concrete slabs of moderate depth and weight is generally limited to spans of 10 to 15 ft. Where it is desired to use concrete for long spans without excessive weight and material, a common type of construction is that shown in Fig. 2-14(*a*). It consists of a relatively thin slab with deep haunched portions or stems at intervals. Figure 2-14(*b*) gives an exaggerated picture of the action of the slab under vertical loads, whereas (*c*) shows the action of the stem if it is simply supported. Instead of considering the stem to be a rectangu-

lar beam which carries the load by itself, it is better and important to realize that all parts of the structure must act simultaneously. In general, the stem and the slab near it can be assumed to act as a unit, forming a "T beam" as shown in Fig. 2-15. The slab and the stem are to be poured monolithically, or they are to be bonded securely together.

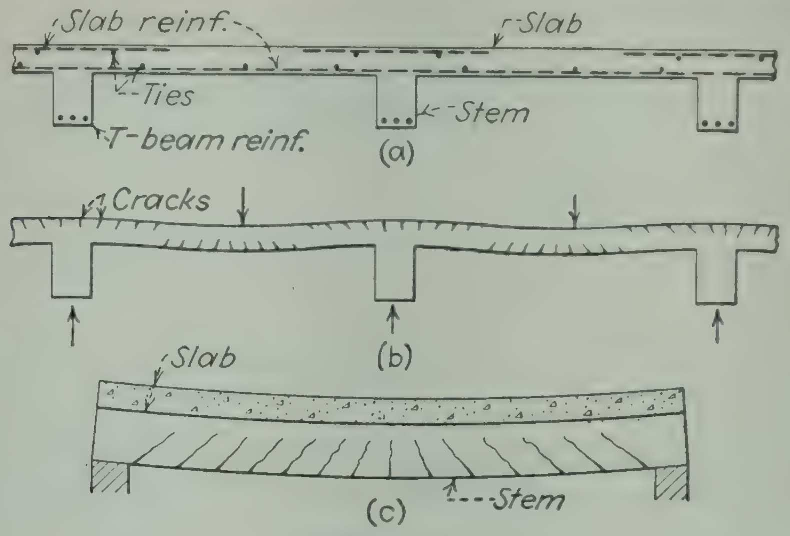


FIG. 2-14.

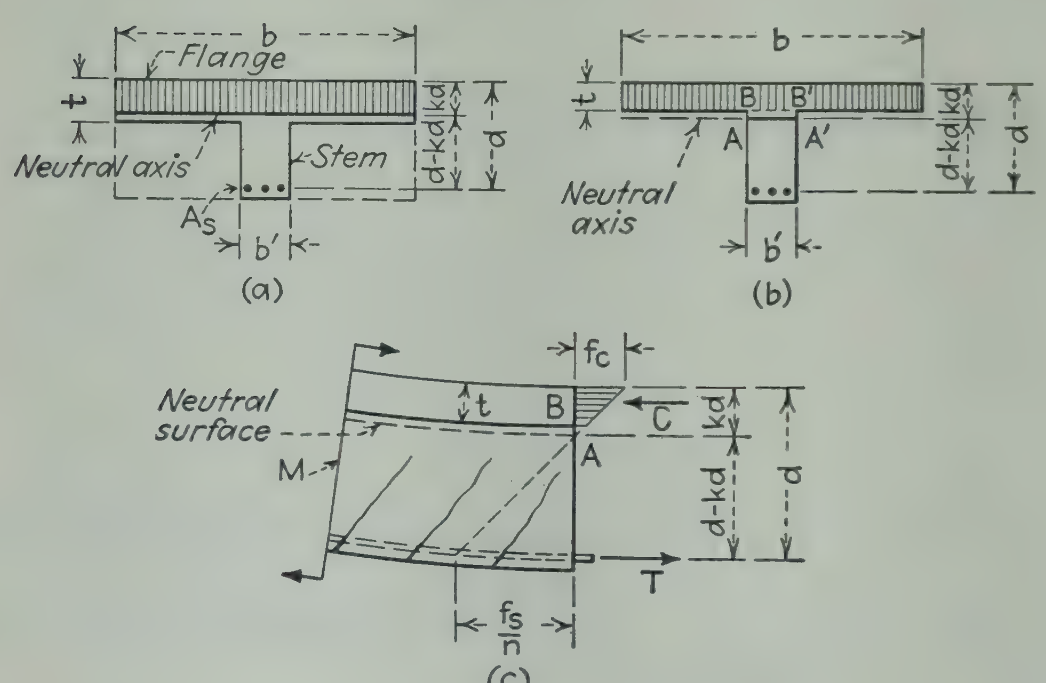


FIG. 2-15.

In this type of construction there are two general cases to consider. In the first, the neutral axis is located in the slab or flange section as shown in Fig. 2-15(*a*). The problem is then the same as for a rectangular beam of the size shown by the dotted lines, since direct tensile stress in the concrete below the neutral axis is neglected anyway. The second case is one in which the neutral axis lies in the stem as pictured in Fig. 2-15(*b*). The diagram representing the compressive unit stresses in the

flange according to the straight-line theory is a trapezoidal wedge instead of a triangular one. This is shown in Fig. 2-15(c). The pressure on the small portion of the top of the stem $ABB'A'$ which is subjected to compression is generally of little importance.

In ordinary practice, as now used, most concrete construction is limited by the strength of the steel except at certain points. For example, few rectangular beams are used alone; they are generally incorporated with floors to form T beams. However, the compression in the stems of continuous T beams where negative bending occurs over and near the columns is one critical point for the concrete. Common remedies for this are sideward flaring of the stem, deepening of the stem, addition of compressive reinforcement, or proportioning of the stem as required at the column and use of this section throughout. The first two cause expensive formwork.

At an interior support a continuous T beam is treated as a rectangular beam the width of which is b' because the flange concrete should not be relied upon to withstand tension in the top.

It is possible to derive formulas for k, j, M_c, M_s, etc., for T beams with tension in the bottom, but they are rather complicated. It is also possible to analyze them by means of the transformed-section method but, as shown in Fig. 2-15(a), there is so much area of concrete in compression compared with that of the tensile reinforcement that the beam is greatly underreinforced. The compressive stress in the concrete for most ordinary T beams may be only 300 to 600 psi, which is far below that allowable. Therefore, computation of f_c is seldom worth the time it takes. The tensile reinforcement is almost always the critical part of such beams when subjected to positive bending moments.

From Fig. 11 of the Appendix it is seen that k for ordinary T beams is very small, and that j varies from about 0.9 to 0.95. It is therefore sufficiently accurate and generally safe to compute M_s, f_s, and A_s for T beams by using $j = 0.9$ in Eq. (2-6).

The Code states that the maximum effective width of the flange on each side of the stem of a symmetrical T beam can be assumed to equal eight times the thickness of the flange or one-half the clear distance between adjacent stems. The total flange width must not exceed one-fourth the span of the beam. For beams with a flange on one side only the assumed effective width of slab beyond the stem shall not exceed six times the thickness of the slab or more than one-half the clear span of the slab to the next beam or more than one-twelfth the span of the beam. These specified rules are empirical and generally are of little interest to the designer. However, the analysis of such beams by the transformed-section method, with d measured to the extreme row of rods, will be given in the next example for the purpose of illustration

Example 2-10. The T beam shown in Fig. 2-16 is to carry a bending moment of 1,300,000 in.-lb. If n equals 12, determine f_s and f_c, assuming that the neutral axis lies below the flange, neglecting the portion of the stem that may be in compression, and using $d = 30$ in. and $b = 48$ in.

$$nA_s = 12 \times 1.32 = 15.8 \text{ for each set of rods}$$
$$48 \times 4(kd - 2) = 15.8(30 - kd + 27 - kd)$$
$$kd = 5.7 \text{ in.} \qquad \text{and} \qquad d - kd = 24.3 \text{ in.}$$

The moment of inertia, computed about the neutral axis, is

$$I_c = 48 \times \frac{4^3}{12} + 48 \times 4 \times 3.7^2 + 15.8(24.3^2 + 21.3^2) = 19{,}390 \text{ in.}^4$$

In computing the moment of inertia of the flange, it must be remembered that the moment of inertia of a section about any axis equals the moment of inertia about its own center of gravity plus the area times the square of the distance between the two axes. However, I for a set of rods about its own center of gravity is so small that it can be neglected.

$$S_c = \frac{I_c}{kd} = \frac{19{,}390}{5.7} = 3{,}400 \text{ in.}^3$$
$$S_s = \frac{I_c}{n(d - kd)} = 19{,}390 \div (12 \times 24.3) = 66 \text{ in.}^3$$
$$f_c = \frac{M}{S_c} = \frac{1{,}300{,}000}{3{,}400} = 382 \text{ psi}$$
$$f_s = \frac{M}{S_s} = \frac{1{,}300{,}000}{66} = 19{,}700 \text{ psi}$$

As a matter of interest and comparison, assume $j = 0.9$ and $d = 28.5$ in., then test for f_s by Eq. (2-6).

$$M_s = A_s f_s jd \qquad \text{or} \qquad f_s = \frac{1{,}300{,}000}{2.64 \times 0.9 \times 28.5} = 19{,}200 \text{ psi}$$

This is close enough for practical purposes, especially when one considers how the magnitude of b was determined empirically.

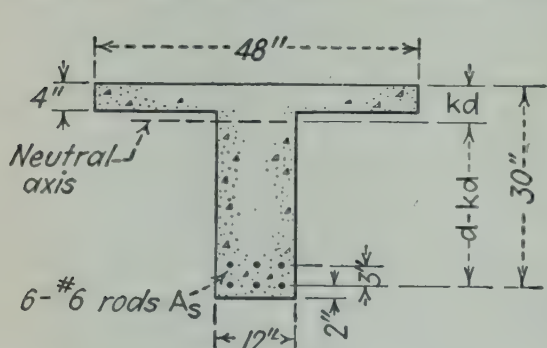


FIG. 2-16.

If one wishes to get a rough check of what the stresses in this beam may be, he may use Figs. 4 and 5 of the Appendix as follows:

1. Assume the beam as shown in Fig. 2-16.
2. Assume d to go to the center of gravity of the tensile reinforcement. Therefore, $d = 30 - 1.5 = 28.5$ in.
3. $A_s = 6 \times 0.44 = 2.64$ in.^{2}, $n = 12$, and $b = 48$ in.
4. Figure 4 has curves in terms of nA_s/b. Therefore, for this beam

$$\frac{nA_s}{b} = \frac{12 \times 2.64}{48} = 0.66$$

Using this and $d = 28.5$ in. in Fig. 4 gives, by interpolation, $kd = 5.5$ in. (approx).

5. Similarly, Fig. 5 gives $S_c/b = 72$ (approx). Then $S_c = 72 \times 48 = 3{,}460$ and

$$f_c = \frac{M}{S_c} = \frac{1,300,000}{3,460} = 376 \text{ psi}$$

Compare this with the previous computation.

6. If $kd = 5.5$,

$$jd = d - \frac{kd}{3}$$

or

$$j = \frac{28.5 - 5.5/3}{28.5} = 0.94$$

and

$$f_s = \frac{M}{A_s jd} = \frac{1,300,000}{2.64 \times 0.94 \times 28.5} = 18,400 \text{ psi}$$

A comparison of these results with those previously computed shows that this value of f_s is less than the one given there. Some difference is to be expected since d in that problem was taken to be the bottom row of rods whereas this approximate method used it as the distance to the center of gravity of the two rows. Furthermore, the curves and the interpolation from them are approximations.

Example 2-11. Design a simply supported T beam to span 25 ft and to carry a uniformly distributed live load of 300 psf plus the dead load. Assume that the framing is such that the slab is 6 in. thick and the stems are 8 ft c.c. Take $n = 10$, the allowable $f_s = 20,000$ psi. Use the formula method.

This problem is another one which requires the making of assumptions. The procedure is as follows:

1. Assume d in inches $= 1.2L$, where L is in feet, $d = 1.2 \times 25 = 30$ in.
2. Assume $b' = 16$ in., a little over $d/2$.
3. Assume two rows of rods, giving a cover of about 4 in. from the center of gravity of the reinforcement.
4. Compute the trial dead load:

$$\text{Slab} = 8 \times \tfrac{6}{12} \times 150 = 600$$
$$\text{Stem} = \frac{16}{12}\frac{(30 + 4 - 6)}{12} 150 = 470$$
$$\text{Total D.L.} = 1,070 \text{ plf}$$

5. Compute the live load:

$$8 \times 300 = 2,400 \text{ plf}$$

6. Compute the total bending moment:

$$M = \frac{wL^2}{8} = (1,070 + 2,400)\frac{25^2}{8} = 270,000 \text{ ft-lb}$$

7. Assume $j = 0.9$, and solve for A_s.

$$A_s = \frac{M}{f_s jd} = \frac{270,000 \times 12}{20,000 \times 0.9 \times 30} = 6 \text{ in.}^2$$

Try six No. 9 rods with four in the bottom row and two in an upper row 3 in. above them, an arrangement similar to Fig. 2-24. From Table 8 of the Appendix, assuming $\frac{3}{4}$-in. aggregate, it is seen that four rods can be used satisfactorily in a beam that is slightly less than the 16 in. selected. With the center of gravity 1 in. above the bottom row, the 4 in. added in item 3 will give adequate cover for the reinforcement.

If the student wishes to do so, he can now check this beam by the transformed-section method in order to prove that it is satisfactory.

If the ultimate-load theory is applied to T beams, the thickness t of the slab in Fig. 2-17 is generally less than the height a of the stress block of Fig. 2-9 ($0.537d$). The average stress of $0.85f'_c$ can be assumed to act

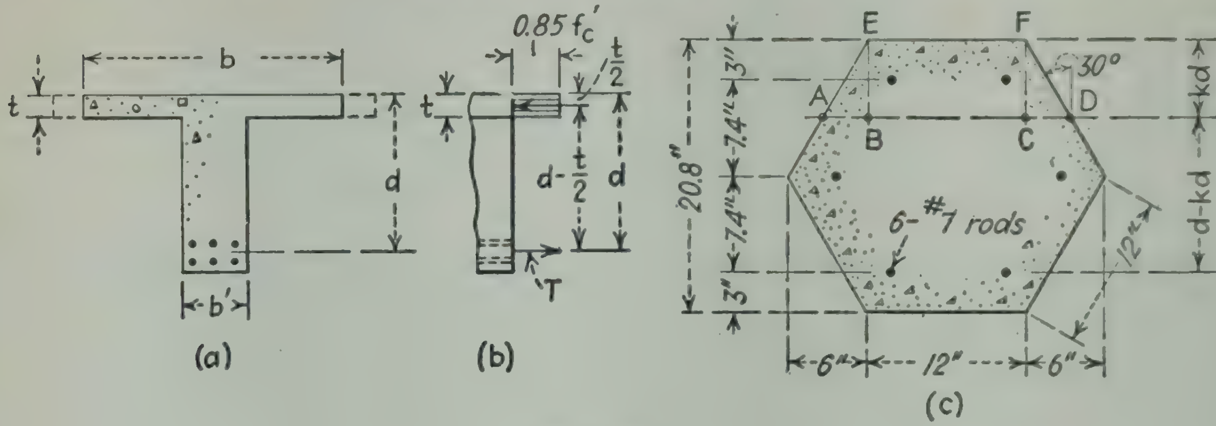


FIG. 2-17.

over the entire flange area. Then the resisting moment in terms of the concrete strength is

$$M = 0.85f'_c bt\left(d - \frac{t}{2}\right)$$

or

$$\frac{M}{bd^2} = 0.85f'_c\left(1 - \frac{t}{2d}\right)\frac{t}{d} \tag{2-37}$$

In terms of the steel strength,

$$M = A_s f_{yp}\left(d - \frac{t}{2}\right) \tag{2-38}$$

and

$$A_s = \frac{M}{f_{yp}(d - t/2)} \tag{2-39}$$

Since the assumed width of the flange is so broad and the concrete area is so large in most cases, Eq. (2-38) is the controlling one. Therefore, with the same safety factors and load factors in both cases, the straight-line and ultimate-load theories will yield practically the same results for tensile steel and positive moments at the centers of T beams. The ends of continuous T beams, as stated previously, are to be designed as inverted rectangular beams.

2-11. Irregular beams. By using the transformed-section method, it is generally possible to analyze and design reinforced-concrete beams of irregular, unusual, and unsymmetrical shapes. Practice in the solution of such problems is the best way in which to fix the methods in one's

mind. In the problem that follows, an unsymmetrical section is analyzed for the purpose of illustration.

Example 2-12. Compute M_c and M_s for the hexagonal section shown in Fig. 2-17(*c*) if $n = 10$ and the allowable f_s and f_c = 18,000 and 1,000 psi, respectively.

The neutral axis lies somewhere in the upper half of the beam. The rectangle *EFCB* has a width of 12 in. and a height equal to kd. The triangles *ABE* and *FCD* have altitudes equal to kd, but the bases *AB* and *CD* = $kd \times \tan 30°$, or $0.577kd$. Therefore, the equation for kd can be expressed as follows:

$$\frac{12(kd)^2}{2} + \frac{2kd(0.577kd)kd}{2 \times 3} + (n-1)A_s'(kd-3) = nA_s(10.4-kd) + nA_s(17.8-kd)$$

where $nA_s = 10 \times 0.6 \times 2 = 12$, and $(n-1)A_s' = 9 \times 0.6 \times 2 = 10.8$. Solving this equation gives

$$kd = 5.2 \text{ in.} \qquad \text{and} \qquad d - kd = 12.6 \text{ in.}$$

$$I_c = \frac{EF \times EB^3}{3} + \frac{2 \times AB \times EB^3}{12} + (n-1)A_s'(kd-3)^2 + nA_s(10.4-kd)^2 + nA_s(17.8-kd)^2$$

$$I_c = \frac{12 \times 5.2^3}{3} + \frac{2(0.577 \times 5.2)5.2^3}{12} + 10.8 \times 2.2^2 + 12 \times 5.2^2 + 12 \times 12.6^2$$

$$I_c = 2{,}920 \text{ in.}^4$$

$$S_c = \frac{I_c}{kd} = \frac{2{,}920}{5.2} = 562 \text{ in.}^3$$

$$S_s = \frac{I_c}{n(d-kd)} = \frac{2{,}920}{10 \times 12.6} = 23.2 \text{ in.}^3$$

$$M_c = f_cS_c = 1{,}000 \times 562 = 562{,}000 \text{ in.-lb}$$

$$M_s = f_sS_s = 18{,}000 \times 23.2 = 418{,}000 \text{ in.-lb}$$

2-12. Distribution of loads on beams. Before proceeding further, it is desirable to consider the distribution of loads to beams. Assume that Fig. 2-18(*a*) represents the framing scheme for one corner section of a large beam-and-girder floor system with the outside walls in the line of column *AEJN* and *ABCD*. Such spandrel beams as *AE* and *EJ* must support the wall construction between them and the next beams above them—in the next higher floor. They must also support a portion of the adjacent floor and its live loads. The wall loads and their distribution are to be determined from the dimensions and character of the wall materials, windows, etc. These loads may or may not be uniformly distributed. The floor loads supported by one of these beams will depend upon the character of the floor reinforcement as well as upon dimensions, materials, and assumed live loads.

For example, refer to Fig. 2-18(*b*). If the principal reinforcement extends across beams *EJ*, *fm*, and *FK*, it is customary to assume that 5–7 and 6–8 are at the middle of their respective portions of the slab between beams and that the loads on the half *E57J* are supported by *EJ*; *5f68m7*, by *fm*, etc. Then loads on girder *EF* (called a girder because it supports

intermediate beams) are assumed to be its own weight plus the reactions of the beams that it supports. Perhaps it would be more correct to assume that 45° lines $A1$, $a1$, etc., to the center line of the slab will represent the distribution of loads so that the loads in area $A13E$ go to beam AE, $a24f31$ to af, $E3f$ and $f4F$ to EF along with the reaction of af, etc. However, this method is generally reserved for use in cases where the intermediate beams are omitted, the slabs are large and nearly square, and the slabs are reinforced both ways, *e.g.*, across AE and BF as well as across AB and EF.

Now refer to Figs. 2-18(*a*) and (*c*). Assume that the slab has been designed and that it is to be 6 in. thick. The loads on beam gn are now

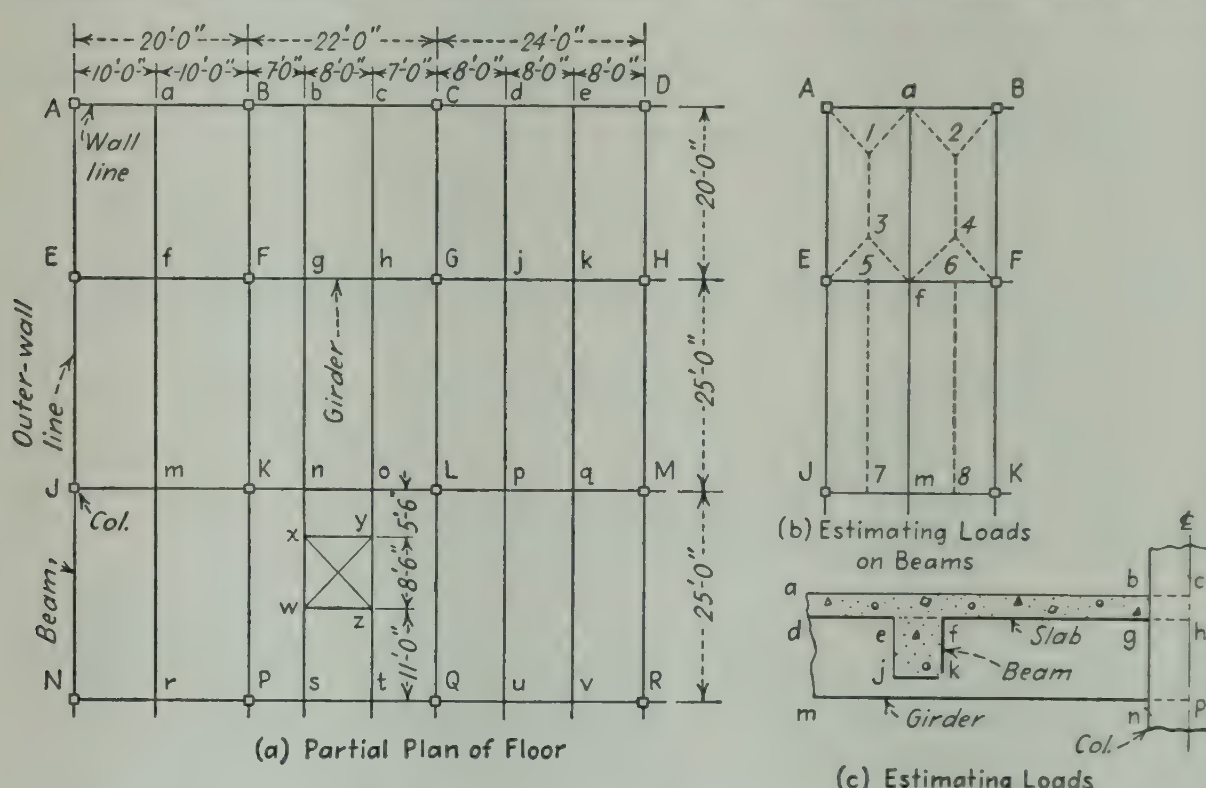


FIG. 2-18. Example of beam-and-girder framing.

to be estimated for design purposes, using the 6-in. slab and an assumed live load of 250 psf. The span is 25 ft. A trial total effective depth may be 28 in. Assume 4 in. of cover below the center of gravity of the rods. Thus fk of Sketch (c) $= 32 - 6 = 26$ in. Assume $jk = 16$ in. Then the stem weighs $26 \times 16 \times 150/144 = 430$ plf. The slab and live load per foot of beam equal $(3.5 + 4.0)\ (75 + 250) = 2{,}440$ plf. The total is 2,870 plf, and it is assumed to cover the full length of 25 ft. If there are partitions resting upon or across the beam, their weights are to be included also.

Next, estimate the loads on girder FG, Fig. 2-18(a). Assume that the depth of haunch or stem below the slab is 28 in.; the width 16 in. The weight of the stem is then 450 plf, applied usually to the full span of 22

ft c.c. of columns. Besides this weight the girder supports the combined reactions of beams *bg* and *gn* at *g*, and *ch* and *ho* at *h*, also the weight of any partition over it.

By referring to Fig. 2-18(*c*), it will be noticed that the preceding methods are somewhat on the safe side, because they neglect the fact that the ends of the stems of the beams within the stem of the girder are included twice. Also, the spans are used as center-to-center distances. In the case of very wide stiff rigid supports, these might be reduced somewhat.

Next, assume beam *ns*, Fig. 2-18(*a*). Portion *xyzw* is a hole for a hatch. Assume that portions *oyxn* and *wzts* are reinforced perpendicularly to *ns*. Assume also that there is a hatch cover over opening *xyzw* and that this cover is supported by beams *xy* and *wz*. Edge beams *xy* and *wz* then hold little load except one-half of the loads on the hatch cover. Compute the reactions of these beams at *x* and *w*, estimate the weight of the haunch of beam *ns*, calculate the uniform load from one-half of *KnsP*, and then add the assumed uniform loads from one-half of *noyx* and *wzts*. These are the loads on *ns* for design purposes.

Practice Problems

Bending-moment formulas for simply supported beams are assumed to be known by the student. The bending-moment diagrams for beams of uniform section with fixed ends and for various conditions of loading are shown in Fig. 1 of the Appendix. These may be substituted for continuous beams in some of the problems. In all cases assume that the weight of concrete plus reinforcement is 150 pcf.

A. The following problems are to be solved by the formulas for rectangular beams according to the straight-line theory:

2-1. Assume a beam of the cross section shown in Fig. 2-19(*a*). It is subjected to a total bending moment of 1,300,000 in.-lb. If $n = 12$, compute the total forces C and T, also the unit stresses f_s and f_c.

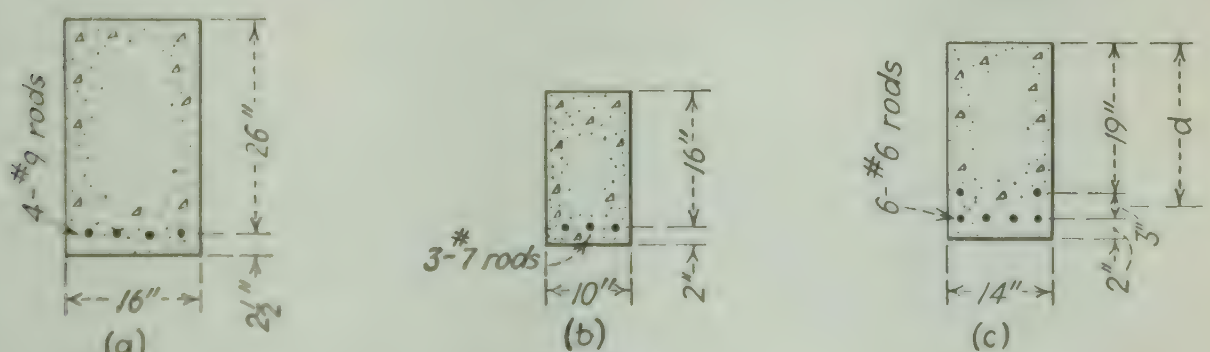


FIG. 2-19.

2-2. Assume that the beam shown in Fig. 2-19(*b*) is simply supported and has a span of 14 ft. If $n = 10$ and the allowable values of f_s and f_c = 20,000 and 1,350 psi, respectively, compute the uniformly distributed live load that this beam will safely support in excess of its own dead load.

2-3. Compute the safe bending moment for the beam shown in Fig. 2-19(*c*) if $n = 12$ and the allowable f_s and $f_c = 18{,}000$ and 900 psi, respectively.

Discussion. The tensile force T is to be considered concentrated at the center of gravity of the group of rods. *Ans.* $M_s = 73{,}000$ ft-lb; $M_c = 75{,}000$ ft-lb.

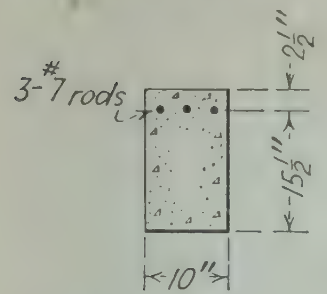


FIG. 2-20.

2-4. The beam pictured in Fig. 2-20 is a cantilever supporting its own weight and a concentrated load of 6,000 lb at its outer end. If $n = 10$ and the allowable f_s and $f_c = 18{,}000$ and 1,200 psi, respectively, compute the safe length for which this cantilever can be used.

Discussion. Since the compression is at the bottom of this beam, the problem is similar to the previous ones except that the section is reversed. Equate the controlling resisting moment to $wL^2 \times {}^{12}\!/\!_2 + 12PL$, and solve for L, remembering that $w = (10 \times {}^{18}\!/\!_{144})1\ 50$ and $P = 6{,}000$ lb.

2-5. Design a simply supported slab to carry a uniformly distributed live load of 300 psf over a span of 8 ft if $n = 10$ and the allowable values of f_s and $f_c = 20{,}000$ and 1,250 psi, respectively.

Discussion. Find K and p from Table 5 of the Appendix. Use $b = 12$ in. Assume a dead load per square foot for the slab itself, and add it to the live load before computing M. Using Eq. (2-8), solve for d, and add $1\frac{1}{2}$ in. to get the total thickness. Then find $A_s = pbd$, and choose rods that will fit into the 12-in. width of slab at not less than 3 in. c.c.

2-6. Assume that the heavy simply supported slab shown in Fig. 2-21(*b*) carries its own weight plus the loads shown in (*a*). Assume that $n = 10$. Compute the stresses in the concrete and steel caused by bending at the center of the span, using the formulas for a rectangular beam 1 ft wide. *Ans.* $f_c = 920$ psi; $f_s = 16{,}900$ psi.

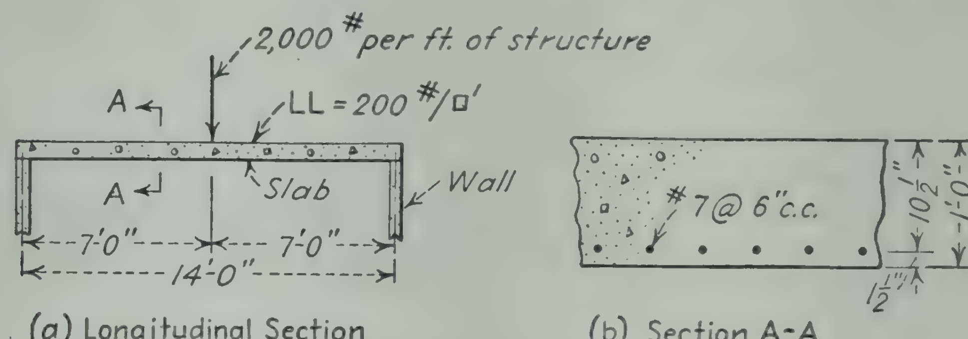


FIG. 2-21.

2-7. Beam AB is simply supported and carries the framing shown in Fig. 2-22(*a*) and a brick wall. The weight of the wall and the reactions of the beams at C and D are shown in (*b*). Estimate the dead load of the beam and add it to the 800 plf. Compute the bending moment at the center of AB, and design this beam as a rectangular section with steel in the bottom only. Use 3,000-lb concrete and maximum unit stresses allowed in the Code. Let $f_s = 20{,}000$ psi.

Ans. $b = 18$ in.; $d = 22$ in.; rods = 6 No. 9.

2-8. Figure 2-23 shows the cross section of a 200-ft-long narrow deep pit that is to be covered by a superstructure. The floor above the pit is to consist of simply supported T beams and a slab. Assume intermediate-grade steel and $f'_c = 3{,}000$ psi. Using the allowable unit stresses given by the Code, design a floor to serve this purpose and to support a uniformly distributed live load of 200 psf. Neglect stairways.

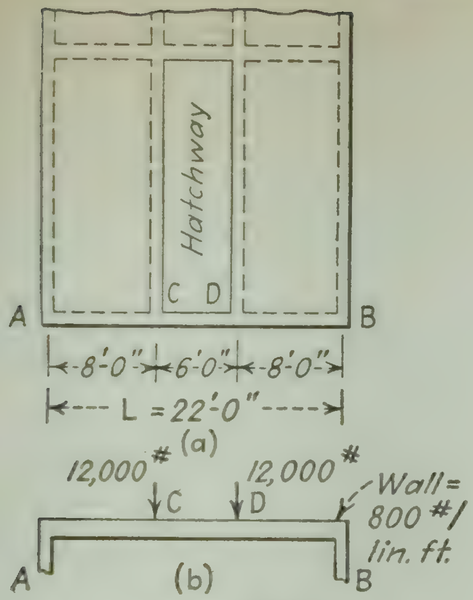


FIG. 2-22.

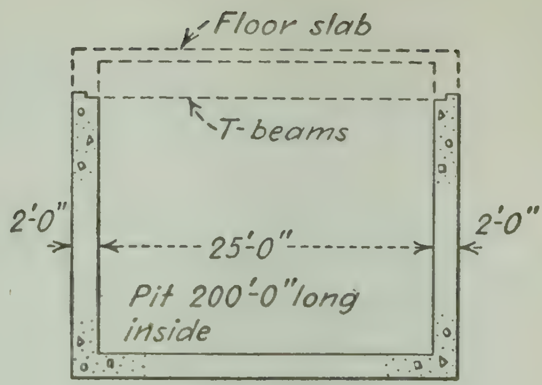


FIG. 2-23.

B. The following problems are to be solved by the transformed-section method:

2-9. The beam shown in Fig. 2-24 is subjected to a bending moment of 800,000 in.-lb. Assume $n = 10$ and solve for f_s and f_c.

Discussion. In this case, use one strip of concrete to replace the upper set of two rods and another strip to take the place of the lower set of four rods. This is more accurate than considering the six rods to be grouped at their center of gravity.

Ans. $f_c = 730$ psi; $f_s = 15{,}600$ psi.

2-10. Compute the resisting moments M_s and M_c for the beam illustrated in Fig. 2-25 if $n = 15$ and the allowable f_s and $f_c = 18{,}000$ and 900 psi, respectively. Assume $d = 18$ in. *Ans.* $M_c = 690{,}000$ in.-lb; $M_s = 780{,}000$ in.-lb.

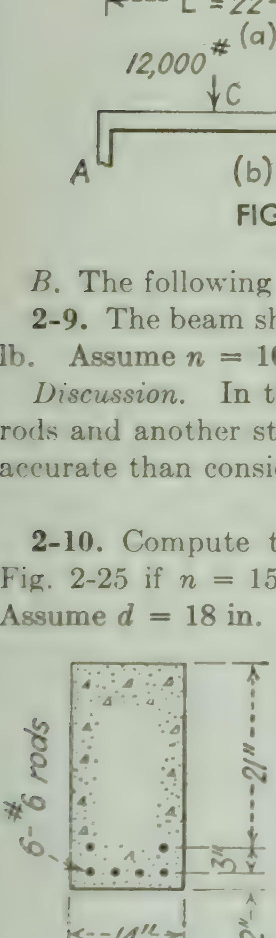


FIG. 2-24.

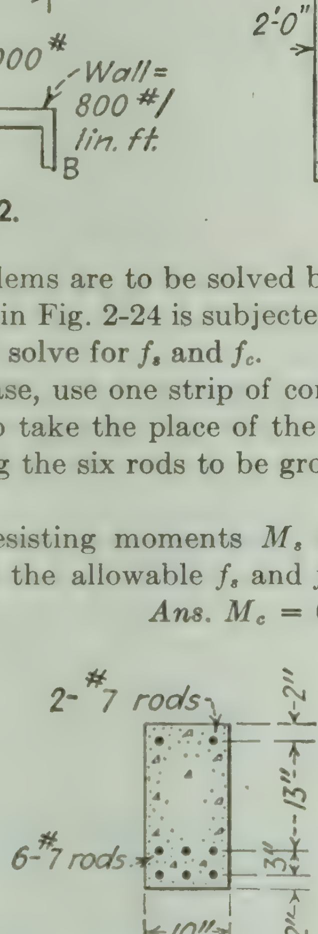


FIG. 2-25.

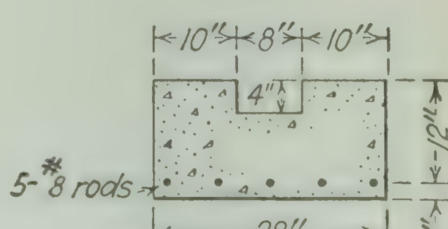


FIG. 2-26.

2-11. A beam is $12\frac{1}{2}$ in. wide and 24 in. in total depth. It runs across a support where it has a negative bending moment of 950,000 in.-lb. There are three No. 7 rods $2\frac{1}{2}$ in. above the bottom. Compute the number of No. 7 rods needed in the top of the beam if $n = 10$ and the allowable f_s and $f_c = 20{,}000$ and 1,350 psi, respectively. Find also the values of f_c, f_s, and f_s'.

2-12. If the beam shown in Fig. 2-26 carries a bending moment of 400,000 in.-lb, compute f_s and f_c, assuming $n = 12$.

Discussion. The 8-in. slot at the center removes all or most of the concrete in the region of compression. Therefore, assume the beam to be 20 in. wide.

2-13. Assume that the T beam shown in Fig. 2-27 is carried continuously across its supports and that the negative bending moment is 2,000,000 in.-lb. The bottom layer of rods is in compression. Determine the rods needed to withstand the tension in the top if they are $2\frac{1}{2}$ in. from the top of the slab. The second row (if any) is 3 in.

below the upper one, and all rods are No. 8. Assume $n = 10$ and the allowable f_s and $f_c = 20{,}000$ and 1,200 psi, respectively.

Discussion. The beam is to be analyzed like a rectangular one 16 in. wide. Make a trial calculation for A_s, using $j = 0.88$ and $d = 27$ in. Thus A_s is found to be 4.26 in.². This requires over five rods, but it will be best to use two rows with four in the top layer and two in the bottom. This is done for symmetry. The trial beam can now be analyzed. *Ans.* $f_c = 940$ psi; $f_s = 19{,}200$ psi.

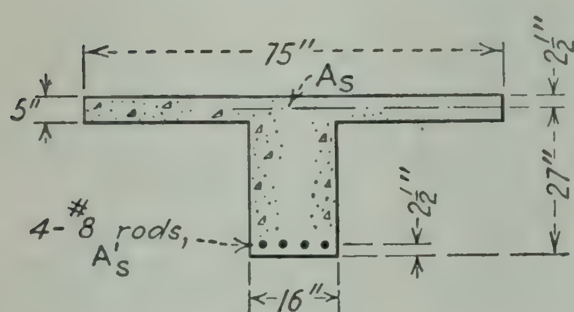


FIG. 2-27.

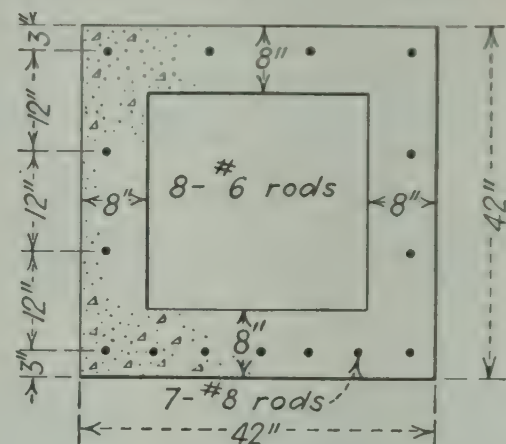


FIG. 2-28.

2-14. Compute the safe resisting moment of the box section shown in Fig. 2-28 if $n = 12$ and the allowable f_s and $f_c = 20{,}000$ and 1,200 psi, respectively. Neglect the compression on the stem portions below the flange (top slab).

2-15. Compute the safe resisting moment of the beam pictured in Fig. 2-29 if $n = 12$ and the allowable f_s and $f_c = 20{,}000$ and 1,000 psi, respectively.

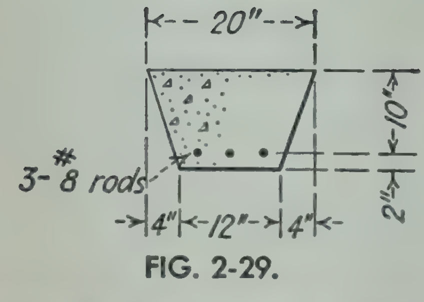


FIG. 2-29.

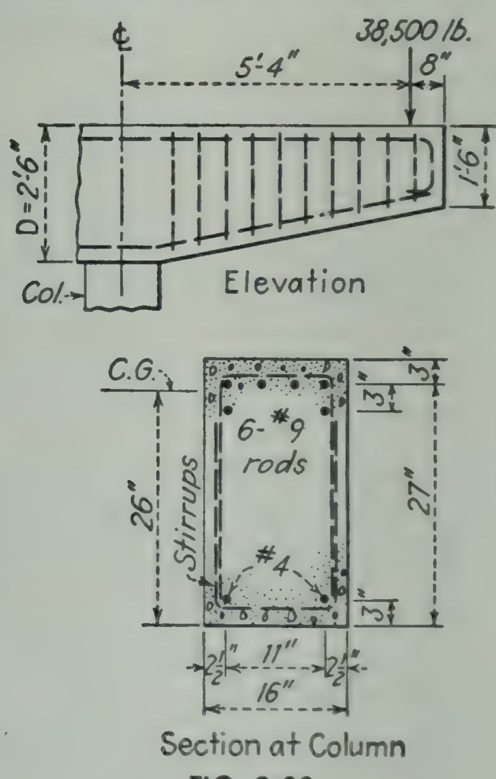


FIG. 2-30

Discussion. Solve for kd as usual, remembering that the trapezoidal area in compression may be considered as a rectangle minus two triangles. The altitude of each triangle is kd; its base is $kd/3$.

2-16. Figure 2-30 shows a cantilevered rectangular beam which it is proposed to use to support the outside stringer or fascia of a viaduct. The allowable f_s and f_c = 20,000 and 1,000 psi, respectively; $n = 10$. Using the loads and dimensions given, check the stresses in the steel and concrete by using the transformed-section method with d = the distance to the center of gravity of the six tensile rods. Neglect the two No. 4 ties. Compute the bending moment at the center of the column, including the weight of the concrete. Is the member safe? If not, what should be done about it? *Ans.* f_s = 18,900 psi, safe; f_c = 1,320 psi, overloaded. Make the section wider or deeper rather than add compressive reinforcement; or perhaps decrease the length of the cantilever if possible.

C. The following problems are to be solved by means of the methods of the ultimate-load theory. For all of them assume f'_c = 3,000 psi, f_{yp} = 40,000 psi, load factor for D.L. = 1.5, and load factor for L.L. = 2.

2-17. Compute the ultimate bending moment for the beam of Fig. 2-31.

2-18. Compute the ultimate bending moment for the beam of Fig. 2-32.

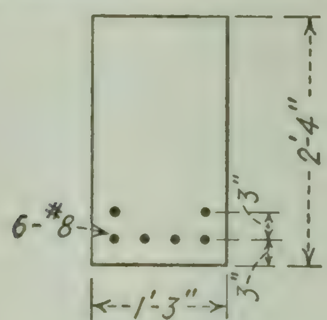


FIG. 2-31.

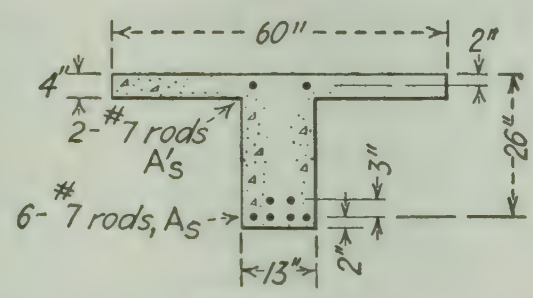


FIG. 2-32.

2-19. Design a simply supported rectangular beam to support a uniformly distributed L.L. of 3,000 plf and a D.L. of 1,500 plf, including its own estimated weight. The span is 22 ft.

2-20. Design a continuous T beam with a span of 25 ft, having a slab 5 in. thick, and supporting a loaded width c.c. of beams equal to 10 ft. L.L. = 250 psf. Include estimated D.L.

2-21. A continuous slab is to support a uniform L.L. of 400 psf over a span of 15 ft. It is 8 in. deep, $d = 6\frac{1}{2}$ in., and the reinforcement is No. 6 bars at 6 in. c.c. Is it safe?

2-22. A designer has proposed the construction shown in Fig. 2-33(*a*) for the floor over a long passageway. The slab has a uniform depth and it is to be simply supported. It is to support a uniformly distributed live load of 300 psf plus its own weight. Is this construction satisfactory?

2-23. Design a simply supported T beam girder to hold the loads shown in Fig. 2-33(*b*) plus its own weight.

2-24. Compute the ultimate bending moment for the continuous T beam shown in Fig. 2-33(*c*). This pictures the conditions where the beam crosses a column and has tension in its top.

2-25. Figure 2-33(*d*) shows the dimensions and reinforcement proposed at the end of a continuous beam (next to a column). The slab is 6 in. thick and the beams are 9 ft c.c. If the span is 24 ft, compute the total safe uniformly distributed load that the beam can support, assuming it to have fixed ends.

2-26. Figure 2-33(*e*) shows the construction proposed for the floor over a narrow basement in an industrial plant. The T beams are simply supported and have a span of 25 ft. The live load is 300 psf. Check the design. If not satisfactory, show what should be done about it.

2-27. A structure has a small boiler room in a basement as pictured in Fig. 2-33(*f*). Design the floor slab and beams over the basement, with simply supported T beams as shown. The live load = 300 psf. Make sketches to show the dimensions and reinforcement to be used for the typical construction.

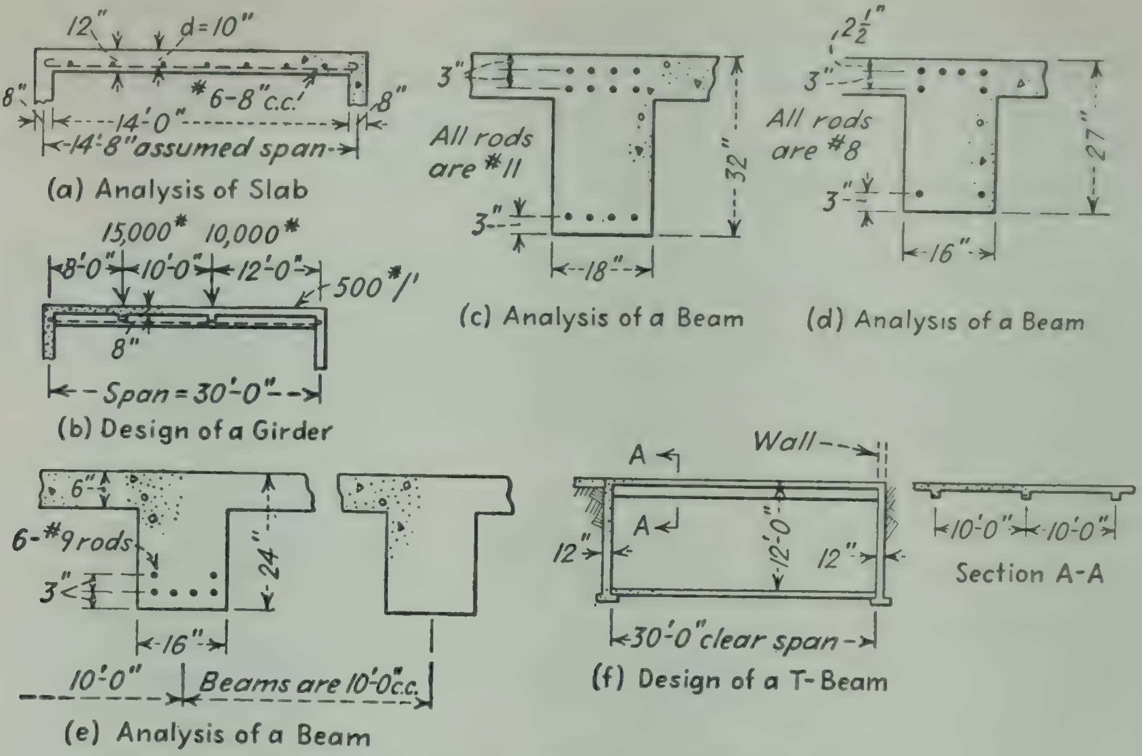


FIG. 2-33. Studies of beams and slabs.

D. The following problem is to be solved by any method that is preferred:

2-28. Design the following members of the floor shown in Fig. 2-18(*a*), assuming a live load of 250 psf on the floor and wall loads along *AD* and *AN* equal to 900 plf of spandrel beams:

a. Slab across the space from beam *EJ* to *fm* with a simply supported edge at *EJ* and an assumed fixed end at *fm*.

b. Slab across beams *fm* and *FK* with both ends fixed.

c. Beam *fm*, assuming both ends fixed.

d. Beam *af*, assuming only end *f* fixed.

e. Beam *EJ*, assuming fixed ends and a uniform wall load of 300 plf above it.

f. Girder *EF*, with only end *F* fixed.

g. Beams *gn* and *ho*, with both ends fixed.

h. Beams *bg* and *ch*, with only ends *g* and *h* fixed.

i. Girder *FG*, with both ends fixed and a partition weighing 300 plf above it.

j. Girder *BC*, with both ends fixed and a uniform wall load of 400 plf above it.

3
BOND

3-1. Nature and magnitude of bond stress. When a reinforcing rod is embedded in concrete, the concrete adheres to its surface, resisting any force that tends to pull or push out the rod. This is called the *bond* between the concrete and the steel. The intensity of this adhesive force is called the *bond stress*, or *bond unit stress*. In reality, this bond stress is a resistance to shearing between the surfaces of the steel and the concrete. The action is that of resistance to forces that try to break the concrete away from the surface of the steel in a direction parallel to that surface and lengthwise of the bar.

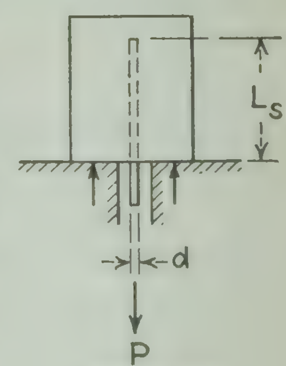


FIG. 3-1.

The function of bond in a reinforced-concrete member is somewhat analogous to that of rivets in structural steelwork. It is the force that holds the two materials together so as to develop their simultaneous and mutually helpful action. If the rods have no change of stress—and therefore no change of length—as a result of the application of a load on the member, then there will be no bond stress set up by it, but as soon as flexural action causes the steel to stretch or to compress, the bond stresses must come into action in order to cause these changes.

When the rod in Fig. 3-1 is stretched, the elongation in the length L_s is greatest at the point where the steel enters the bottom of the concrete block. It then decreases to zero at or somewhere below the top end of the rod. A little reflection will show that the intensities of the bond stresses along the rod must vary somewhat in proportion to the stretching of the rod inside the concrete block unless the bond is broken. Probably the bond stresses are very high near the point at which the rod enters the concrete. It is also likely that they are very high (probably to the point of local failure) at the cracks pictured in Fig. 3-7(*b*). The distribution of the bond stresses is very uncertain; yet, for analysis and design, it is generally considered to be uniform over some length that is necessary to develop the strength of the bar. However, one must realize

that the bond will develop the rod as quickly as possible so that a part of the rod which is a long way (15 to 30 diameters) from the point of entry of the rod in Fig. 3-1 may have no stresses at all; in other words, anchorage far from the point where the rod is needed may not be brought into action if the bond can develop the required resistance before tensile stresses can reach the anchor.[1]

An expression for the magnitude of the average bond stress can be found readily. Referring to Fig. 3-1, let o = the perimeter of the cross section of this rod, let d = its diameter, let L_s = the length of embedment, and let u = the average bond unit stress. It is clear that the total strength of the bond per inch of rod equals ou. It is also apparent that the embedded length of the rods should be great enough to develop the required tensile (or compressive) strength of the bar in order to avoid having it pull out of the concrete. Then, for a round rod that is subjected to a tensile force P,

$$P = \frac{\pi d^2 f_s}{4} = \pi d u L_s \qquad \text{or} \qquad L = \frac{df_s}{4u} \tag{3-1}$$

This shows that the length of embedment that is needed to develop the strength of the bar in tension (or in compression) increases with the tensile stress in the steel and decreases with an increase in the magnitude of the permissible bond stress. Notice that Eq. (3-1) gives the average stress and does not take into account the local and severe increases in bond stresses at and near cracks like those pictured in Fig. 3-7(b).

Theoretically, many small rods are better than a few very large ones as far as bond is concerned. This is evident if one studies Eq. (3-1), bearing in mind that the cross-sectional area of a bar increases as the square of d, whereas the surface of the bar per inch of length varies as the first power of d. However, there must be adequate space between adjacent rods—also between the rods and the forms—so that the concrete will completely fill the forms and thoroughly encase the steel if the bond is to be developed fully. See Table 8, Appendix, for recommended spacing of rods in beams.

The American Concrete Institute adopted in 1951 the allowable bond unit stresses shown in Table 1-8. In connection with beams the reader should notice the difference between the assigned values of $0.10f'_c$ for bond on horizontal bottom rods and $0.07f'_c$ for horizontal top rods—having more than 12 in. of concrete cast under them. This is apparently due to the tendency of wet concrete to settle, or at least for the coarse aggregate to do so. This may cause voids under horizontal rods that

[1] R. M. Mains, Measurement of the Distribution of Tensile and Bond Stresses along Reinforcing Bars, *J. ACI*, November, 1951.

are supported in fixed positions when the depth of concrete under them is considerable, thus weakening the bond on the underside of the bars. Vibration may or may not increase this action, and it is likely to be worse in wet mixes than in dry ones.

Of course, in design work one should follow whatever specifications govern for a given job. On the other hand, one does not violate the spirit of a specification when he uses limiting values that are more conservative than the maximum limit permitted. Because of the great importance of bond strength, and in order to avoid a double standard that may be confusing and incorrectly applied, the author prefers to use one specified allowable bond stress for all rods in beams whether top or bottom, except in special cases. He therefore prefers that $0.07f'_c$ be used as a limit for the best modern deformed bars unless this causes an undue hardship. If the loss caused by a failure is not likely to be serious, and if the $0.07f'_c$ limit seems to be an unnecessary handicap, the higher stress may be considered where applicable. Also, if the live load is relatively small and the dead load is accurately determinable, there may be more justification in raising the bond limit. If the live load is relatively large, and especially if impact is severe, one will be wise to adhere to a conservative limit in so vital a matter.

When it is practicable, the longitudinal rods in a beam should be "anchored" in regions where the concrete is under compression. They may be bent into the compression side like those in Fig. 4-10(a) or extended to the support like bar a of Fig. 3-11(c). In such places as the latter, the pressure itself may help to hold the bars to some extent. When they must be anchored in the portion of the beam that is subjected to tensile stresses, the rods should extend across the cracks as in Fig. 3-7(b); they should not be even approximately parallel to the lines in which the tensile cracks in the concrete will occur; in other words, the rods must not depend upon the tensile strength of the concrete to anchor them. In the case of a beam such as that which is shown in Fig. 3-7(b), the tensile reinforcement runs across these cracks, and the bond stresses along the rods develop the tensile forces T by transferring the longitudinal shear in the concrete to the rods.

All rods must be clean and free from dirt, grease, scale, and loose rust. Anything that destroys the ability of the concrete to grip the steel may prove to be serious because it will prevent the stress in the latter from being fully developed, and therefore it will keep the steel from performing its function properly.

The permissible bond stress along the reinforcement can be increased by making the surface of a rod rough or irregular. Such rods are called "deformed" rods, and they are used generally. Some of the modern types of such bars are pictured in Fig. 3-2. The lugs, or corrugations,

produce a mechanical bond which helps to lock the concrete and the steel together. Although the lugs do not seem to cause much increase in the bond stress at which a rod will have its initial slip[1] when tested, the wedging or locking action of the projections has a considerable effect in raising the ultimate strength of the bond provided the cover of concrete over the steel is sufficient to prevent stripping or spalling.

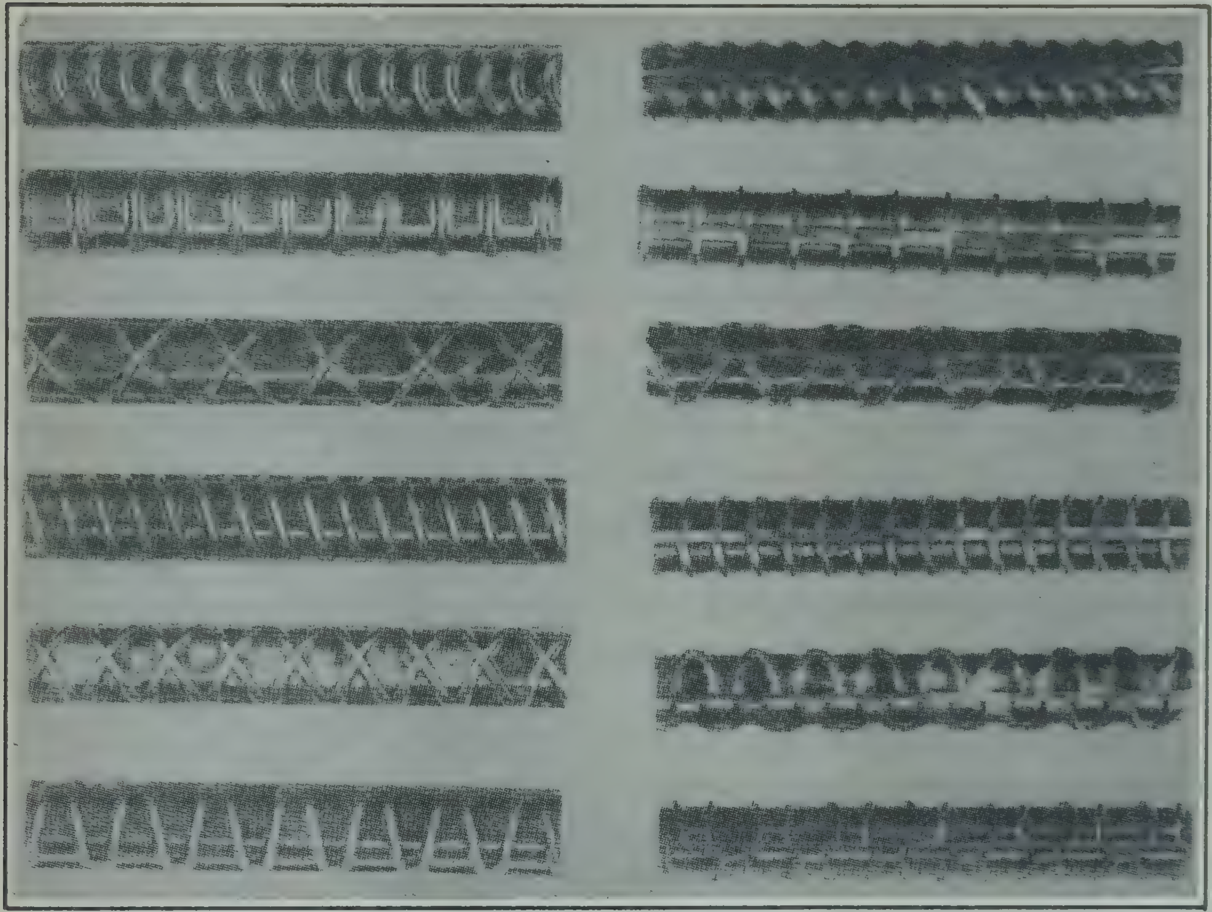


FIG. 3-2. These reinforcing bars are manufactured to meet the requirements of ASTM Tentative Specifications for Minimum Requirements for the Deformations of Deformed Steel Bars for Concrete Reinforcement A 305—50T. (*Courtesy of the American Iron and Steel Institute.*)

Tests of the bond stresses that are developed by plain and deformed rods of types used in 1950 and before have been made at Lehigh University.[2] These experiments indicated that those deformed bars developed 45 to 50 per cent more ultimate bond stress than did plain rods in beam tests with 3,000-lb concrete. Furthermore, the form of the lugs seems to be important in "pull-out" tests, but this variation is not so apparent in the tests of beams. Any type of deformation that is equivalent to a

[1] Gilkey, Chamberlin, and Beal, The Bond between Concrete and Steel, *J. ACI*, September, 1938.

[2] George Robert Wernisch, Bond Studies of Different Types of Reinforcing Bars, *J. ACI*, November–December, 1937.

raised rib that extends directly or diagonally around the rod appears to be better than narrow ridges running lengthwise of the bar.

New bars meeting the ASTM A 305 specifications adopted in 1951 develop better bond than do those of previously used types. At least, by implication, Table 1-8 indicates that the older types are to be classed as plain bars, with correspondingly low values of allowable bond stresses. However, when one is compelled by necessity to use them, there seems to be no justification for reducing the formerly specified limit of $0.05f'_c$. The A 305 rods will probably replace the other types. The allowable bond stress on plain bars, as shown in Table 1-8, is relatively low, and the use of such rods is to be avoided because they tend to slip easily and to cause localized serious cracking. The deformed bars tend to distribute the deformation in the concrete in a series of hair cracks closely spaced.

The magnitude of the bond stress probably does not vary directly with the ultimate compressive strength of the concrete f'_c in actual structures. The report of pull-out tests by Arthur P. Clark[1] gives bond-slip curves that show considerably higher values of bond for a given slippage in the case of 6,000-lb concrete than for 2,000-lb concrete at slippages of 0.001 in. but less difference at very small slippages. Concrete of 3,500-lb strength gave bond-slip curves that do not differ greatly from those for 2,000-lb concrete. For his own use, the author prefers to set an upper limit for bond stresses based upon $f'_c = 3{,}000$ psi except for prestressed-concrete structures. If the concrete is stronger, the bond may have a somewhat larger safety factor.

In the short members tested, especially those with loads at or near the third point of the span, a relatively large portion of the bars is affected by the compression at and near the supports. This may be a very helpful feature in obtaining a relatively high value for the bond resistance.

The maximum bond stresses that may be used as a limit with the ultimate-load theory are uncertain. Clark's tests indicate that the maximum bond resistance increases by varying amounts as the slippage increases. To be determined are the amounts of slippage that are permissible without becoming the direct or indirect cause of failure of the members. For the present, bond will be considered in terms of the magnitudes that experience seems to justify for working-load conditions.

In light floors, pavements, and other construction where welded wire mesh can be used to advantage, bond is no problem. This is partly because of the large surface area of the wires in comparison with their cross-sectional area but mainly because of the effective anchorage provided by the welded junctions of the cross wires. Woven mesh and the ordinary wiring of crossed bars do not provide effective anchorages.

[1] Arthur P. Clark, Bond of Concrete Reinforcing Bars, *J. ACI*, November, 1949.

3-2. Hooks. In many cases it is not feasible or possible to extend straight bars far enough to develop their strength sufficiently by bond alone. A common way to remedy this trouble is to bend or hook the rods so as to obtain additional length for their development through bond. When a hooked rod is pulled, it tries to slip or slide around the curve. The hook provides a certain amount of mechanical locking of the steel into the concrete, but this is too indefinite to be relied upon.

Considerable publicity has been given to the idea that the use of A 305-type bars will completely eliminate the need for hooks to anchor the reinforcement. The use of judgment is still desirable. If a bar has to resist large tension very near its end, and if failure of the structure would be a serious matter—as it usually is—an engineer is unwise in failing to provide some means for ensuring suitable anchorage for the end of the rod.

To illustrate this idea in just one example, assume that the ground floor of a large warehouse is to be constructed as shown in Fig. 3-3(a). This design is used so that the floor can be poured after the walls and roof are completed. Straight bars a project

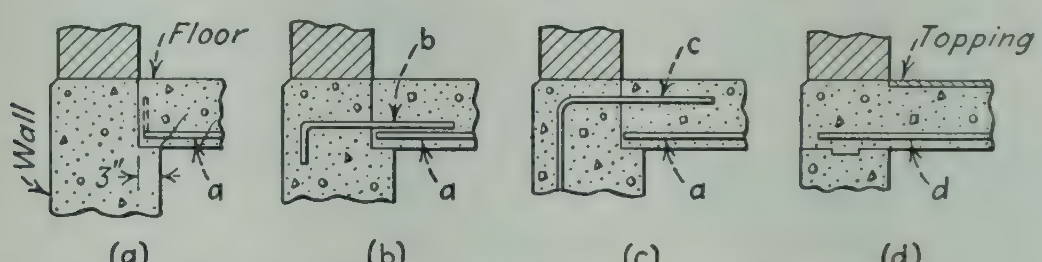


FIG. 3-3. A floor slab seated on a foundation wall.

over the 3-in. seat but end there. Heavy loads on the floor will tend to cause cracks as shown because of both bending and shear. It would be much better if the ends were bent up as shown by the dotted lines. Another and better remedy is shown in Fig. 3-3(b) where small dowels b lap over bars a and tie the floor to the foundation wall as well as serving like an extension of a. If the foundation is strong enough to restrain the end of the floor slab, rods c shown in Sketch (c) will resist the tension in the top, thereby transferring to the right the point of inflection and therefore the place where bar a is really needed. Still another scheme is shown in Sketch (d) where rods d and the structural floor extend across the foundation wall and a topping is later placed over the slab.

Strong development of the tensile steel is especially important at the fixed ends of cantilevered beams, at the ends of simply supported beams, and at the tops of continuous or restrained beams where they pass over their supports. This action will be discussed more fully in Arts. 3-7 and 3-8.

Hooks are generally necessary for plain bars in tension. One exception is when such bars terminate at the interior supports of continuous beams because they are then in a region of compressive stresses.

Certain principles should be followed in designing hooks. Figure 3-4 shows some types of anchorage that are often specified in designs. Sketch (a) is a sharp right-angular bend that may be used when a little mechanical anchorage is desired but full development of the rod is unnecessary. When the rod is pulled downward, the bent portion produces a compressive stress in the concrete. However, this arm usually has insufficient strength as a cantilever to spread the load over the entire length AB. It therefore tends to crush the concrete locally at A. It is clear also that a downward pull on the rod cannot produce a horizontal motion of the portion AB and therefore cannot develop its bond resistance until the rod begins to pull out below A and until it crushes a fillet in the concrete. Consequently, when a large tension is to be developed in the bar,

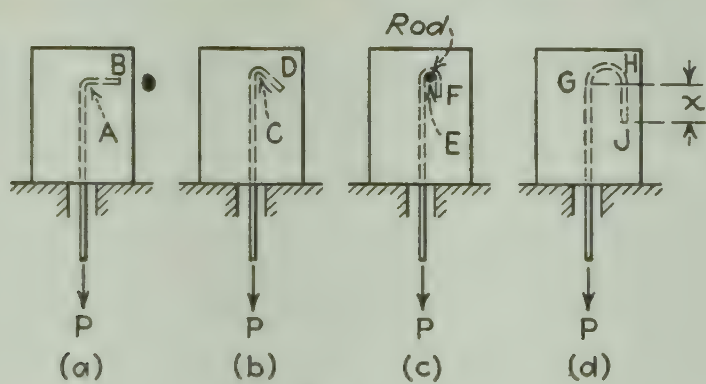


FIG. 3-4.

this type of anchorage should be made with a reasonably large radius at A—usually at least $3d$ for the inside of the curved rod for minor anchorages but larger for heavy longitudinal rods, such as those at the knees of rigid-frame bridges and the bottoms of retaining walls.

The anchorage shown in Fig. 3-4(b) is a modification of that pictured in (a). It is not an improvement, because the acute angle tends to make the compression at point C even worse than that at A in Fig. 3-4(a). Also, the concrete may not fill the triangular space at C completely, so that the portion CD may be of little use until the bond below C is broken. In fact, the larger the rod the worse the condition becomes.

Figure 3-4(c) shows another kind of bend in which the rod is hooked back upon itself with a small radius at the bend. This is not much better than the two previous types unless bent around a rod as shown; this, or the bend in Sketch (b), then makes a good anchorage. However, such sharp bends cannot be made with hard steels. It is preferable to go a little farther and to bend the rod as pictured in Fig. 3-4(d). This gives sufficient concrete area inside the bent portion of the rod to withstand the compression which is caused by the tension in the steel. It is also desirable to provide a straight portion beyond the bend as an additional anchorage. For the smaller sizes, the diameter of the inside of

the bend should be about six times that of the rod, and 8 diameters for the larger ones. The portion HJ should be about four times the diameter of the rod, or at least $2\frac{1}{2}$ in. If the details are arranged in this way, the bond strength of the full length of the rod may be considered to be effective. In fact, it is important to make all bends in reinforcing bars so that they will have a reasonably large diameter. The Code states that a standard hook should not be assumed to develop more than 10,000 psi in the bar. Table 10 in the Appendix shows typical hooks and 90° bends for bars.

When the hooks pass around a longitudinal rod, as pictured in Fig. 3-4(c), the pull of the former is transferred at least partially into the latter as a small beam. This is a sort of mechanical connection. In most cases, however, bond must be the chief means of developing the hooked rod.

A hook should not be depended upon to anchor a bar subjected to compression, because the bend merely accentuates the tendency of the bar to buckle.

Example 3-1. Analyze the anchorage of a plain round rod if the type is that shown in Fig. 3-4(d) with a radius of $3d$ inside the curve. Assume that the rod has a diameter of $\frac{1}{2}$ in., a tensile stress of 20,000 psi, and an allowable bond stress of 125 psi.

From Eq. (3-1), the length of rod required for anchorage is

$$L_s = \frac{df_s}{4u} = \frac{0.5 \times 20{,}000}{4 \times 125} = 20 \text{ in.}$$

The length of HJ is $4d = 2$ in.

$$GH \text{ is } \frac{\pi(6d + d)}{2} = \pi \times 1.75 = 5.5 \text{ in.}$$

Therefore, the length required below G is

$$L_x = 20 - (2 + 5.5) = 12.5 \text{ in.}$$

Using Eq. (3-1), the maximum tensile stresses at H and G that can be developed by bond are

$$f_s \text{ at } H = \frac{4uL_s}{d} = \frac{4 \times 125 \times 2}{0.5} = 2{,}000 \text{ psi}$$

$$f_s \text{ at } G = 2{,}000 + \frac{4 \times 125 \times 5.5}{0.5} = 7{,}500 \text{ psi}$$

The greatest pressure on the concrete at the inside of the hook of this rod is likely to occur near G. The actual stress condition is uncertain, but an arbitrary and theoretical maximum value for this pressure may be found by assuming that the action is similar to that of a hoop or pipe which is subjected to normal pressure from the inside for which

$$T = pr$$

T = the tension in the rod, p = the normal pressure in pounds per linear inch of the

rod, and r = the radius of the curve in inches—in this case r is assumed to be the radius of the inside of the rod. From the foregoing calculations, at G,

$$T = A_s f_s = pr \tag{3-2}$$

$$0.2 \times 7{,}500 = p \times 1.5 \qquad \text{or} \qquad p = 1{,}000 \text{ pli}$$

Then the unit compressive stress in the concrete is

$$f_c = \frac{p}{d} = \frac{1{,}000}{0.5} = 2{,}000 \text{ psi}$$

This is a large compressive stress, but it is localized and will soon be dissipated through the body of the concrete.

3-3. Bond of multiple layers of rods. Let Fig. 3-5 represent a beam with two layers of rods having diameters equal to d. Let the cover of concrete on the sides be s, and let the clear distance between rods be m. When the beam is loaded it bends, and the portion below the neutral axis O-O elongates, thus stretching the steel. The increment of tension must be transferred to the rods by the concrete. If the bond strength of the bottom half of the lower rods is to be developed, it must be done by shear in the concrete across the section AB. The bond in the top half of these rods is developed directly. Therefore, let v_L = the allowable unit stress in the concrete (longitudinal shear), $(T_1' - T_1)$ the increment of tension in the bottom rods, and $(T_2' - T_2)$ the increment in the upper rods per inch of length; then

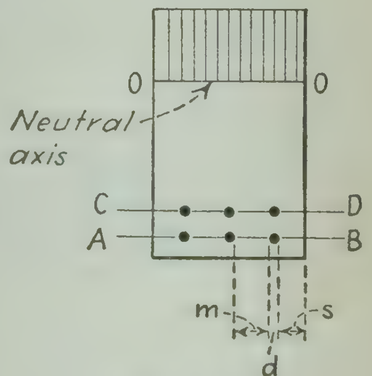


FIG. 3-5.

$$0.5(T_1' - T_1) = 3\left(\frac{\pi}{2}\,du\right) = (2s + 2m)v_L \tag{3-3}$$

The section CD must develop the entire lower set of rods and half the stress in the upper set, or

$$(T_1' - T_1) + 0.5(T_2' - T_2) = 3\left(\pi d + \frac{\pi}{2}\,d\right)u = (2s + 2m)v_L \tag{3-4}$$

It must be noted that u in the foregoing cases may not be the full allowable value for the bond stress, but it will be as much as is required to develop the increment of tension. However, the concrete must be strong enough in shear to act in the manner shown. This is a possible weakness in short beams with heavy reinforcement in two or more rows. In such cases, it may not be desirable to use the close spacing shown in Tables 8 and 8A of the Appendix for inner rows of bars.

A further explanation of the bond stresses resulting from beam action will be given in Art. 3-6 and in the next chapter.

3-4. Splices. The ordinary method of splicing reinforcement is by the lapping of the bars past each other so that the bond stresses will transfer the load out of one bar into the concrete and thence into the other bar. These rods might be hooked, but it is not always practicable or desirable to bend them. The length of lap should be at least that given for L_s in Art. 3-1. However, such splices should not be made at points of maximum bending unless they are in the portions of the beam in which compressive stresses exist and in which the steel is not the principal stress-carrying part of the section. In general, it is best to locate splices near points of contraflexure. Many times, they can be staggered so that all splices do not come at the same point.

Splices of reinforcement made by means of the bond strength of the concrete may theoretically be somewhat more effective if the rods are staggered as shown in Fig. 3-6(a). This is satisfactory in a wide slab or

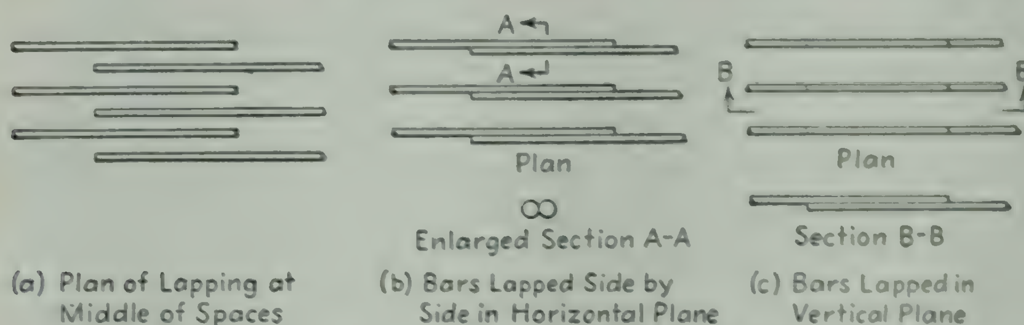


FIG. 3-6. Arrangements for splicing reinforcement by means of bond.

mat where the spacing of the bars is large but it is undesirable in a beam or other member having several closely spaced bars because the overlapped section acts as a screen to interfere with proper encasement of rods and filling of forms. In the case of a horizontal beam, the rods may be lapped in a horizontal plane as pictured in Sketch (b) or one above the other as illustrated in (c). These two facilitate wiring of the rods to hold them in position during concreting. The wiring does not add appreciably to the strength of the splices. The arrangement in Sketch (b) is likely to cause air pockets and poor bond in the space just below the junction between bars. Encasement in (c) is better, but the top rods do not fit the stirrups properly, and they cause the beam to have a smaller effective depth at one place than at another one. In actual practice, bars in such a position are likely to be knocked down into the position shown in (b); hence the latter is the more practicable arrangement if the spacing provides proper clearance for the passage of aggregate.

A lawsuit in which the author was a witness illustrates how bad results can be caused by "a little knowledge." A man engaged a contractor friend to build a commercial garage for him, letting the contractor design the structure. The second floor was to have a series of three 20-ft bays

made of T beams. The rods were ordered 30 ft long. The contractor laid them in the bottoms of the beams. At the center of the middle span, a point of maximum bending, he butted them end to end instead of lapping them, thus making the total just the right length to reach across the structure. Of course, when he tried to remove the forms, the beams opened up in the middle and came down with them. Since it was the contractor's design, the court compelled him to tear down the damaged side spans also and to replace the entire floor at his own expense.

Sometimes it is desirable (but expensive) to splice large rods by welding. Butt splices or lapped splices may be used, the latter being made by having welds along both sides of the rods at their junction. Such splices may be conservatively designed on the assumption that 1 lin in. of fillet weld has a strength of 800 to 1,000 lb for each ⅛ in. of thickness

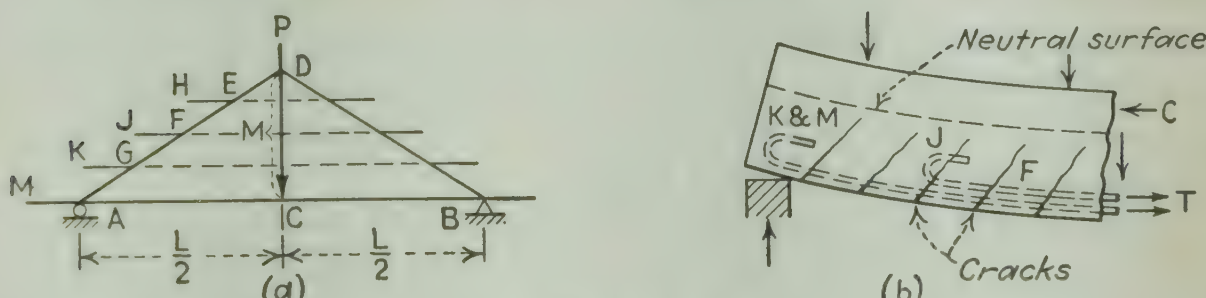


FIG. 3-7.

of the weld itself. Ordinary fillet or bead welds are ¼ to ½ in. thick. The welds should develop the full strength of the rod, not depend upon the help of bond.

Still another method of splicing heavy plain rods is by threading them and connecting the parts by means of couplings—similar to piping. However, this arrangement is expensive, and it is used only in special cases, if at all.

3-5. Development of longitudinal reinforcement. The longitudinal reinforcement in a beam should be fully developed if the design is to be economical and if the steel is to be effective. Of course, the required number of rods is determined by the maximum bending moment. All of them need not be extended for the full length of the beam. They can be discontinued as fast as the decrease in the bending moment will permit—somewhat in the same manner as the cover plates on steel girders. This will be illustrated in Arts. 3-7 and 3-8.

For instance, if a simply supported beam has a concentrated load at its center, the bending-moment diagram is as shown in Fig. 3-7(a). Assume that four No. 6 rods are required. Then it is customary to assume that each rod carries an equal share of the maximum bending moment DC. It is therefore theoretically possible to stop one rod at

each of points E, F, and G without overloading the remaining ones. However, it is also customary (and generally advisable) to extend the rods as shown by EH, FJ, and GK so as to develop them by bond [at least partially as given in Eq. (3-1)] before they reach the point at which they are really needed to help in carrying the load. These rods may be either straight or bent, as shown in Fig. 3-11. It is customary to stop or to bend them in pairs (for symmetry) unless the number is odd or unless the conditions are unusual.

To illustrate the anchorage of these rods more clearly, let Fig. 3-7(b) represent the anchorage of the longitudinal rod at point F of Fig. 3-7(a). Assuming that it is hooked, then the hook FJ causes a pressure on the concrete of the lower portion of the beam, which is in tension and may be cracked. The longitudinal shearing strength of the concrete must resist this force. Thus it is easy to see that the hook is of little or no advantage because of the concentration of the pull at an inadvisable point in the beam. It is therefore better to have the rod straight and extending an adequate distance beyond F or to bend it up and hook it into the compression side of the beam.

The rods of Fig. 3-7(a) which extend to K and M should be hooked as shown in Fig. 3-7(b), or A 305 bars should have a length of 8 to 12 in. over a stiff support in order to keep them from being pulled out. The hooks should have adequate cover on the sides because, otherwise, the concrete may spall off owing to its inadequate strength in tension due to the wedging effect of the hook.

A report of tests[1] made to ascertain the bond strength along horizontal bars gives, in part, the following tentative but interesting conclusions:

1. Straight horizontal rods fixed in position near the tops of beams may be weak in bond because the wet shrinkage or settlement of the concrete tends to cause voids under the bars. Vibration of the forms may help to remedy the situation. This feature may be a source of weakness in continuous beams.

2. Straight horizontal rods released so as to "float" in the concrete near the tops of beams develop good bond strength.

3. Horizontal rods with hooked ends but rigidly held vertically may develop harmfully large slip before the anchorage will be effective.

4. Straight horizontal rods in the bottoms of beams (and vertical ones) develop good bond.

5. End anchorages beyond the points of inflection of continuous beams are likely to permit large deflections of the beams before these anchorages become effective.

[1] R. C. Robin, P. E. Olsen, and R. F. Kinnane, Bond Strength of Reinforcing Bars Embedded Horizontally in Concrete, *J. Inst. Engs., Australia*, Vol. 14, No. 9, September, 1942; and *Highway Research Abstracts*, February, 1943.

6. Completely watertight forms tend to improve the bond along the top rods.

The data in item 5 are especially worthy of thought in connection with such beams as those pictured in Figs. 3-9 to 3-11, inclusive. They seem to show the desirability of bending the rods so that they can, before

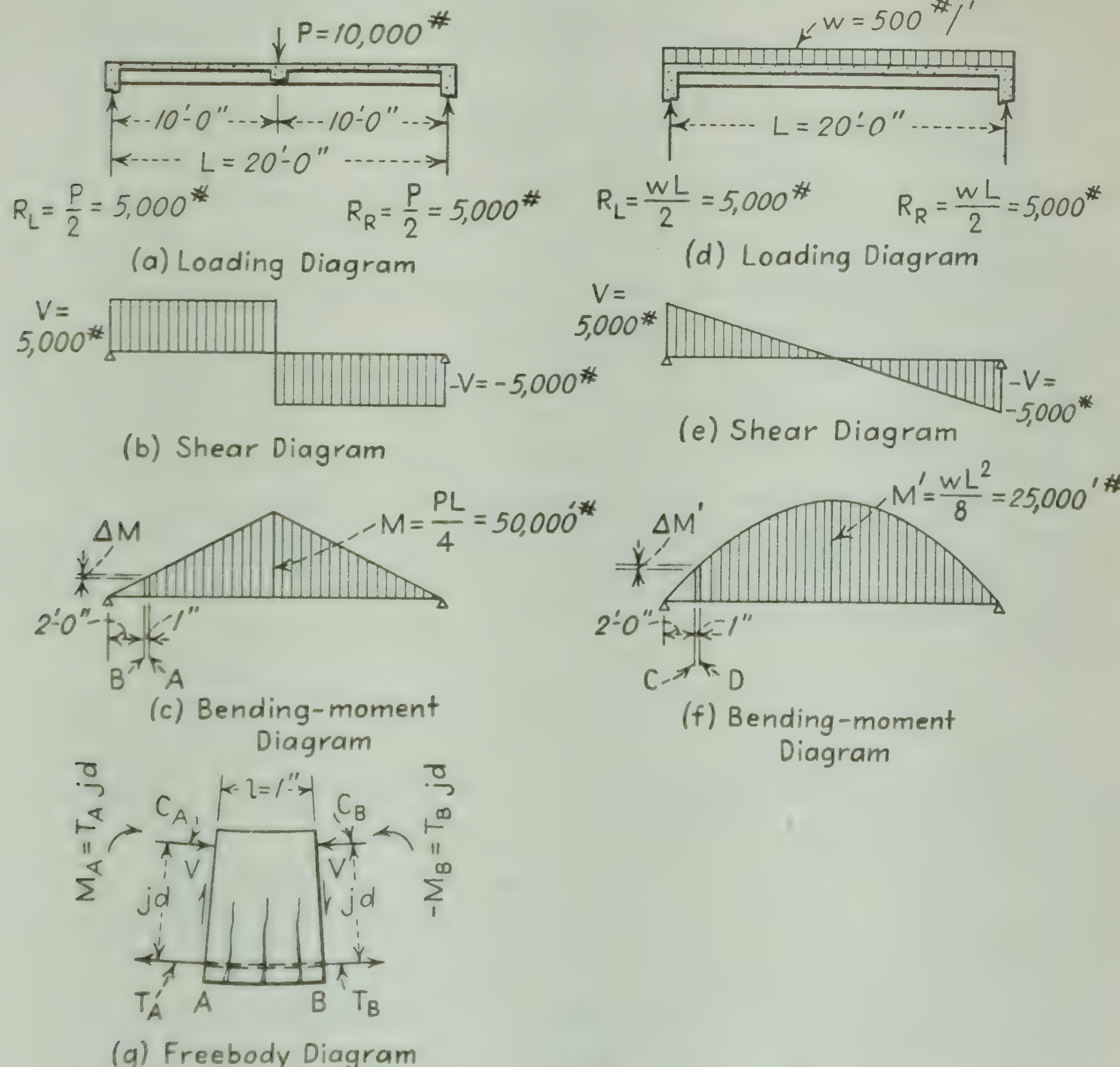


FIG. 3-8. Relation of bond to changes in bending moment.

failure, serve somewhat the same function as the cables in a suspension bridge.

3-6. Bond stresses on longitudinal bars in beams. It is important for a designer of concrete structures to be able to visualize clearly the action of bond on the reinforcement in a beam. As an example to aid in the understanding of this action, assume the two simply supported beams shown in Figs. 3-8(*a*) and (*d*). The respective shear and bending-moment diagrams are pictured in Sketches (*b*), (*c*), (*e*), and (*f*).

One way to investigate the bond stresses is to determine the change in the bending moment M in a short length of the beam. This change in

M produces a corresponding change in the tension in the reinforcement, and this change of tension is a measure of the bond stress required to develop it in the given distance. Therefore, for a distance Δx,

$$\Delta M = \Delta Tjd \qquad \text{but} \qquad \Delta T = u(\Sigma o)\Delta x$$

Therefore,

$$u = \frac{\Delta M}{(\Sigma o)jd(\Delta x)}$$

For example, using Fig. 3-8(c), the bond from A to B is

$$u = \frac{125{,}000 - 120{,}000}{(\Sigma o)jd} = \frac{5{,}000}{(\Sigma o)jd} \qquad \text{psi}$$

Since the change in ΔM is constant, u remains constant for one half of the beam if Σo and jd are constant also.

However, there is a more customary and convenient method for computing u. Let Fig. 3-8(g) represent to exaggerated scale a small piece of the beam of Sketch (a). Treating this as a free body in equilibrium, and balancing moments about C_B at the right-hand face,

$$(T_B - T_A)jd = V(\Delta x) = u_{AB}(\Sigma o)x(jd)$$

or, in general terms, and for tensile reinforcement,

$$u = \frac{V}{(\Sigma o)jd} \tag{3-5}$$

where V equals the total transverse shear in the beam at the cross section being considered and Σo equals the total surface area of the rods per inch of length of beam at that location—numerically equal to the total perimeter of the bars. From Eq. (3-5) it is clear that the bond stresses are greatest where the shear is the largest, where the total surface area of the rods is the least, and where jd is the smallest.

Referring to Fig. 3-8(b), Eq. (3-5) gives

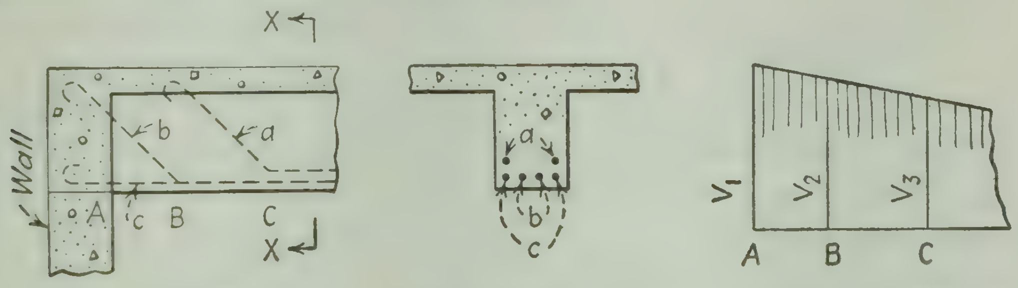
$$u = \frac{5{,}000}{(\Sigma o)jd} \qquad \text{psi}$$

This agrees with the previous calculation, as it should, and the shear diagram itself shows that the magnitude of u remains constant for one half of this beam if Σo and jd do not change.

Figures 3-8(e) and (f) produce similar results except that ΔM and V are changing from a maximum at the end to zero at the center.

To reiterate, regardless of how the computations are made, a difference in the stress in the reinforcement at two neighboring points along a particular bar requires that there be bond stresses acting upon the rods

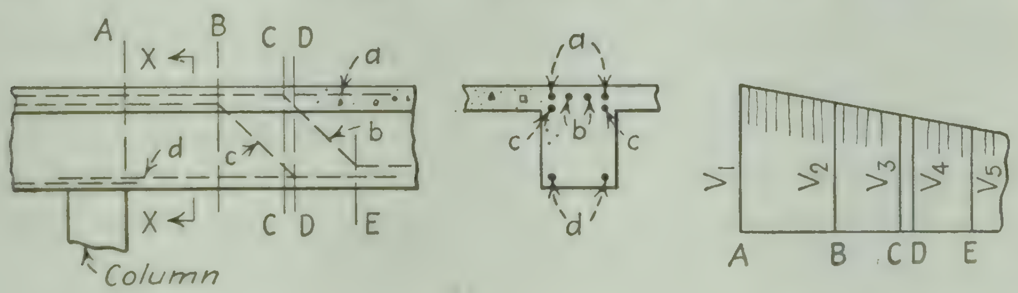
between these points in order to develop the change in stress. This is true whether the stresses are tensile or compressive, and even whether one is of the first type and the other of the second. The only danger to guard against is that of a required change of stress that is more rapid than the bond can provide. When the bond strength is exceeded the bar may slip, and then the member will no longer act in the manner intended by its designer even if it does not fall down.



(a) Side Elevation (b) Section X-X (c) Shear Diagram

FIG. 3-9. Beam with bent-up bars.

Assume a simply supported beam in which the rods are bent up as pictured in Fig. 3-9(a), this being done for purposes that will be explained in Chap. 4. In the portion near A of Sketch (c), the two rods marked c are the only ones that can be fully relied upon to resist the tension and



(a) Side Elevation (b) Section X-X (c) Shear Diagram

FIG. 3-10. Continuous beam.

the effect of the shear V_1 on the bond—and to develop the change of bending moment—although a and b help some in the region near their bends. In the portion from B toward C it is safest to assume that the four bars b and c resist the tension and the effect of V_2. To the right of C, the six bars a, b, and c are available for V_3. It is therefore obvious that the bond on rods c near A will be critical, and it may be serious. If the latter, it may be undesirable to bend bars b as intended, but they should be extended and bonded or hooked as are rods c.

Again, assume a continuous beam with rods bent up as pictured in Fig. 3-10(a). The tension is in the top near the column. Six rods a, b, and c are therefore available to resist the bond stresses in the tensile reinforcement caused by V_1 in the portion AB. It is advisable to rely upon only four rods a and b to resist the effect of V_2 in BC. Similarly,

the two rods a are available and trustworthy to resist V_3 and any tensile stresses at the top of the beam to the right of C.

In the bottom of the beam of Fig. 3-10(a), rods d are in compression. The bond stress on them may be tested by computing the stress in the concrete alongside them near A, then similarly 1 ft to the right of A. Then n times these computed stresses will represent sufficiently well the compressive stresses in the bars, and their difference is the amount of stress to be developed in the steel by bond in that 1-ft distance. This is seldom if ever critical.

In the vicinity of D and E of the beam of Fig. 3-10(a), or even at the left of D, tensile stresses may occur in the bottom of the beam for certain conditions of loading. If so, the two bars d are the only ones to be relied upon in the portion from A to near D; four bars c and d in DE; and six bars b, c, and d at E and to the right of E. The worst case for any particular portion of the beam may have to be found by determination of the loading conditions, bending moments, and shears that produce the highest bond stresses at that place. This may require some trial computations. At any rate, satisfactory bond stresses at one point may not mean that they are equally satisfactory at all other points. Furthermore, it should be remembered that rods d in the compression zone do not help bars a, b, and c perform their duties in the tensile region.

In order to illustrate the computation of bond unit stresses in a beam, assume the case shown in Fig. 3-11, which pictures half of a heavy continuous beam 24 ft long. Using Eq. (3-5) and the values in Fig. 3-11, but with d measured to the center of gravity of the group of rods, and with no reduction of the shear due to the width of the column, the bond stress on the eight No. 9 rods at A is

$$u_A = \frac{V}{(\Sigma o)jd} = \frac{59{,}600}{28.4 \times 0.872 \times 28.5} = 85 \text{ psi}$$

Also, approximately,

$$u_C = \frac{53{,}200}{14.2 \times 0.88 \times 30} = 142 \text{ psi just left of } C$$

There is another way of looking at this question of bond unit stresses. Figure 3-11(a) shows that a point of contraflexure (zero moment) is at F, the distance x from A to this point scaling 5.3 ft. Obviously, the stress in the rods at F should be zero. Using the moment at A and the properties of the section given in Fig. 3-11(d), the unit stress in these rods at A is

$$f_s = \frac{304{,}000 \times 12}{182} = 20{,}000 \text{ psi}$$

Therefore, this stress must be imparted to the steel through bond in the distance x. Since the shear is nearly constant from A to F, assume that

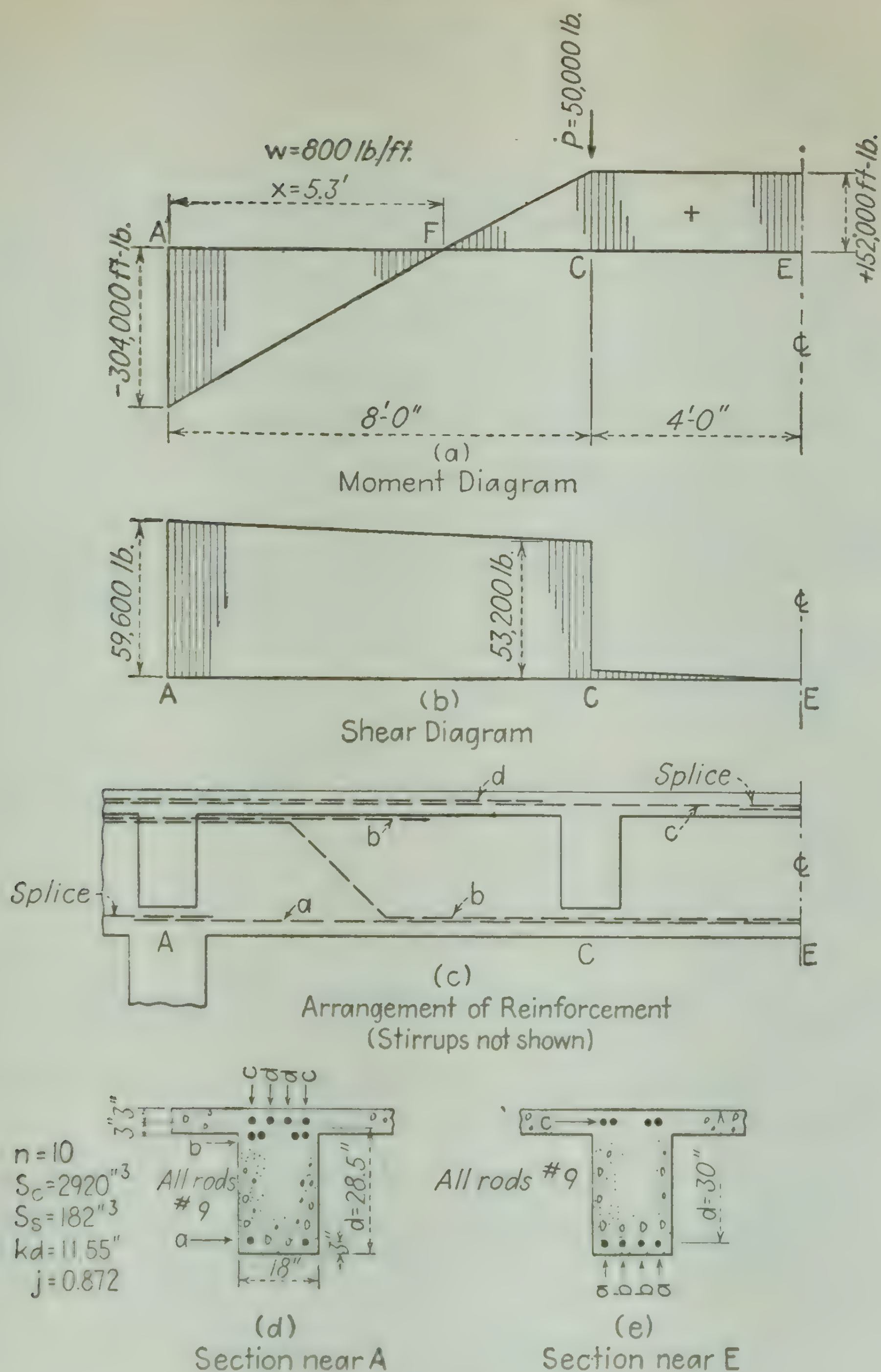


FIG. 3-11.

this "pickup" of tension is at a constant rate. Therefore, on this basis, the bond stress on one top rod is

$$u = \frac{f_s A_s}{x(\Sigma o)} = \frac{20{,}000 \times 1}{5.3 \times 12 \times 3.54} = 89 \text{ psi}$$

This is only slightly different from the result given by Eq. (3-5). Exact agreement of these figures is not to be expected nor is it generally important.

The foregoing method is particularly useful in computing bond stresses in members subjected to longitudinal thrusts or tensions combined with

bending. The basic idea is to compute the stress in the steel at a given point, then calculate it a foot (or two) from the first point. Obviously, the bond stress must be

$$u = \frac{\text{difference in unit stress} \times \text{area of a rod}}{\text{distance between points in inches} \times \text{perimeter of rod}} \tag{3-6}$$

Unless the direct forces are large compared with the bending, their effects upon the bond stresses are not important.

One must not forget that bond stresses also act upon rods in compression. However, the lower unit stresses generally existent in such bars

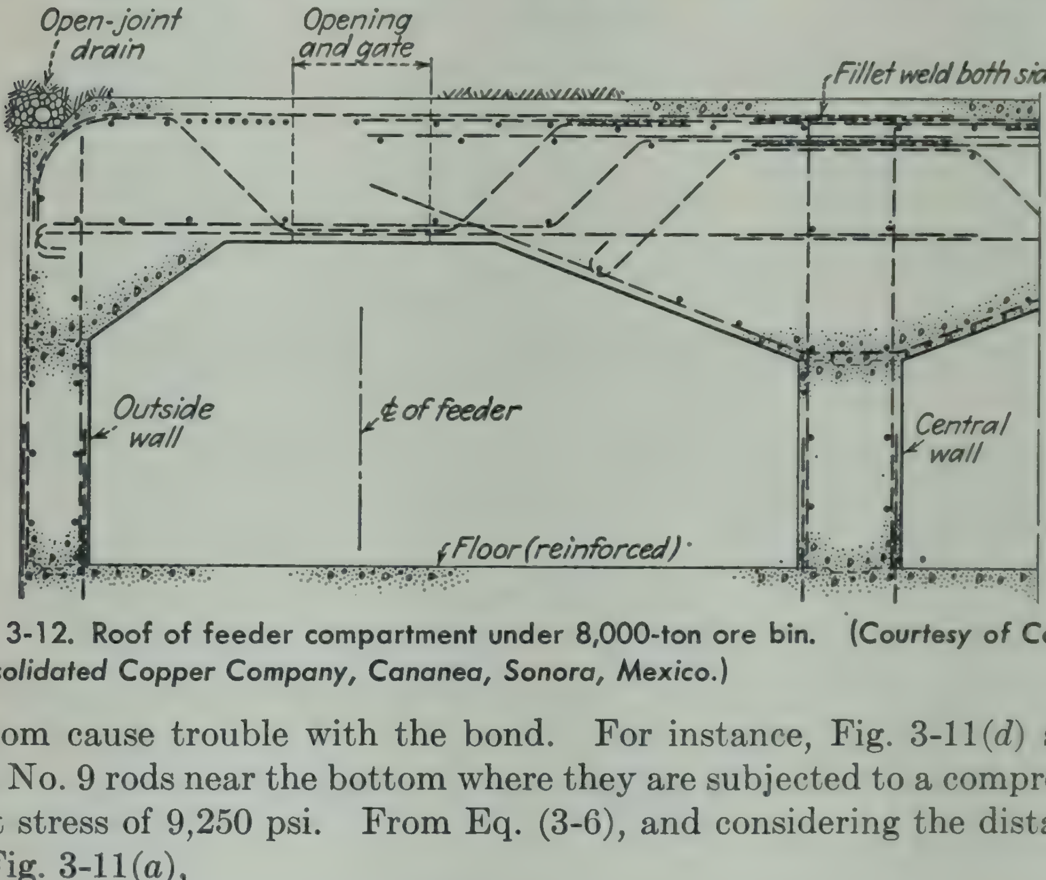


FIG. 3-12. Roof of feeder compartment under 8,000-ton ore bin. (Courtesy of Cananea Consolidated Copper Company, Cananea, Sonora, Mexico.)

seldom cause trouble with the bond. For instance, Fig. 3-11(d) shows two No. 9 rods near the bottom where they are subjected to a compressive unit stress of 9,250 psi. From Eq. (3-6), and considering the distance x in Fig. 3-11(a),

$$u = \frac{9{,}250 \times 1}{5.3 \times 12 \times 3.54} = 41 \text{ psi}$$

The designer is not so much interested in learning what the magnitudes of the bond stresses are as he is in making sure that they are not excessive. Bond generally becomes critical only in the case of short heavily loaded members such as footings, beams carrying offset columns, and other cases where the shears are relatively large compared with the bending moments.

When bond stresses are too high, or when the designer does not want to depend upon bond alone, it is often feasible mechanically to fasten rods to something that anchors them thoroughly. An example of this in industrial construction is shown in Fig. 3-12. The roof of this feeder

compartment must support a possible depth of ore of about 70 ft; hence it must be very strong. In order to ensure against bond failure, some of the critical splices are welded so as to make the rods continuous from one side to the other. Short diagonal rods might also be welded to the main bars to tie the structure together.

The Code states the following general rules for the anchorage of tensile reinforcement in beams:

1. Negative reinforcement in continuous, restrained, or cantilevered beams, and any member of a rigid frame, shall be anchored by bond, hooks, or mechanical anchors in or through the supporting member. Both the positive and negative reinforcement shall extend at least 12 diameters beyond the point where it is theoretically no longer needed to resist tension. Bars may be bent somewhat as shown for b in Fig. 3-11(c).

2. In continuous beams, not less than one-fourth of the positive reinforcement shall extend along the same face of the beam into the support a distance of 6 in.

3. At simply supported ends of beams, at least one-third of the required positive reinforcement shall extend along the same face of the beam into the support a distance of 6 in.

3-7. Distribution of bond stresses in simply supported beams. In the preceding article the bond stresses in beams have been discussed without regard to the cracked condition of the members when they are highly stressed. The situation, from a practical standpoint, deserves a further examination.

Assume the simply supported beam AB of Fig. 3-13. Bars a are shown extending the full length and terminating over the supporting columns. When a heavy load is applied at the center C, the beam deflects and may crack somewhat as pictured to exaggerated scale in (b). The length of the neutral axis remains unchanged, but the bottom corners and the slender columns each move outward some distance Δ. Rods a, being held by bond, are stretched until they offer sufficient resistance to produce equilibrium.

Now cut out an imaginary cracked piece of this beam as pictured in Fig. 3-13(c). Let the bending moment be computed about an axis in the vertical plane $X - X$ through the uncracked portion EO above the point where the crack FO or its projection will intersect the neutral axis of the beam. Then, neglecting the weight of the beam, the external bending moment at E is

$$M_E = R_A x = Vx$$

since $R_A = V$ in this instance. This causes a clockwise rotation that is counteracted by the internal forces represented by C' and T'. There-

fore, taking moments about C' in the face EO, and assuming that no stresses cross the crack except the tension in the steel,

$$M_E = T'jd \qquad \text{or} \qquad T' = \frac{M_E}{jd}$$

On the other hand, notice that this tension must be physically developed by bond on the rods from the end A to F. Similarly, the tension at crack OG is

$$T = \frac{M_D}{jd}$$

and T must be developed by bond between A and G. The difference $T' - T$ must be developed in the rods by bond in the distance GF. The action of the bond is similar for all portions from A to the center C.

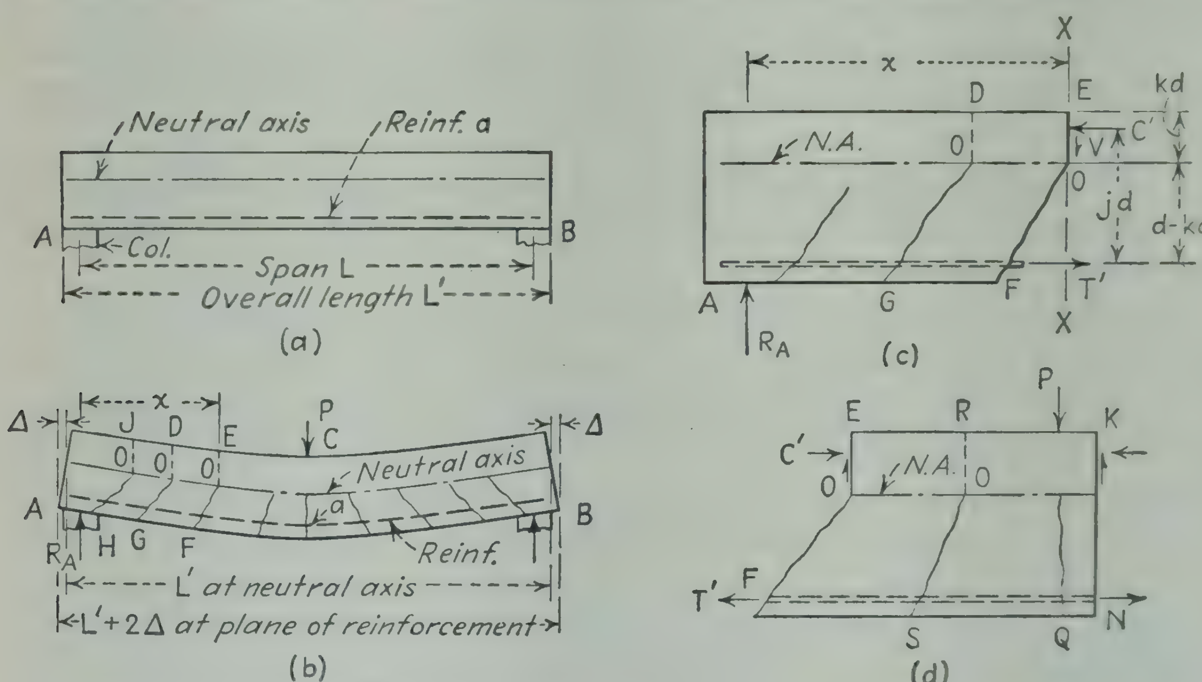


FIG. 3-13. Action of a simply supported beam.

Now notice the offset between the axis of moments and the point where the rods "emerge" from the assumed free body in Fig. 3-13(c). As assumed pieces closer and closer to the end of the beam are investigated, such as the piece from A to JOH, it is seen that, if the beam does crack close to the support, the bond length AH becomes very short whereas the computed tensile force needed for equilibrium is based upon a considerably longer distance from the reaction to the center used in computing the moment. Therefore, it seems that the bond strength needed near the end of a simply supported beam is greater than that indicated by the bending-moment and shear diagrams at that particular spot. This is a qualitative statement. The exact magnitude of the bond stress will be difficult to ascertain.

There may be some question regarding the effect of the portions of the beam at the right of the crack FO of Fig. 3-13(b). A triangular portion of this piece is on the left side of a plane through EO.

The remainder of the beam from EOF to the center is pictured in Sketch (d) as a free body. One may imagine that, as the beam bends, the cracked part $OFSO$ tends to thrust toward the left so that the bond adds to the tension in the bars, causing their stress to increase from F to S. There is a compressive reaction in the upper part $OERO$, but this compression is in a region that is at the right of section EO in the picture. Because of these things it seems to be justifiable to consider that the free body pictured in Fig. 3-13(c) is satisfactory and that bond stresses in portion FS of Sketch (d) do not affect conditions at the left of EOF.

If the cracks near the end are at 45 deg with respect to a vertical plane, the uncracked concrete that grips the reinforcement is offset a distance $d - kd$ toward the support from the center of moments. Actually, a crack may or may not exist close to the support. However, for design purposes, it seems to be advisable to make the beam so that it cannot fail even if such a crack does occur.

One might look upon Fig. 3-13(b) as though the compressive forces in the upper part of the beam tend to thrust downward and toward the ends, producing something that approaches the action of a tied arch. This concept also indicates that there is probably a tendency to cause large bond stresses near the ends of tensile bars in simply supported beams. The author therefore recommends the following procedure for simply supported ends of beams:

1. Assume that a crack crosses the tensile reinforcement a little beyond the edge of the support, as at H in Fig. 3-13(b).

2. Assume that the crack has a slope approximately 45 deg from the vertical.

3. Assume a center of moments, as in OJ of Sketch (b), that is located $\frac{2}{3}d$ beyond where the crack crosses the tensile reinforcement.

4. Compute the tension in the steel as required by the bending moment computed at this assumed center.

5. From the end of the beam to the assumed crack, provide enough length of reinforcement to develop the computed tensile force, using the full allowable bond stress u permitted by the specifications.

6. Elsewhere, compute the bond stress by use of Eq. (3-5).

7. Let this procedure be assumed to apply to concentrated or uniform loads or to any combination of loads on the beam, including its own weight.

8. Since the Code limitations for the allowable bond unit stress are based upon tests that have been interpreted by means of Eq. (3-5), it is desirable to retain them as controlling values even when one uses the extra allowances for lengths suggested here.

The preceding discussion indicates that, when the width of the support is small, some form of hook may be desirable in order to make sure that the ends of the reinforcing bars cannot slip. Of course, if the first actual crack is considerably farther from the end than assumed here, the intensity of the bond stress required is reduced.

Adequate hooks or other anchorages of bars at simply supported ends of beams are an insurance against failure by pulling out of the bars. Even though the bars may slip elsewhere and cause bad local cracking, the anchorages will enable the bars to hang on. The failure of the beam will then be likely to occur in flexure or in shear.

By similar reasoning, it seems that the required bond stress near the center of the beam of Fig. 3-13(*b*) may be less than that computed by the use of the shear diagram and Eq. (3-5). However, this has no important significance.

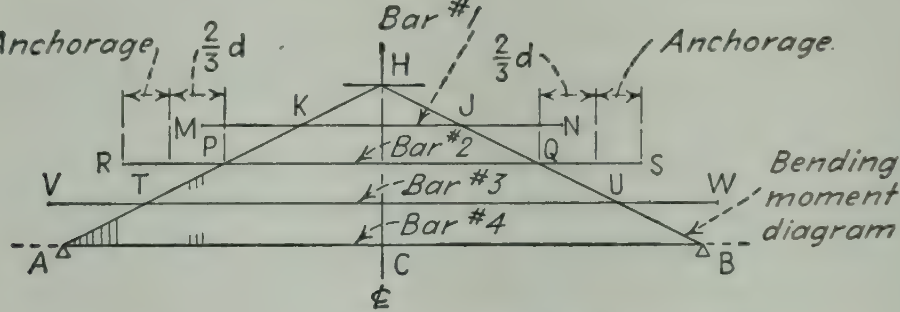


FIG. 3-14. Cutoff of longitudinal reinforcement.

On the other hand, if the beam acts as the cracked member of Fig. 3-13(*b*) indicates, it is advisable to be conservative in terminating any tensile reinforcement that does not extend for the full length of the member. If four bars are needed at the center of the beam of Fig. 3-13(*b*), draw the bending-moment diagram to scale, as in Fig. 3-14; then divide the middle ordinate of the diagram into four equal parts. Draw lines parallel to AB until they intersect the bending-moment diagram at K, J, P, Q, T, and U. Scale the lengths of KJ, etc.; then add a distance equal to about two-thirds of the effective depth of the beam to each end to allow for the offsetting effect. This is a minimum length for a particular bar. However, it is good policy to extend each bar a moderate distance beyond the point where it is supposedly needed in order to have a little chance to develop some of its strength through bond. Also, this reserve length will allow for some unusual loading that may increase the bending moment. This second addition to the length is to be determined by judgment for any given case. The author prefers to make this extra length 12 in. or sufficient to develop 50 per cent of the allowable tensile stress in the reinforcement, using the maximum allowable bond stress. In the case illustrated by Fig. 3-14, it is apparent that at least two of the bars should extend the full length of the beam.

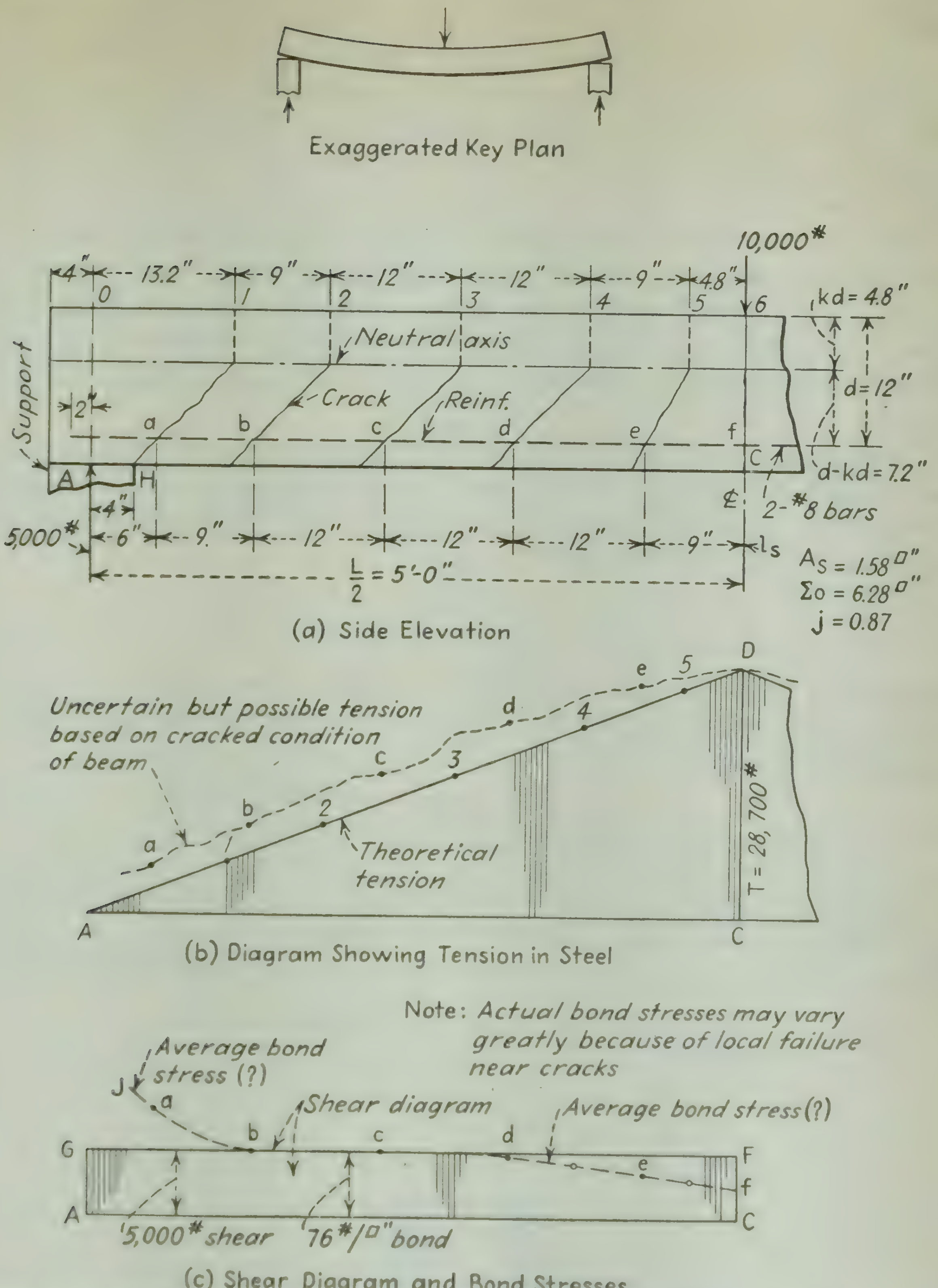


FIG. 3-15. Study of a simply supported beam with a concentrated load at the center.

Now, with the preceding ideas in mind, investigate the bond stresses in a particular beam in order to get some scale on what the offsetting may mean. Figure 3-15(*a*) shows a beam like that of Fig. 3-13(*a*). It is loaded by applying a concentration of 10,000 lb at the center, and the weight of the beam is neglected in order to simplify the computations.

Assume that $b = 10$ in., $d = 12$ in., $kd = 4.8$ in., $d - kd = 7.2$ in., $L = 10$ ft, and the reinforcement is two No. 8 bars. Assumed cracks are shown. The purpose is to get qualitative data. The cracks may occur at various positions and at various slopes. Those shown seem to be possible and reasonable, but there should probably be more of them near the center.

The bending moments are computed at sections 1, 2, etc. From these, the tensile forces are computed but they are assumed to be offset to points a, b, etc., as pictured in Fig. 3-15(a). The triangle ADC in Sketch (b) shows the diagram of tensile forces T as plotted for points 1, 2, 3, etc., and they are naturally proportional to the triangular bending-moment diagram.

To investigate the bond on the rods of the beam in Fig. 3-15(a), assume that the shear diagram is as pictured by the rectangle $ACFG$ in Sketch (c). According to Eq. (3-5), the bond stress will be uniform along the bars from A to C. Therefore,

$$u = \frac{V}{(\Sigma o)jd} = \frac{5{,}000}{6.28 \times 0.87 \times 12} = 76 \text{ psi}$$

However, if the tension at a is to be developed by the bond on the 8-in. length of bars shown at the left of this crack, the average bond stress along this length must be

$$u = \frac{T_a}{(\Sigma o)L_s} = \frac{6{,}300}{6.28 \times 8} = 125 \text{ psi}$$

Table 3-1 shows that the computed bond, or rate of pickup of tension, is substantially the same as given by Eq. (3-5) for points b, c, and d. Then it declines somewhat at e and f, as would be expected from the

TABLE 3-1. Study of Bond in Beam of Fig. 3-15

Point	M, in.-lb	$T = M/jd$, lb	ΔT, lb	l_s, in.	$u = \Delta T/(\Sigma o)l_s$, psi
1	66,000	6,300	6,300		
2	111,000	10,600	4,300		
3	171,000	16,400	5,800		
4	231,000	22,100	5,700		
5	276,000	26,400	4,300		
6	300,000	28,700	2,300		
To a			6,300	8	125
a-b			4,300	9	76
b-c			5,700	12	76
c-d			5,800	12	77
d-e			4,300	12	57
e-f			2,300	9	41

magnitudes of the tensile forces. These values for the average bond unit stresses between cracks are plotted in the position of the centers of the lengths l_s between cracks, as shown by the dotted lines in Fig. 3-15(*c*), and they indicate that a critical situation may exist near the ends of the beam considered. Close to the cracks themselves, the bond stresses must be considerably larger, with probable failure locally.

As a second case, assume that a 15,000-lb load is placed on the beam of Fig. 3-15(*a*) at a distance of 2 ft from the left end A instead of at the center, causing nearly the same bending moment as before. Then the reaction at A = 12,000 lb, and the bending moment at point 1 is 158,400 in.-lb. By Eq. (3-5),

$$u = \frac{12{,}000}{6.28 \times 0.87 \times 12} = 183 \text{ psi}$$

By considering the extreme cracked condition,

$$T = \frac{M}{jd} = \frac{158{,}400}{0.87 \times 12} = 15{,}200 \text{ lb}$$

and

$$u = \frac{\Delta T}{(\Sigma o)L_s} = \frac{15{,}200}{6.28 \times 8} = 302 \text{ psi}$$

This indicates that one should be more conservative than Eq. (3-5) may permit when the shears are relatively large.

As a third illustration, assume the same beam as before, with the same cracks, but apply a uniformly distributed load to produce the same maximum bending moment. This is pictured in Fig. 3-16(*a*). The end reaction is 10,000 lb. The results of the computations for tensile forces and bond are shown in Table 3-2. The tensile forces based upon sections 1,

TABLE 3-2. Study of Bond in Beam of Fig. 3-16

Point	M, in.-lb	$T = M/jd$, lb	ΔT, lb	l_s, in.	$u = \Delta T/(\Sigma o)l_s$, psi
1	117,500	11,200	11,200		
2	180,900	17,300	6,100		
3	244,500	23,400	6,100		
4	284,000	27,200	3,800		
5	298,000	28,500	1,300		
6	300,000	28,700	200		
To *a*			11,200	8	225
a-b			6,100	9	108
b-c			6,100	12	81
c-d			3,800	12	50
d-e			1,300	12	17
e-f			200	9	4

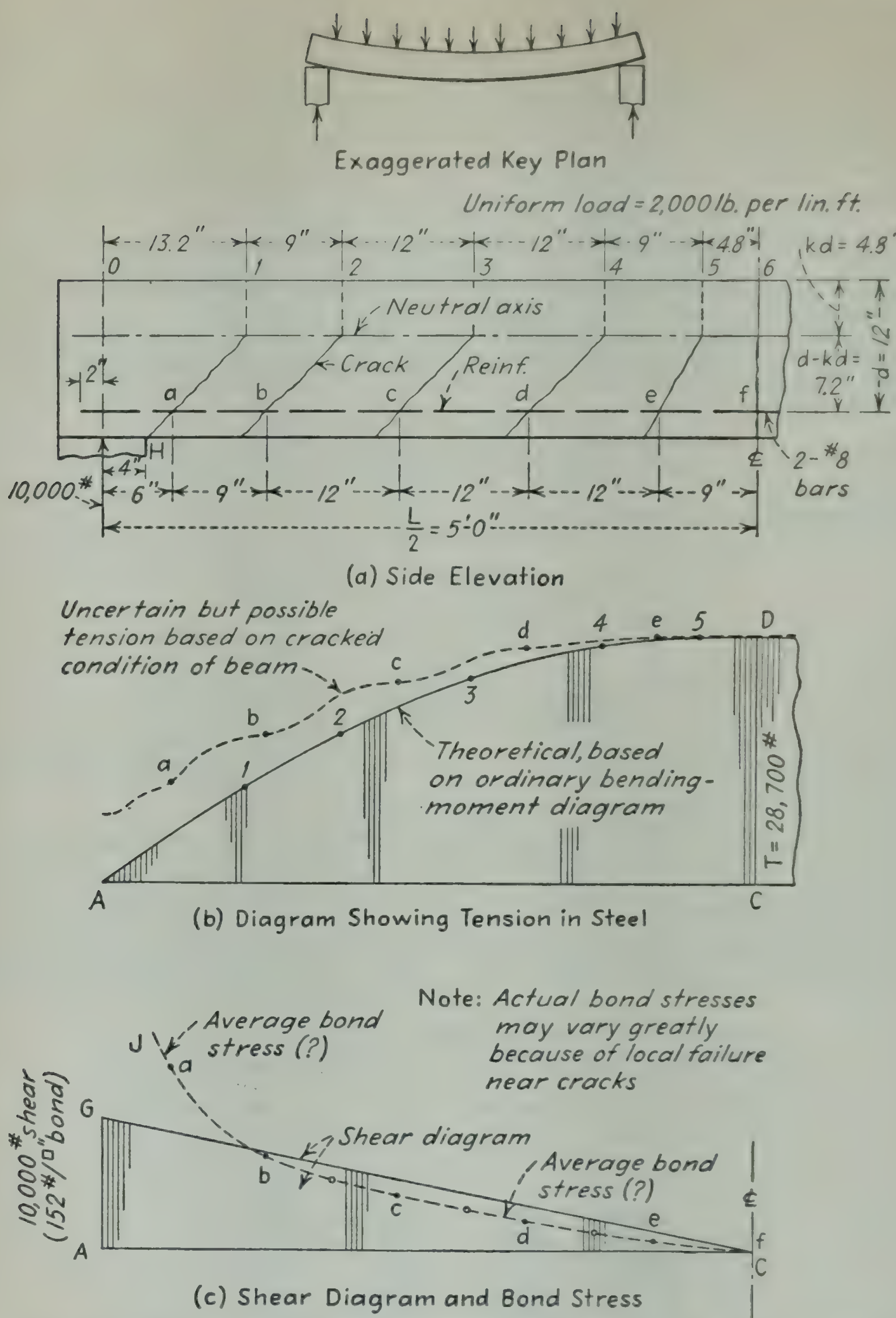


FIG. 3-16. Study of a simply supported beam with a uniformly distributed load.

2, etc., are given by the curved line AD in Sketch (b) which is founded upon the conventional bending-moment diagram. Figure 3-16(c) shows, by the triangle ACG, the magnitudes of the bond if computed by Eq. (3-5). The dotted line in (c) pictures the magnitudes of the average bond stresses when based upon the pickup of the tension for the assumed

conditions. Again it shows the larger stresses near the end, and the need for good anchorage of the reinforcement.

As a rough guide, assume that the distance from the center of the reaction to the first possible crack across the reinforcement = $\frac{1}{2}d$. Next assume that $d - kd = \frac{2}{3}d$. Then the bending moment to be provided for is the reaction

$$V(\tfrac{1}{2}d + \tfrac{2}{3}d)$$

Let

$$T = \frac{M}{\frac{7}{8}d} = \frac{\frac{7}{6}Vd}{\frac{7}{8}d} = 1.33V$$

Also,

$$T = (\Sigma o)uL_s$$

Therefore,

$$L_s = \frac{1.33V}{(\Sigma o)u} \text{ or } L_s = \frac{0.75V}{(\Sigma o)u} \text{ left of point } A \text{ (approx.)} \qquad (3\text{-}7)$$

In this equation, assume that L_s is the length of bar needed over the support itself, and it is a minimum. It may be increased if this seems to be desirable.

Remember that Eq. (3-7) is to be used only for simply supported ends of beams. The bars may be straight or hooked, provided that sufficient length is made available.

Adequate hooks or other anchorages of bars at simply supported ends of beams are an insurance against failure by pulling out of the bars. Even though the bars may slip elsewhere and cause bad local cracking, the anchorages will enable the bars to hang on. The failure of the beam will then be likely to occur in flexure or in shear.

3-8. Distribution of bond stresses in continuous beams. When a horizontal, continuous, reinforced-concrete beam is subjected to a small load, the tensions in the top over a support and in the bottom near midspan may be sufficient to cause only a few hair cracks in these regions. However, as the load is increased, the cracking intensifies and spreads until the beam may be cracked somewhat as shown to exaggerated scale in Fig. 4-16 when the customarily allowable unit stresses exist in the longitudinal bars. This condition is the one to be investigated to see what the bond stresses may be and to determine the proper locations of cutoff or bend points of the rods. Some qualitative data are desired; exact quantitative information is probably unattainable.

As an illustration, assume a continuous rectangular beam with a concentrated load at the center, as pictured in Figs. 3-17(a) and (b). Neglect the dead load of the beam in order to simplify the computations. Imagine that the beam has the cracks shown, with approximately 45° slopes except at the center and over the support at A. Rods R_1 and R_3 extend the full

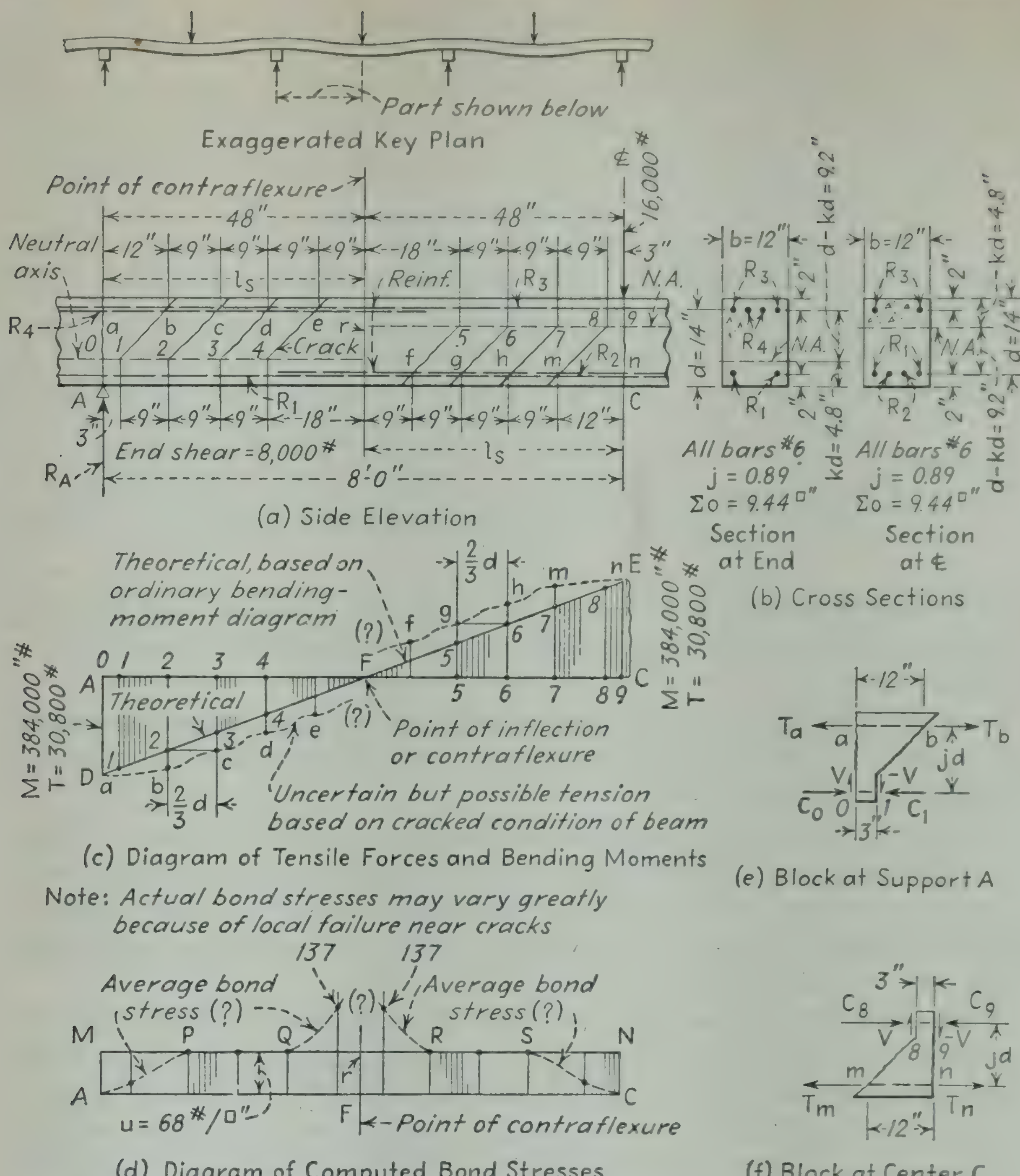


FIG. 3-17. Study of a continuous beam with a concentrated load at the center.

length of the beam but R_2 and R_4 are assumed to terminate as shown. Assuming balanced loads that cause the equivalent of a fixed-end condition for the beam, the bending moments at A and C are

$$M = \frac{WL}{8} \times 12 = \frac{16{,}000 \times 16 \times 12}{8} = 384{,}000 \text{ in.-lb}$$

The bending-moment diagram is shown by the triangles ADF and FEC of Sketch (c).

Analyze the beam as though it were made of a series of blocks, with

each one in equilibrium. Each block is bounded on the sides by a crack and a vertical plane through the intersection of the crack or its projection and the neutral axis. The end and central blocks are pictured in Figs. 3-17(*e*) and (*f*), respectively.

In Sketch (*e*), the left side is assumed to be cut just off the theoretical support so that V acts as shown. The tension T_a is

$$T_a = \frac{M}{jd} = \frac{384{,}000}{0.89 \times 14} = 30{,}800 \text{ lb}$$

If the moments of the forces shown in (*e*) are taken about the center of compression C_1, in the right-hand face,

$$\begin{aligned} -T_a jd + V \times 3 &= T_b jd \\ -30{,}800 \times 0.89 \times 14 + 8{,}000 \times 3 &= T_b \times 0.89 \times 14 \\ T_b &= 28{,}900 \text{ lb} \end{aligned}$$

The change in tension $T_a - T_b = 1{,}900$ lb. It must be produced by bond stresses on the embedded length *ab* of the tensile reinforcement in Fig. 3-17(*e*). This offsetting of T_b means that there is more distance in which bond can develop the change in tension near the support than the distance from the latter to the center of moments. Therefore, the magnitude of the required bond stress here is less than the bending-moment diagram indicates.

Similarly, the tensions at *c*, *d*, and *e* are offset *toward* the point of inflection with reference to their respective centers of moment, points 2, 3, and 4. By corresponding analyses the tensions at *f*, *g*, *h*, and *m* are seen to be offset *toward* the point of inflection with reference to their centers of moment, points 5, 6, 7, and 8.

This offsetting due to cracks on both sides of the inflection point at *r* in Fig. 3-17(*a*) and in a direction toward this point indicates that the required bond stresses in the vicinity of the inflection point must be larger than required by the bending-moment diagram in order to develop the tensions fast enough. Of course, the completeness, location, and direction of the cracks in the vicinity of the point of contraflexure are problematical. Table 3-3 merely shows the moments, tensions, and bond stresses computed upon the basis of the stated assumptions for this problem.

Figure 3-17(*d*) has been prepared to show the results of this problem and to picture the general idea graphically. It is recommended that the tensile reinforcement for negative bending over the supports of a continuous beam be designed by the use of Eq. (3-5) since this is conservative, but that special attention be given to the provision of adequate lengths for bond near the points of inflection. Therefore, considering the bending-moment diagram of Fig. 3-17(*c*) the tensile reinforcement

TABLE 3-3. Study of Bond in Beam of Fig. 3-17

Point	M, in.-lb	$T = M/jd$, lb	ΔT, lb	L_s between cracks, in.		$u = \Delta T/(\Sigma o)l_s$, psi
				Section	Length l_s	
0	−384,000	30,800	1,900	a-b	12	17
1	−360,000	28,900	5,800	b-c	9	68
2	−288,000	23,100	5,800	c-d	9	68
3	−216,000	17,300	5,700	d-e	9	67
4	−144,000	11,600	11,600	e-r	9	137
r	0	0				
5	+144,000	11,600	11,600	r-f	9	137
6	+216,000	17,300	5,700	f-g	9	67
7	+288,000	23,100	5,800	g-h	9	68
8	+360,000	28,900	5,800	h-m	9	68
9	+384,000	30,800	1,900	m-n	12	17

should be bent or terminated about $\frac{2}{3}d$ beyond the requirements of the bending-moment diagram. This is indicated in Fig. 3-18. The actual

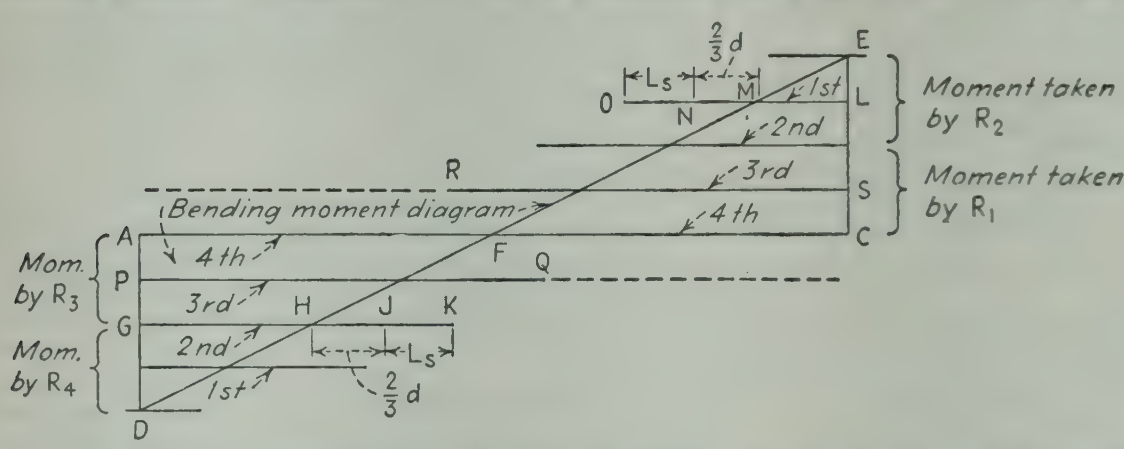


FIG. 3-18. Diagram showing cutoff or bend points of longitudinal bars in a continuous beam with a concentrated load at the center.

conditions and requirements in the vicinity of the point of inflection are unknown, but some extension of the bars here seems to be desirable for the top as well as for the bottom. Of course, when the bars are terminated, an extra anchorage allowance L_s of 12 in. or enough to develop 50 per cent of the allowable tensile stress is desirable for conservatism.

Conclusions of similar character will be found if one analyzes a continuous beam with a uniformly distributed load upon it or one with both concentrated and uniform loads.

When moving loads are applied, the bending-moment diagrams should be those which show the maximum bending over the support for various critical positions of loads as one extreme and the maximum bending in the central portion of the span as the opposite extreme. Bars should

then be bent or cut off as determined by these maxima individually with the $\frac{2}{3}d$ added for offsetting, plus any extra needed for anchorage before cutoff. Since such loading causes a shifting of the point of inflection, these critical diagrams are needed to make sure that none of the bars is terminated too soon.

The statements made for the end portion of continuous beams—from the support to the point of inflection—apply equally for cantilevered beams except that the point of zero bending is at the outer end or at the outer load. Good anchorage, and probably hooks, are desirable at the outer ends of the longitudinal tensile reinforcement of such beams.

3-9. Bond under impact and reversal of stress. Apparently the resistance of the bond of concrete to reinforcement under static or gradually applied loads is one thing; under shocks from suddenly applied loads it seems to be something else—and far less reliable. More data are needed on this subject, but experiences with reinforced-concrete structures under bombing appear to show that the concrete is likely to disintegrate and fly to pieces, leaving the rods behind as a grille of steel. This action may be due to the brittleness of the concrete and to the action of severe and rapidly traveling deformations of the steel, causing progressive local bond failures and cracking which "run" along the rods from point to point as the "wave" of sudden tensile elongation progresses through the steel. At any rate, structures designed to resist any such loads should be knitted together with a sort of steel cage having the junctions of main rods mechanically fastened in all directions—preferably by welding. In other words, the concrete should be a sort of filler between the rods, but the bond strength should be depended upon only slightly.

Rapid increases in the stresses in reinforcement, and quick reversal of these stresses, may also cause breaking of the bond. This may occur in continuous girders carrying heavy trucks, railroad trains, and cranes. Pending more reliable information than is now available, the designer should be unusually careful in checking into the bond situation. He should be conservative, basing his decisions upon the nature and importance of each special case.

3-10. Bond of concrete to structural plates and shapes. When a large I beam is covered with concrete, a strange combination results. The former is a strong ductile member, whereas the latter is relatively stiff and brittle. Obviously, the two materials do not willingly act in unison. The bond of the concrete to the steel must be great enough to compel the latter to act as a piece of reinforcement in the former, or else the two will break apart.

There is much uncertainty regarding the magnitude of the bond stress that can be developed when such heavy steel members are encased in this

way. The cross section of the steel may be large compared with its surface area; the surface of the steel is generally flat and smooth; there may be dirt, rust, or grease on the steel when it is embedded; the details of the concrete section may be such that it tends to break into isolated parts or chunks so that it becomes merely a filler. Therefore, the steel member must be properly anchored to the concrete. The greatest permissible value for the bond stress in such a case is the same as for plain rods.

The methods to use in designing such members are described in Chap. 5.

Practice Problems

3-1. Compute the necessary length through which two No. 8 rods must be lapped in order to splice them fully if f_s in the rods = 18,000 psi and the allowable bond stress = 150 psi.

3-2. Two No. 10 rods are lapped 20 in. If the allowable bond stress = 250 psi, what stress can be developed safely in the rods by such a lapped splice?

3-3. Two No. 11 bars in a beam extend 10 in. over their support, as pictured for bar d of Fig. 3-3(d). If the allowable bond stress is $0.07f'_c$, what unit stress can be developed in the rods? Assume $f'_c = 2{,}500$ psi for one case; then compare the result if $f'_c = 5{,}000$ psi. *Ans.* $f_{s1} = 4{,}970$ psi; $f_{s2} = 9{,}940$ psi.

3-4. Two No. 7 bars are to be spliced by lapping. If $f_c = 20{,}000$ psi, $f'_c = 3{,}000$ psi, and $u = 0.07f'_c$, what length of lap is needed?

3-5. Two No. 5 rods are lapped 24 in. The rods are stressed to 20,000 psi. What is the intensity of the bond stress that must be developed at the splice?

3-6. A No. 6 rod has a hooked end similar to that of Fig. 3-4(d). If $HJ = 3$ in., the allowable bond stress = 150 psi, and the radius of the inside of the bend = $4d$, locate the point at which the bond alone can be said to have developed 18,000 psi in the rod. *Ans.* $L_s = 8.9$ in. from tangent point.

3-7. Compute the maximum crushing stress on the concrete at the beginning of the hook of the rod in Prob. 3-6, point G of Fig. 3-4(d), using principles illustrated in Example 3-1.

3-8. Assume the beam shown in Fig. 2-24. If the maximum shear = 46,000 lb and $f'_c = 3{,}000$ psi, is the bond stress satisfactory?

Discussion. From Table 1-8, the allowable $u = 0.07f'_c$. Assume d to the center of gravity of the group of rods, 23 in. Call $j = 0.88$.

Ans. $u = 160$ psi and is therefore satisfactory.

3-9. Assume a continuous beam having the dimensions shown in Fig. 2-27. The top row of steel is four No. 8 rods; two other No. 8 rods are located 3 in. below these. Is the bond stress on the top tensile steel satisfactory if $f'_c = 3{,}000$ psi, max $u = 0.07f'_c$, $j = 0.88$, $d = 26$ in., and $V = 42{,}000$ lb? *Ans.* Yes, since $u = 98$ psi.

3-10. Using the beam of Prob. 3-9, assume that the bending moment at the support is 2,000,000 in.-lb and that it is 1,500,000 in.-lb 12 in. away from that support. If $n = 10$, $kd = 8.86$ in., $d - kd = 18.14$ in., $S_c = 2{,}130$ in.^{3}, $S_s = 104$ in.^{3}, and the top rods are as given in Prob. 3-9, compute the bond stresses on one of the top rods and on one of the rods in compression near the bottom, using the idea of bond developing the "pickup" of stress. *Ans.* $u = 100$ psi at top; $u = 35$ psi at bottom.

3-11. Assume a beam like that of Fig. 3-9(a). If the shear V near $A = 50{,}000$ lb, bars c are two No. 9, $d = 22$ in., $j = 0.9$, and the allowable $u = 210$ psi, is the bond safe? *Ans.* No. $u = 356$ psi vs. 210 psi allowed.

3-12. For the beam shown in Fig. 3-9(a), assume that the shear V in the section BC = 62,000 lb, bars c = three No. 9, bars b = two No. 9, d = 26 in., j = 0.88, and the allowable u = 210 psi. Is the bond safe?

3-13. Assume that rods a of Fig. 3-3(a) are No. 6 at 8 in. c.c., d = 8 in., j = 0.9, f_s = 18,000 psi, and u = 210 psi. The engineer in charge says that the bars are to be 50 per cent developed by the time they pass the edge of the 3-in. shelf. Design a means of doing this.

3-14. Assume the same slab and other conditions given in Prob. 3-13, except that the engineer wants these rods to be 50 per cent developed by the means shown in Fig. 3-3(b). The shelf is 3 in., the wall beyond it is 13 in. Design and dimension the rods b.

3-15. The rods at the bottom of a retaining wall are similar to the bent ones shown in Fig. 8-28. Assume that they are No. 11, the inner radius of the 90° bend is 15 in., the tensile stress at the top of the bend is 20,000 psi, the tensile stress at the tangent point at the bottom is 16,000 psi, and f'_c = 3,000 psi. Compute the average bond stress along the curved portion of the rod, and calculate the approximate pressure on the concrete supporting the top portion of the curved bar.

Discussion. For finding the pressure under the curve, assume that the tensile force at the top of the curve is continued around a semicircle with a 15 in. radius. From $T = pr$ and with a width of rod equal to 1.43 in., find p.

3-16. A simply supported T beam like that of Fig. 2-16 has a span of 24 ft c.c. of supports. It rests upon foundation walls 12 in. thick. Assume d = 28 in., b' = 14 in., bottom cover = 4 in., f'_c = 3,000 psi, u = 210 psi, and reinforcement is three No. 7 bars in one layer and three more 3 in. above it. The beam supports a uniformly distributed load of 1,500 plf. Determine the length and anchorage of the bars, and detail them in accordance with Art. 3-7.

3-17. Assume that the beam of Prob. 3-16 supports concentrated loads of 5,000 lb at two points 8 ft from each end, also a uniformly distributed load of 1,000 plf. Investigate the bond stresses and determine the points of cutoff and the anchorage of the reinforcement.

3-18. Assume the beam and loading conditions given in Fig. 3-11. Considering Art. 3-8, determine the proper bend points, cutoffs, and splices of the longitudinal reinforcement.

3-19. Assume the continuous beam shown in Figs. 3-11(d) and (e). The span is 30 ft, the load is 4,200 plf uniformly distributed, the ends are assumed to be fixed and the bending-moment diagram as given in Fig. 1 of the Appendix. Determine the bend points, cutoffs, and splices of the reinforcement.

4

SHEAR AND WEB REINFORCEMENT IN BEAMS

4-1. Introduction. The determination of the magnitudes and the distributions of the shearing stresses in reinforced-concrete beams is a very troublesome problem. Many tests have been made, and these have determined the ultimate shearing strengths of certain beams. Their results can be accepted as being correct for those particular cases only. It is clear that the total safe shear for any given member must be that which the tests show it to be. However, the question to settle is that of the probable action of the beam that one is analyzing or designing, and the best method of finding the shearing stresses in it or of predicting the shearing resistance that it can develop safely. In fact, much more experimental information is needed by engineers in order to ascertain the actual conditions of shearing stress in such members.

A large part of the testing that has been done in laboratories until recently has been performed upon beams that were much deeper with respect to their spans than those commonly used in actual construction. Furthermore, most of these beams have been simply supported. It is hoped that much more will be done to investigate the shearing stresses in beams of ordinary proportions and in those which are continuous.

Nevertheless, engineers have designed an almost countless number of structures of reinforced concrete that, in general, have proved to be safe. Because experience is a valuable teacher it seems that the principles used in their design should not be discarded lightly.

Reinforced-concrete beams are not homogeneous. When they are subjected to shearing forces, they therefore seem to behave in a manner that is peculiar to themselves. It should be admitted that this behavior is not well understood. The subsequent portions of this chapter are parts of a general attempt to clarify what may be the nature of this behavior, to present a reasonable and understandable theory to be used in estimating the shearing strength of such beams, and to determine how to proportion the reinforcement for them. The explanations given here are designed to make it possible to visualize more clearly what may be the

action of each element of the member—or at least to understand what that action may become before the beam will fail.

So little is now known about the application of the ultimate-load method to the shearing resistance of beams that this chapter will attack the problem on the basis of working loads. Allowable magnitudes for shearing stresses under various working conditions have been tested fairly well by long experience. Ultimate values may be assigned later, but, if so, the general method of analysis is likely to remain the same.

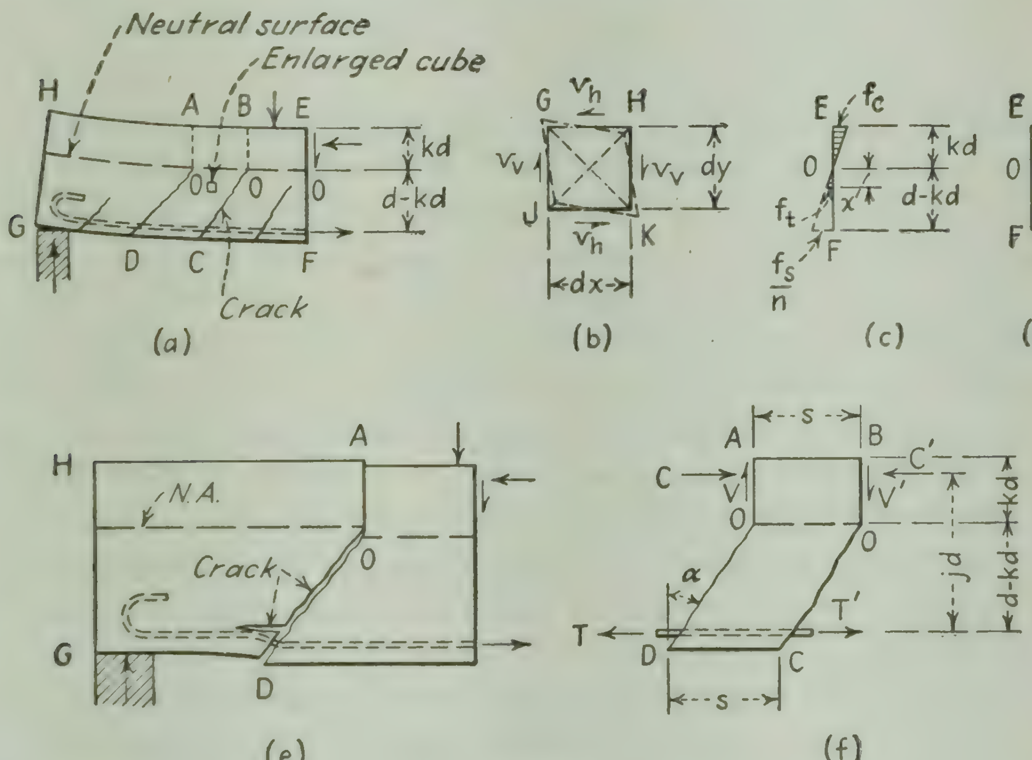


FIG. 4-1.

4-2. Determination of shearing stresses. Assume a reinforced-concrete beam that has transverse loads that cause shearing and bending stresses in it. Under this action, the beam will curve as shown in Fig. 4-1(*a*), producing compression in the top and tension in the bottom part of the member. The portion of this beam above the neutral axis will not crack open prior to real failure. However, as pictured for the lower region, cracks will form when the concrete is unable to elongate sufficiently to equal the deformation of the rods.

It seems that there are two cases to consider when studying the behavior of reinforced-concrete beams, *viz.*, the shearing stresses in the uncracked beam and those in the member after the cracks have formed. The latter condition is the one of practical importance. As stated in the preceding chapter, there must be transverse shearing forces to be resisted, and there are longitudinal shearing stresses that accompany them.

First, investigate the *uncracked* condition, neglecting the reinforce-

ment. Let Fig. 4-1(*b*) represent a small prism of concrete having a length equal to b, the width of the beam. Assume a vertical or transverse shearing unit stress v_v acting upon the face HK and an equal but opposite stress acting on GJ. These two forces constitute a couple having a moment equal to $v_v b(dy)(dx)$. Since the conditions for equilibrium require that there must be no rotation of this prism, there must be another couple which counteracts the first one. The latter must be composed of another set of shearing forces which act longitudinally on faces GH and JK. Assuming the intensity of the latter to be v_h, then

$$v_v b(dy)(dx) = v_h b(dx)(dy) \qquad \text{or} \qquad v_v = v_h \tag{4-1}$$

Equation (4-1) indicates that at any point in the uncracked beam there must be vertical and longitudinal shearing stresses of equal intensity. These tend to distort the material as shown in Fig. 4-1(*b*), causing the diagonal GK to lengthen and the other diagonal JH to shorten. Therefore, compressive stresses must exist on a plane passed through GK perpendicular to the paper, and tensile stresses must act on such a plane through JH. The latter is called *diagonal tension,* and its critical direction is assumed to be inclined at an angle of 45° from the beam's axis unless affected by other conditions. The intensities of these diagonal compressions and tensions on planes through GK and JH are each equal to v_h. Before any cracking of the concrete occurs, these shearing stresses may be distributed over the entire depth of the member somewhat the same as in homogeneous beams.

Now, consider the *cracked* condition of the beam. Referring to Fig. 4-1(*a*), it is clear that, when the beam bends sufficiently, the concrete will fail in tension somewhere below the neutral axis. To illustrate this, let $n = 10$, the allowable $f_s = 20{,}000$ psi, and the ultimate tensile strength of the concrete $f_t = 400$ psi. Then, from similar triangles in Fig. 4-1(*c*),

$$f_t : \frac{f_s}{n} :: x : (d - kd)$$

$$400 : \frac{20{,}000}{10} :: x : (d - kd) \qquad \text{or} \qquad x = \frac{d - kd}{5}$$

It is obvious that the concrete will crack below this limit x.

There are two kinds of tensile cracks to consider. The first ones are those which occur in regions of large bending moments but where the shear is very small or is zero. They are due to the elongation of the rods and to the concrete's inability to stretch equally. These cracks are usually normal to the axis of the beam, or nearly so. Figure 4-5[1] shows

[1] Frank E. Richart, An Investigation of Web Stresses in Reinforced Concrete Beams, *Univ. Illinois Eng. Expt. Sta. Bull.* 166.

some of these cracks between the two loads on the beams. The second kind of crack consists of those which are caused primarily by the combined action of the longitudinal tension and the transverse shearing forces applied to the beam. Henceforth, this combined action will be referred to as *diagonal tension*.

When a crack has formed it seems that the shearing and diagonal tensile stresses cannot be transmitted across the opening by the concrete alone, even though the opening is only a hair crack. Therefore, the solid part of the beam above the crack must be the thing which prevents the transverse failure of the member, since the shearing resistance of the tensile reinforcement as dowels cannot be relied upon or, at best, is relatively small and should be neglected. The resistance to the transverse shearing forces will therefore be assumed to be confined to the uncracked portion when the only reinforcement in the beam is longitudinal, as in Fig. 4-1(a).

Figure 4-1(e) is an attempt to illustrate this. The right-hand portion of the beam of Fig. 4-1(a) is pictured as though it had moved downward slightly with respect to the part beyond section AO and the assumed crack OD. The cracked surfaces will separate under this movement so that it is difficult to see how they will offer resistance to it. The dowel action of the bars will tend to spall off the concrete from D toward G, but this resistance is not trustworthy. The surfaces at AO have to fracture and then slide past each other to cause failure. The forces which cause this movement must overcome the shearing resistance of section AO. Even frictional resistance under the compression acting on AO will try to resist such transverse movement.

According to these assumptions, if v_T is the average transverse shearing stress in the portion above the neutral axis of Fig. 4-1(e), and if V is the total transverse shear at section AO,

$$v_T = \frac{V}{bkd} \tag{4-2}$$

However, this is seldom critical because the resistance of the concrete to such shearing seems to be very large. In short deep members like cantilevered beams and footings where the shearing forces are very large compared with the bending moments, and in simply supported ends of T beams where k is perhaps 0.2 to 0.3, this feature may deserve investigation. This is sometimes called *punching shear*. The allowable intensity for v_T is problematical. Perhaps the following is satisfactory as a guide:

$$\max v_T = 0.2f'_c \tag{4-3}$$

One unusual case of failure of a slab by punching or diagonal tension

is illustrated in Fig. 4-2. This structure is a tunnel under a coal pile at an industrial plant. The original design of the floor as made by the engineer was somewhat as shown in Sketch (*a*). Notice that the pile reaction at the center of the floor was supposed to be spread through a thick cap; then the floor slab itself was to have nominal bending, and compression from pressure on the side walls. Somehow or other the contractor omitted the thick pile cap, he deepened the floor somewhat, and he placed the same reinforcement near the bottom, as shown in Sketch (*b*). The piles in some places punched conical holes through the floor, somewhat as pictured by the indicated cracks.

Next, consider the longitudinal forces that are set up in the member when it is loaded. As the beam of Fig. 4-1(*a*) bends, it must develop the tension in the rods. In Chap. 3 it was stated that this change in tension in the reinforcement is produced by the bond stresses acting on the steel. To study this action further, assume that the portion of this beam marked *ABCD* is cut out as pictured in Fig. 4-1(*f*). Assume that the length s is short. The cracks are at some angle α, the maximum value of which is assumed to be 45° near the support. The section above the neutral axis *O-O* is cut by imaginary vertical planes. It seems to be obvious that, in horizontal planes above the rods, there must be a horizontal shearing force that equals the change in tension, $T' - T$. If the intensity of this horizontal or longitudinal shear is called v_L, the total shear just above the steel for a length s and a width b is

$$T' - T = v_L bs \tag{4-4}$$

However, for the same horizontal area, the total bond strength is

$$T' - T = u(\Sigma o)s \tag{4-5}$$

or

$$v_L bs = u(\Sigma o)s \tag{4-6}$$

Then, substituting $u = V/(\Sigma o)jd$ from Eq. (3-5),

$$v_L bs = \frac{V(\Sigma o)s}{(\Sigma o)jd}$$

or

$$v_L = \frac{V}{bjd} \tag{4-7}$$

This is the conventional and general formula for the magnitude of the shearing stresses in a reinforced-concrete beam in terms of the transverse shear V and the dimensions of the section, remembering that b is the width of the stem of a T beam or the full width of a rectangular beam. It means that one may then design or analyze a reinforced-concrete beam

for practical purposes as though there were a vertical shearing stress, also equal to v_L, that acts upon j times the width of the beam times its effective depth However, in this text v_L will be associated primarily with the longitudinal shearing stresses in order to avoid confusing it with Eq. (4-2). It also indicates that v_L may be considered to be distributed over the cross section of the beam somewhat as pictured in Fig. 4-1(d), this being in contrast to the assumed distribution of v_T. Notice that v_L and v_T are not equal. The effective depth d used in Eqs. (4-2) and (4-7) should be measured to the center of gravity of the groups of tensile bars. Table 1-8 shows the allowable values for v_L.

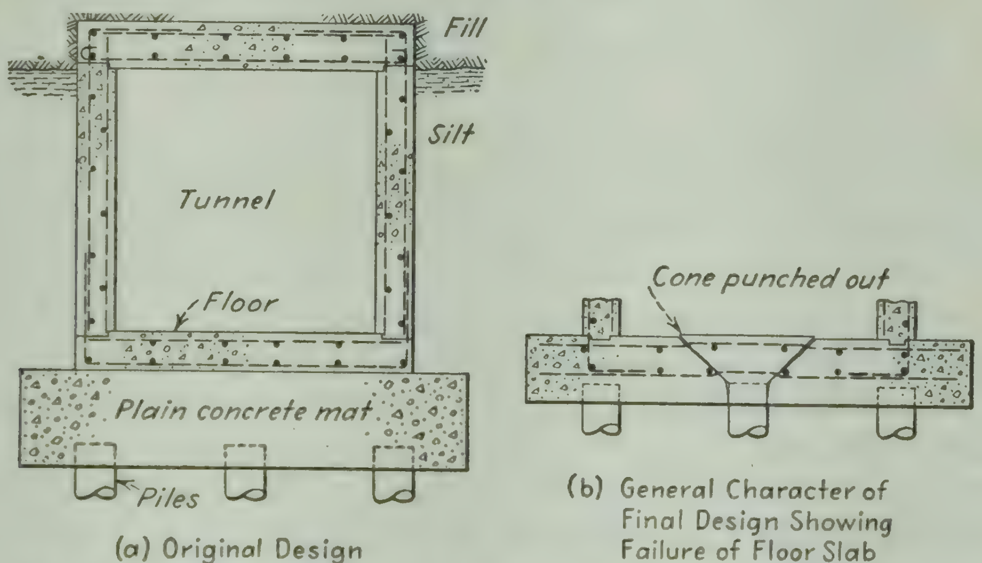


FIG. 4-2. Unusual example of failure by diagonal tension or punching.

Consider again the isolated piece of a cracked beam shown in Fig. 4-1(f), acting as a free body in space. The fact that a reinforced-concrete beam, with longitudinal rods only, does not always fail as soon as one of these cracks occurs seems to be due largely to the strength of the portions like $OCDO$ acting as short cantilevered blocks below the neutral surface O-O (locus of neutral axes), each block resisting the bending caused by the corresponding increment of tension $T' - T$. If the increment of tension per foot of length is not very great, and if the distance s between cracks is considerable, the bending stresses at O-O do not seem to be unreasonable for any particular piece. However, as the cracking increases and more cracks form close together the effect upon the safety of the member may become serious, and it can be seen that the strength of the member is severely limited. At any rate, this concept of the local action of portions of a cracked beam seems to be beneficial for the designer in visualizing what may happen to his structure and in planning how to prevent its failure.

Briefly reviewing the matter, notice that the real weakness of concrete in beams seems to be its lack of ductility. Its strength in true punching

shear is very great. However, when the steel elongates and the cracks open, the latter seem to extend farther and farther toward the compression side of the beam until it fails. This increasing lengthening of the cracks really accompanies a shifting of the neutral axis, and it causes a decrease of the real area of the concrete which can resist the transverse shear and the compression.

In the design of a reinforced-concrete beam with longitudinal reinforcement only, and when a maximum shearing stress V/bjd is specified, a trial value can be assumed for j equal to 0.87. When this is substituted in Eq. (4-7) along with the known or trial values of v_L, V, and b, the magnitude of a trial value for the effective depth d is readily found. After the dimensions and materials for the final beam are decided upon, v_L can be checked to see that it is satisfactory. Great refinements in computing the magnitude of j are not justified, however, because of the uncertainties that surround this whole question of shearing stresses. Unless the member is unusual, it is sufficiently accurate for one to assume that $j = 1 - k/3$, or even $j = 0.87$ or 0.88, when calculating v_L.

The reader may as well accept the fact that, regardless of all the theorizing, the true stress conditions for bond, shear, and diagonal tension are not readily determinable and, considering the variability of concrete, great accuracy is not to be expected. A clear visualization of probable action is a great help to the designer, however, but empirical and somewhat conservative methods of design are the best that can be expected.

Example 4-1. Assume a simply supported slab 9 in. thick which has a span of 9 ft and which has No. 5 rods 5 in. c.c. located 2 in. above the bottom. It carries a uniformly distributed live load of 600 psf. If $k = 0.37$, $j = 0.88$, $n = 12$, and $f'_c = 2{,}500$ psi, calculate the maximum vertical and longitudinal shearing stresses in the slab.

The weight of the slab equals

$$\tfrac{9}{12} \times 150 = 112 \text{ psf}$$

$$\max V = \frac{wL}{2} = (112 + 600)\frac{9}{2} = 3{,}200 \text{ lb}$$

From Eq. (4-7),

$$v_L = \frac{V}{bjd} = \frac{3{,}200}{12 \times 0.88 \times 7} = 43 \text{ psi}$$

From Eq. (4-2),

$$v_T = \frac{V}{bkd} = \frac{3{,}200}{12 \times 0.37 \times 7} = 103 \text{ psi}$$

Both of these are satisfactory.

4-3. Vertical stirrups. If the loads acting upon a beam are great enough to exceed the safe shearing stress of the concrete when it is used with longitudinal reinforcement only, then it becomes necessary to

strengthen the beam. This can be done by adding "web reinforcement" which will prevent the cracks from spreading and from causing failure of the structure.

One type of such web reinforcement is shown in Figs. 4-3(a) and (b), which represent a reinforced-concrete beam with vertical rods, called *stirrups*, placed in it as illustrated by EF. These stirrups are carried under the longitudinal reinforcement at E and are anchored into the concrete near F. The presence of these vertical rods makes considerable difference in the shearing strength of the beam.

This discussion of the action of vertical stirrups will include any web reinforcement that is placed perpendicular to the main longitudinal reinforcement of a beam. An ordinary horizontal beam is used here for convenience.

Examine what happens as the beam of Fig. 4-3(a) bends under increased loading. Before the concrete cracks, the stirrups are subjected to a stress of only n times that which is in the concrete beside and parallel to them. They have very little effect because of their small area compared with that of the concrete. In this case, the latter resists a large part of the longitudinal tension which is caused by bending. It also resists most of the diagonal tension which is caused by the shearing forces. However, as previously explained, the deformations accompanying the usual tensile stresses in the steel are so great that they compel the concrete to open up in "hair" cracks. Then the stirrups come into action prominently, and they resist the spreading of the cracks.

Next, analyze the action of these stirrups. To do so, assume that Fig. 4-3(c) represents the piece $ABCD$ of the beam of Fig. 4-3(a). Imagine that it is cut out along with the top portion of the adjacent parts of the beam and that it is placed between fixed supports. Then assume that the increment of tension $(T' - T)$ is applied as a horizontal pull on the longitudinal rods. This pull tends to break off the portion $OCDO$ about the neutral surface O-O, or about the faces AO and BO, an action that the concrete alone cannot resist well. Then, if the stirrup EF is in place, it will serve as an anchor or tie to stop this rotation. Thus it becomes clear that the force $(T' - T)$ sets up a diagonal compression D_c in the concrete and a tension T_s in the stirrup. The horizontal component of the diagonal compression will be called F_L, whereas its vertical component is F_T. Therefore, the stirrup EF, if acting without help from the concrete, must resist a pull T_s equal to F_T. In other words, $(T' - T)$, D_c, and T_s are three forces that are in equilibrium and that meet at a point. Therefore, their magnitudes must be proportional to the sides of a triangle, as shown in Fig. 4-3(d).

From all this, referring to Fig. 4-3(a), it is apparent that each stirrup acts as a tie to prevent the portion or portions that it occupies from break-

ing off. Each stirrup must withstand a force equal to the sum of the stresses acting upon the area that affects it, or $A_vf_v = F_T = F_L$ when D_c is inclined at 45°. Therefore, assuming cracks at 45°,

$$A_vf_v = v_L(bs) = \frac{Vs}{jd} \tag{4-8}$$

where A_v is the area of the rods composing one complete stirrup, and f_v is the intensity of the tension in the stirrup. This assumes that the concrete does not help the stirrup. However, this matter will be discussed further in the next article.

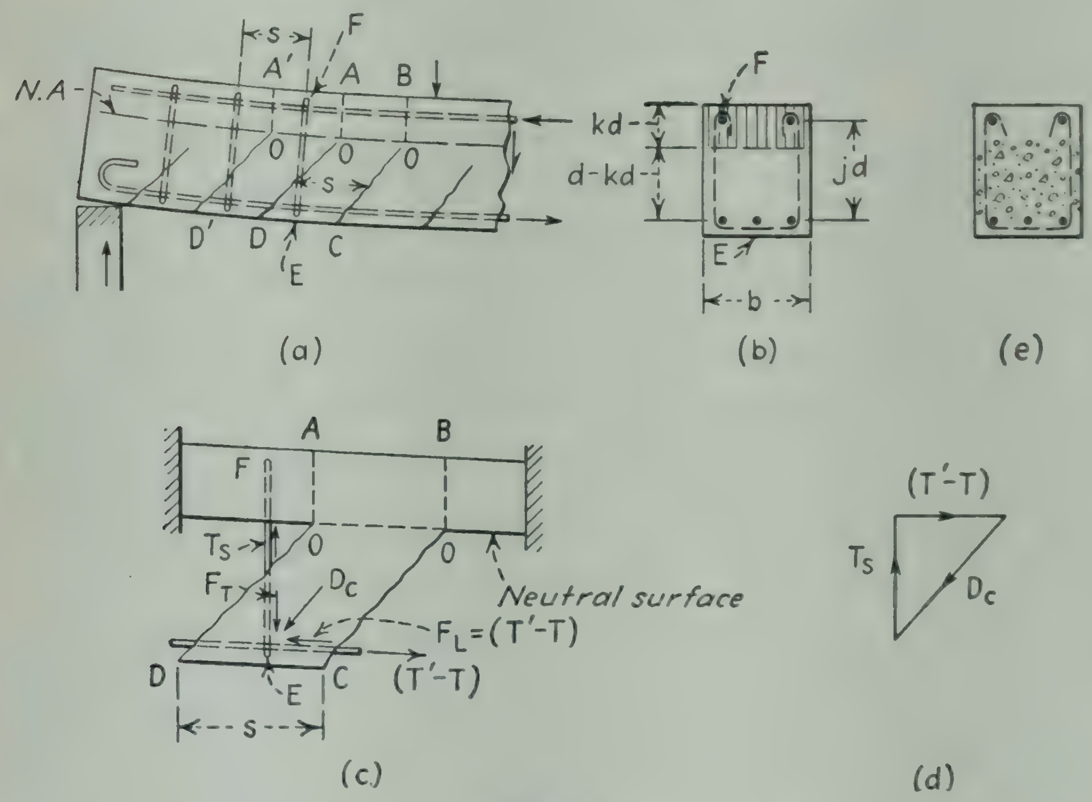


FIG. 4-3.

Equation (4-8) shows that the loads on the stirrups must vary with their spacing and increase with the vertical shear. Furthermore, from Fig. 4-3(c), one can see that the stirrups must not be too far apart. A practical maximum value for their spacing where stress or load conditions warrant is $s = d/2$.

The details of one type of stirrup can be seen by examining Fig. 4-3(b). The bottom is looped under the longitudinal reinforcement so as to get a mechanical grip around it and thus obtain a better chance to develop the stirrup. The top ends are hooked but are not overlapped, thus permitting the placing of the main rods after the stirrups are set in the forms. The reason for these hooks is obvious. The ability of the concrete to develop the necessary bond along the stirrup below the neutral axis is decreased because of the cracks. The rods should be developed by bond

in and near the region of compression. It seems to be safe to assume that the maximum depth of this portion that can be relied upon for bond is $0.5d$. Since this distance is relatively short, the hooks are needed in order to provide the necessary length of rod, which may be found from the formula

$$T_s = A_v f_v = (\Sigma o) u L_s \tag{4-9}$$

where L_s is the length of embedment which is needed to develop the allowable tensile unit stress in the stirrup. Generally, the requirements for bond strength compel the use of rather small rods for stirrups so as to have a large ratio of surface area to cross-sectional area. The close spacing needed anyway will seldom require the use of large rods as stirrups. The small ones also are easier to bend, and the hooks are not so large. It is also advantageous to use longitudinal tie rods as pictured in Fig. 4-3(b) so as to help develop the stirrups and hold them in position during the placing of the concrete. If such rods are used, the stirrups may be bent as shown in Fig. 4-3(e), a method that may help to hold them in better line at the top.

Vertical stirrups are so simple, they can be arranged so readily in cases of varying shear, and they can be set in the forms with the other rods so easily that they constitute one of the most practical systems of web reinforcement.

Figures 4-3(a) and (c) suggest that the beam may act somewhat like a truss. This truss action of stirrups is likely to modify the transverse shearing stresses in the uncracked portion of the beam, but this change is difficult to compute with any certainty. Therefore, v_T will be assumed to be confined entirely to the uncracked concrete.

Sometimes, specifications refer to the percentage of web reinforcement in beams. This simply means the cross-sectional area of the whole stirrup divided by the area (in plan) of the portion of the beam that it reinforces. In the case of vertical stirrups,

$$p_v = \frac{A_v}{bs}$$

4-4. Spacing of vertical stirrups. The Code states that web reinforcement should be used when the computed longitudinal shearing stress ($v_L = V/bjd$) exceeds $0.03f'_c$. Furthermore, tests reported by Prof. Richart[1] seem to indicate that the stirrups do not become stressed so highly as Eq. (4-8) would indicate. This may be caused by the fact that the concrete continues to resist the shearing forces and the diagonal tension as much as it is able to do, acting somewhat as it would if no stirrups

[1] Frank E. Richart, An Investigation of Web Stresses in Reinforced Concrete Beams, *Univ. Illinois, Eng. Expt. Sta. Bull.* 166.

were present. However, this combined action is very uncertain. Furthermore, it seems that the concrete will be less effective in its resistance as the cracking increases under the action of larger and larger loads. Eventually, the stirrups may have to hold the member together primarily as a sort of trussing.

These tests seem to indicate also that the stirrups near a load, as pictured in Fig. 4-4, have smaller stresses than those farther away. This probably is due to local compression under the load and to the decrease of the bond stresses and the "pickup" of the tension at this position as explained in Art. 3-7. Furthermore, when the ends of a simply supported beam are approached, as shown in Fig. 4-4, there seems to be another decrease in stirrup stresses.

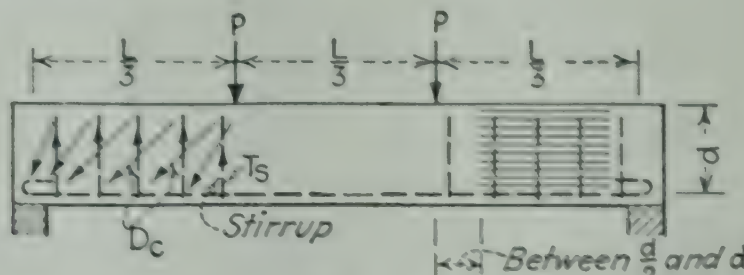


FIG. 4-4.

This may be caused by the fact that some of the inclined pressure lines may reach the support without fully affecting the adjacent stirrups. The left-hand portion of Fig. 4-4 is an attempt to picture this action by indicating the lines of diagonal compression in the concrete and the tension in the stirrups, which appear to form the web system of a sort of Howe truss. The shaded area of the right-hand part of this beam shows the region where the stirrup stresses may be the greatest.

Tests made by Arthur P. Clark[1] on beams with and without web reinforcement seem to indicate that, for the beams tested, the resistance to shear increased with an increase in the compressive strength of the concrete, with the percentage of tensile reinforcement, with the closeness of the loads to the ends of the beams, and "as the square root of the ratio of the web reinforcement" A_v/bs. He concluded that "the strength in shear varied as the compressive strength multiplied by a factor representing the ratio of depth of beam to distance from the plane of load to the plane of support." However, it seems that the results of these tests on short beams cannot be applied unquestioningly to the long ones customarily used in practice. Furthermore, much ordinary construction is made with continuous or restrained beams that are very different from the short simply supported ones tested.

[1] Arthur P. Clark, Diagonal Tension in Reinforced Concrete Beams, *J. ACI*, October, 1951.

Now examine Fig. 4-5 more carefully. It pictures the cracking of beams reinforced with the types of bars formerly used.

1. Beam 221.1 is one without web reinforcement that seemingly failed in bond by stripping at the left end. The crack probably then progressed up through the compression region as the chunk at the left broke off. The cracks between the loads are substantially vertical.

2. Beam 222.1 with hooked rods apparently failed because of diagonal tensile cracks that progressed up to the top. Some local slipping seems to have occurred near the left support.

3. Beams 223.1, 224.1, and 225.1 have vertical stirrups at varying spacing. In these, notice the multitude of diagonal tension cracks, especially in beam 223.1 where the cracks are close together and where each inclined crack generally passes across more than one stirrup. In beam 225.1, the stirrups at wide spacing obviously resulted in fewer but probably wider cracks, and there is more likelihood that some cracks can occur between the stirrups. This feature will be discussed further.

4. Beams 2210.1, 226.1, 227.1, and 228.1 have a flange to simulate T beams. A glance at the cracks and a comparison of them with those in the rectangular beams show that, at or near failure, the neutral axis of the T beams is much higher, even way up at or into the slab, as is to be expected.

In designing stirrups in reinforced-concrete beams, it is customary to assume that the concrete and the stirrups will act together and that the former will resist shearing forces up to the allowable working stress of $0.03f'_c$. Obviously the concrete itself is of some value. However, it seems that the cracking in a beam with strong closely spaced stirrups, as in beams 223.1 and 226.1 of Fig. 4-5, will render the concrete in the tensile region nearly worthless in shear at high loads when there are large unit stresses in both longitudinal steel and stirrups.

Figure 4-6 is made from Clark's report to show approximately the general range of the relation between the stress in the stirrups and the computed shearing stress—called v_L here—as he measured or determined them. f_v at first increases only moderately, as the loads and v_L increase, but it jumps rapidly after v_L reaches the vicinity of 200 psi. This seems to agree with the preceding speculation that the concrete is helpful at first but that the shearing resistance of the beam depends more and more upon the stirrups as the beam approaches failure.

The Code sets a maximum limit of $v_L = 0.12f'_c$ for beams with properly designed web reinforcement and with well-anchored longitudinal bars. This is too optimistic for most cases. It is far better to limit the computed shearing stress to more nearly $0.06f'_c$. Even this may not apply when f'_c exceeds 4,000 psi. In any case, the author prefers to limit the computed v_L to 250 psi as an extreme case.

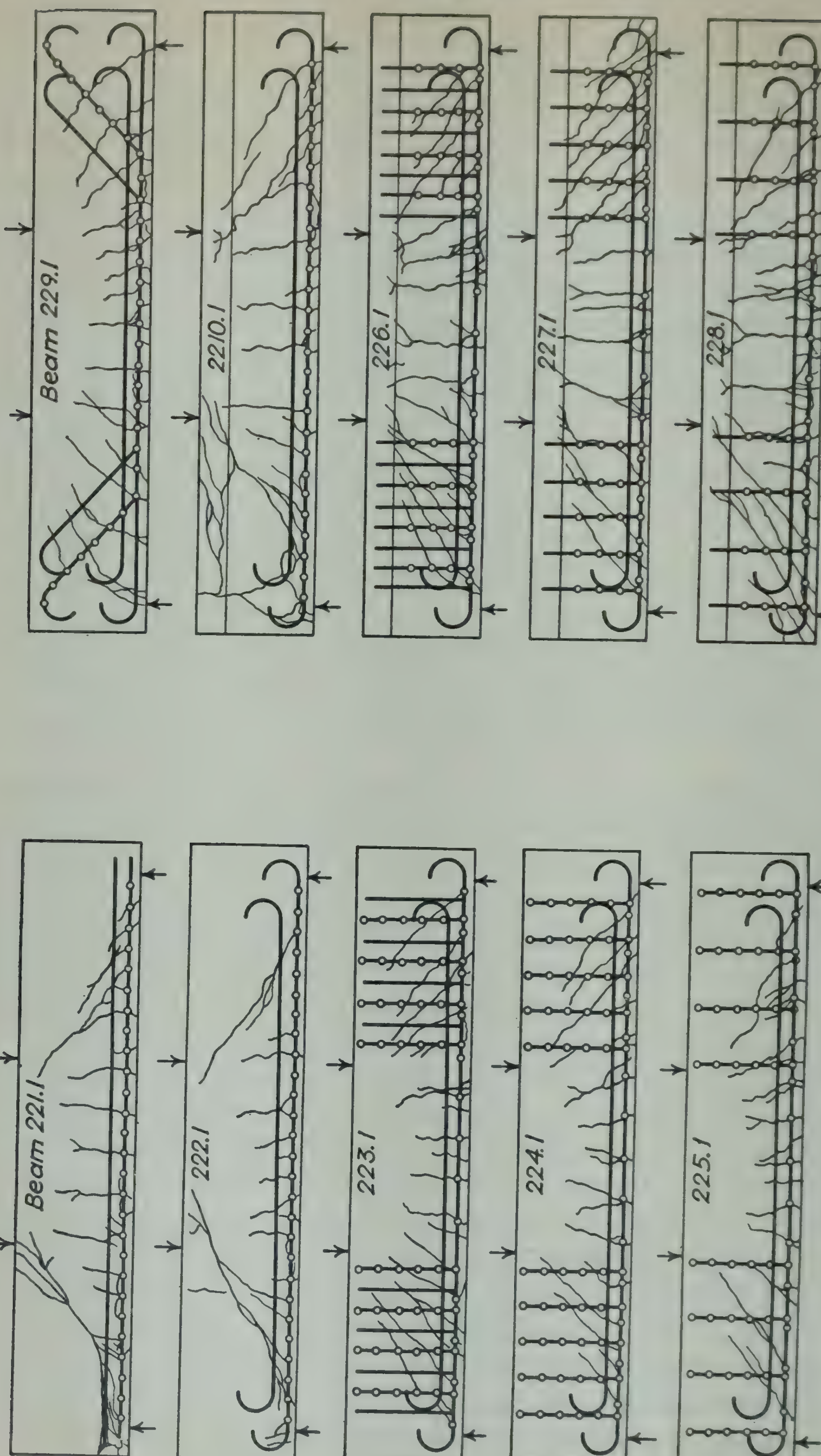


FIG. 4-5. Sketches of beams that were tested at the Engineering Experiment Station, University of Illinois.

Probably it is wasteful to disregard the concrete entirely in resisting shearing stresses at moderate working loads. However, the author prefers to do so when the computed v_L exceeds 200 to 250 psi. In such cases, the web reinforcement should be designed to support all the shearing forces, although a larger unit stress than 18,000 psi may be used for f_v.

It is worth while to emphasize again that any serious elongation of the reinforcement or any slip of the rods that causes excessive cracking of the concrete will automatically affect the shearing resistance that the beam

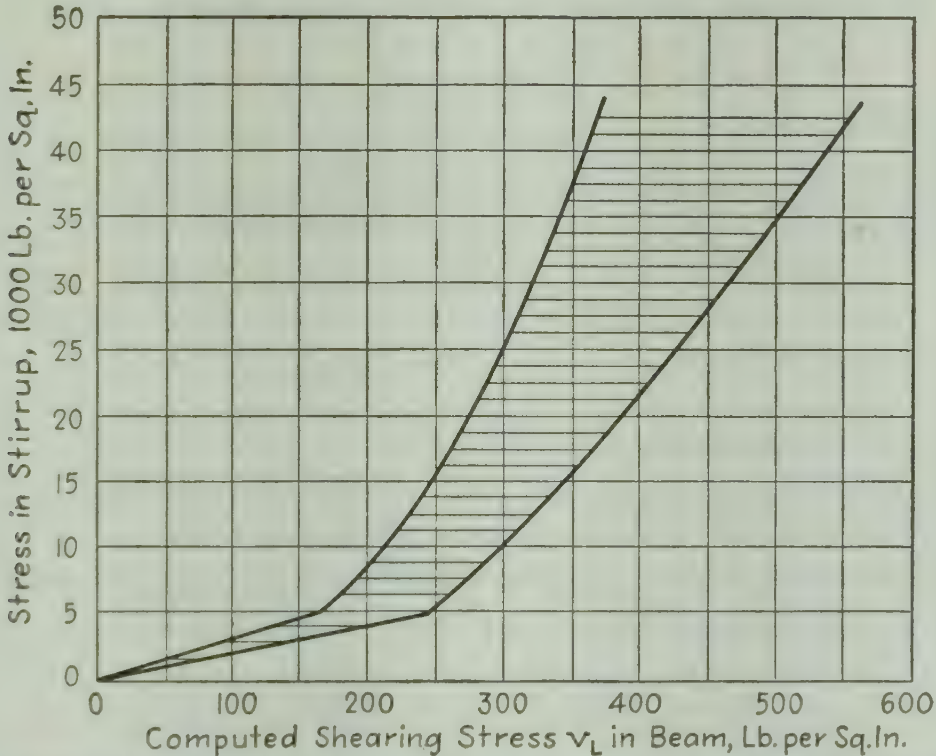


FIG. 4-6. Range of relationship of stirrup stress and shearing stress in beams as reported by Arthur P. Clark.

can offer. This is partly because the cracks will cause the neutral axis to approach more closely to the compressive edge, thus reducing the area of the uncracked concrete that can resist compression and transverse breaking. This is one of the disadvantages of the use of high tensile stresses in the longitudinal rods. With them goes more cracking and more need for effective web reinforcement. When web reinforcement is not desirable in members, as in footings, this feature should be considered, and a conservative value should be specified for the allowable tensile stress.

To illustrate the procedure when part of the shearing resistance is allotted to the concrete, let Fig. 4-7(a) be a diagram that shows the intensity of the longitudinal shear in one-half of a beam which carries a uniform load. (It must be remembered that the diagonal tension is the

force that really makes the stirrups necessary.) The value of the maximum ordinate AC is

$$v_L = \frac{V}{bjd}$$

If v'_L is the intensity of the longitudinal shearing stress that the specifications permit in the concrete without stirrups, then this value can be represented by AB. The stirrups must therefore resist the remaining shear which is pictured as the triangle BCD. It is clear that, from the center E to the point D of Fig. 4-7(a), no stirrups are needed theoretically. From D to B the longitudinal shear steadily increases. Therefore, it is

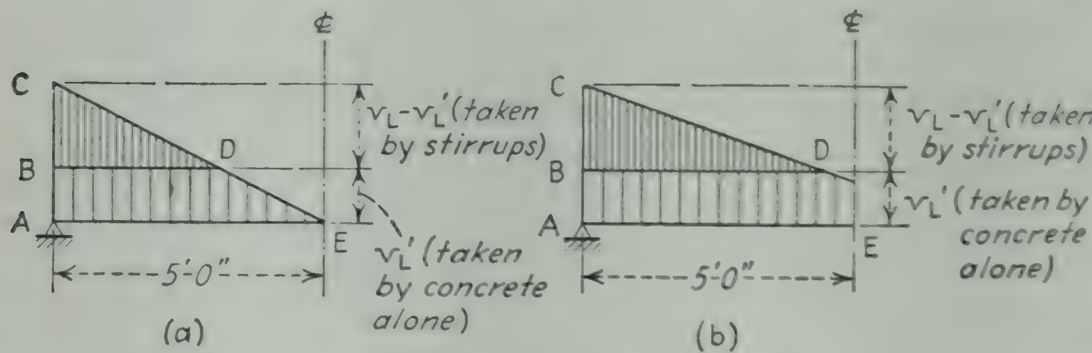


FIG. 4-7.

theoretically necessary to use stirrups of different strengths at uniform spacing or else to use stirrups of one given size with varying spacing. The former method is not generally advisable. To apply the latter plan, assume a size of rod, and find A_vf_v. Then from Eq. (4-8),

$$A_vf_v = (v_L - v'_L)bs \tag{4-10}$$

where $(v_L - v'_L)$ is the excess longitudinal shearing stress for which the stirrups are to be proportioned at any particular point.

Any beam that carries relatively large moving live loads or is subjected to a reversal of stress should have stirrups throughout its length, their spacing being not over $\frac{1}{2}d$. In extreme cases, the stirrups may be designed for the total longitudinal shear (or diagonal tension), but the permissible unit stress in them may be increased to about two-thirds to three-fourths of the yield-point stress. However, the results of computations based upon such an assumption should not control if the web reinforcement thus determined is less than that based upon Eq. (4-10), with f_v as specified in the Code.

The Code also states that, when v_L exceeds $0.08f'_c$, vertical stirrups should not be used alone. They should be combined with bent-up rods or inclined stirrups. The former combination is preferable. In such cases, the maximum spacing of the stirrups should not exceed about $\frac{1}{4}d$; the bent-up rods should be spaced at about $\frac{3}{8}$ to $\frac{3}{4}d$.

Some engineers assume that the minimum shearing stress in any beam is from 25 to 50 per cent of the maximum stress. In such a case, the shear diagram would be a trapezoid, as shown in Fig. 4-7(*b*), and the length in which stirrups are required would be extended. Apparently it is customary in some European countries to depend upon the resistance of the concrete until the diagonal tensile stress reaches the limit that is considered to be safe. Beyond that, when stirrups are needed at all, they are designed to withstand all the shear without dependence upon the concrete. This seems to be a sensible and conservative procedure.

Some additional data specified by the Code and applying to vertical stirrups, which also means any stirrups that are perpendicular to the longitudinal steel, are the following:

1. Stirrups must be anchored at both ends.

2. Adequate anchorage may be secured by welding to longitudinal steel, by hooking tightly around longitudinal reinforcement [as shown top and bottom in Fig. 4-3(*b*)], by having sufficient embedment above the mid-depth of the beam to develop the required stress by bond, and by using a standard hook (Table 10 of the Appendix) assumed to develop 10,000 psi together with enough more length between the hook and mid-depth of the beam to develop the required stirrup stress by bond.

3. The allowable bond stress for stirrups made of deformed bars is $0.10f'_c$; for plain bars, $0.045f'_c$.

If a beam is very deep compared with its span—perhaps 3 ft deep for a span of 10 ft—because the loads are extremely large and because stiffness is desired, the cracking due to bending will not generally be so severe as in slender members. Part of the shearing forces may be transmitted through the member to its supports by a sort of arch action if the supports are able to resist the spreading tendencies. The computed $v_L = V/bjd$ can be found, but the magnitude of v'_L for the concrete alone may be questionable. One should be careful how he trusts such uncertainties.

Furthermore, a beam that is also subjected to a large longitudinal thrust will probably be stronger in resisting transverse shearing forces than it will when it resists flexure only.

In any case, the student should remember that an adequate system of web reinforcement will give a beam considerable "toughness." It may crack and deflect badly, but it will not be likely to fail suddenly in diagonal tension as plain beams may do.

Example 4-2. Design the vertical U-shaped stirrups for the simply supported T beam shown in Fig. 4-8(*a*). It is 20 ft long, and it has a uniformly distributed total load of 3,000 plf. Assume $b' = 12$ in., $d = 20$ in., $j = 0.9$, and $f'_c = 2{,}500$ psi. Assume that the stirrups are to be made of No. 3 deformed bars. Let $f_v = 18{,}000$ psi, $u = 250$ psi, and $v'_L = 75$ psi.

$$v = \frac{wL}{2} = 3{,}000 \times 10 = 30{,}000 \text{ lb}$$

$$\max v_L = \frac{V}{bjd} = \frac{30{,}000}{12 \times 0.9 \times 20} = 139 \text{ psi}$$

Then, referring to Fig. 4-8(b), $BC = 139 - 75 = 64$ psi.

To find the point D where the stirrups must start, utilize the similar triangles, CDB and CEA. Then

$$BD:AE::CB:CA$$

$$BD = \frac{64 \times 120}{139} = 55.2 \text{ in., say, } 56 \text{ in.}$$

From Eq. (4-10),

$$A_v f_v = (v_L - v_L')bs$$

or

$$2 \times 0.11 \times 18{,}000 = 64 \times 12 \times s$$

Therefore, $s = 5.2$ in. at the support. Call it 5 in. Assume that the supporting column or beam is 12 in. wide as shown in Fig. 4-8(a). The first stirrup should be

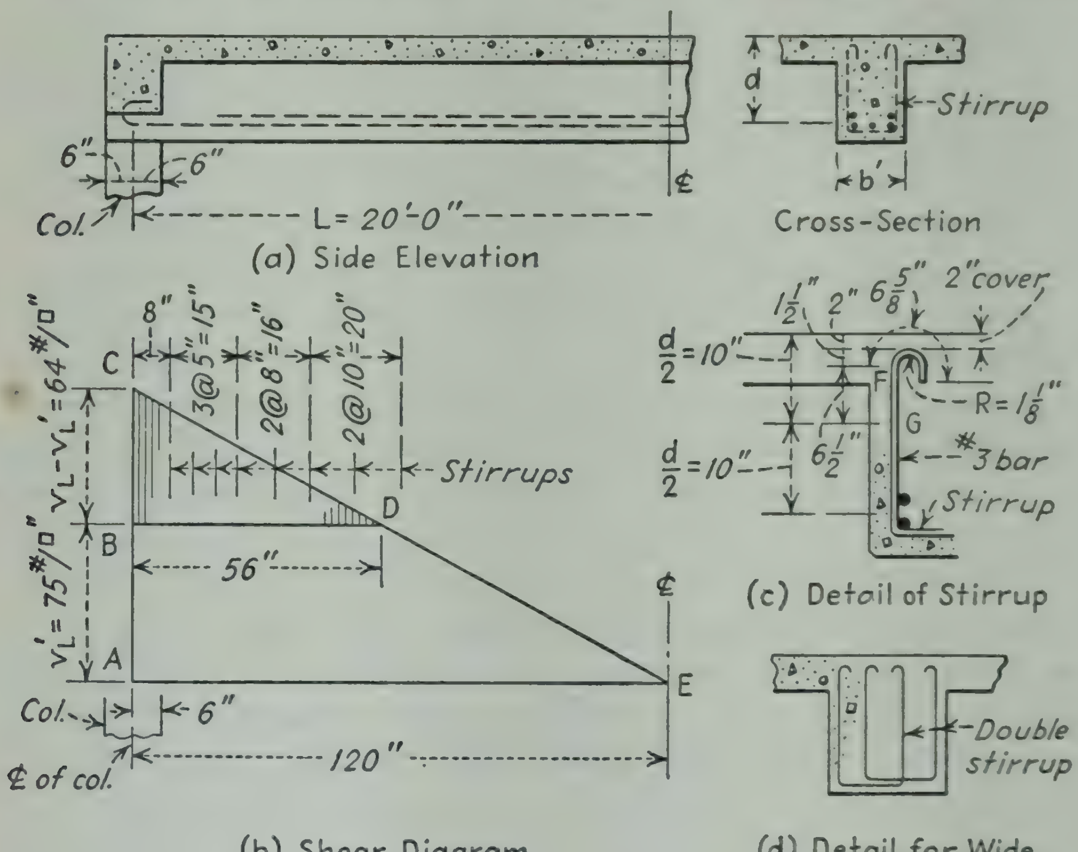


FIG. 4-8. Problem in spacing of stirrups.

placed near the edge of the support. It is customary to place this stirrup 2 in. or $\frac{1}{2}s$ or less from this edge. In this case, assume that it is 8 in. from the center of the column. Halfway along BD, the spacing can be twice as great as at the end, or 10 in., because $v_L - v_L' = 32$ psi. Since the stirrups should not be farther apart than $\frac{1}{2}d$, 10 in. is the maximum for this case. The spacing might be varied from stirrup to stirrup, but this refinement is not desirable in practice. It is customary to use a few stirrups at the minimum spacing, then a few more at a larger spacing, and finally the balance at $\frac{1}{2}d$ or a little less until no more are needed. However, it is advisable to extend the stirrups close to or somewhat beyond the theoretical terminus D in

Fig. 4-8(b), because of the possibility that heavier uniform or concentrated live loads may cause greater shears than are expected. The suggested arrangement is shown in the diagram.

Figure 12 in the Appendix can be used to advantage in finding the allowable spacing of stirrups. Having $(v_L - v'_L)$ at a given point, as obtained from Fig. 4-8(b), multiply by b or b'. Then, using the size of stirrups intended, the spacing is quickly found from the diagram. If a given spacing is desired, the required size of stirrup can be determined similarly. However, it is customary to use one given size of bar for stirrups in any particular beam. If the theoretical varying spacing of stirrups is desired for a triangular diagram, the methods shown in Figs. 15 and 16 of the Appendix can be used to advantage.

The length of embedment of the stirrups must be, from Eq. (4-9),

$$0.11 \times 18{,}000 = 1.18 \times 250 \times L_s \qquad \text{or} \qquad L_s = 6\tfrac{3}{4} \text{ in.}$$

Table 10 of the Appendix shows that $C + F$ for a standard hook for No. 3 bars is $6\frac{5}{8}$ in. However, the Code limits the value of a standard hook to the development of 10,000 psi in the bar. With $u = 0.10f'_c$, standard hooks would develop approximately 15,000 psi. This restriction required by the Code is desirable and conservative. In this case then the required length below point F is

$$L'_s = \frac{A_s(f_s - 10{,}000)}{(\Sigma o)u}$$

or

$$L'_s = \frac{0.11 \times 8{,}000}{1.18 \times 250} = 3 \text{ in.}$$

Since $6\frac{1}{2}$ in. is available in the distance FG, this detail is safe.

As a matter of interest, assume $k = 0.25$, as it may be for a T beam. Then compute v_T for this beam.

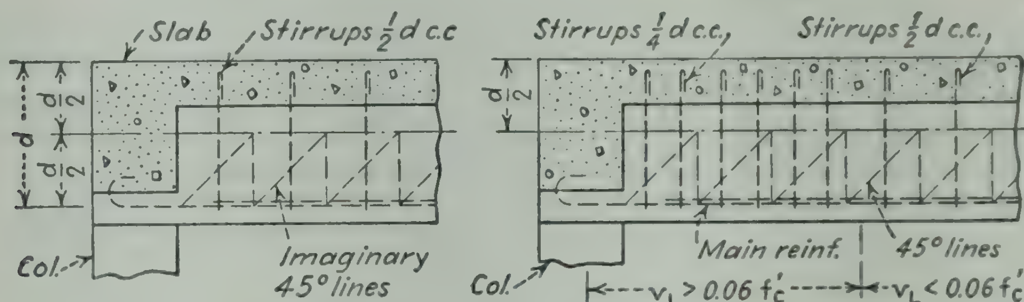
$$v_T = \frac{V}{b'kd} = \frac{30{,}000}{12 \times 0.25 \times 20} = 500 \text{ psi}$$

This is the assumed limit of $0.2f'_c = 0.2 \times 2{,}500 = 500$ psi. Since the flange or adjacent slab should not be relied upon to resist transverse shearing forces, this beam might well be widened or deepened somewhat.

Notice that the area of a U-shaped stirrup is $2A_s$ because both parts are stressed simultaneously. Sometimes, as in wide and heavy beams, it is desirable to obtain more area without having the spacing too small or the rods too large. Stirrups shaped like the letter W might be used in order to have four rods that will be effective in the stirrups. However, the radius of the bend at the top of the central loop must be adequate. The hooks of the outer rods must be just the same as for a U-shaped stirrup. Their chief disadvantage is the fact that they do not tie clear across the bottom of the beam. A better arrangement of stirrups for heavy beams is the use of two U-shaped stirrups placed in the same general cross-sectional plane with the two inner sides overlapping by several inches, as shown in Fig. 4-8(d), thus providing four effective rods for A_v. This is helpful since it is not wise to use stirrups of more

than ⅝ or ¾ in. diameter because they are difficult to bend, the hooks become large, and the anchorage may be difficult to secure. Furthermore, it is not advisable to use U-shaped stirrups that are too wide. There is no specified limit between the upstanding legs of a stirrup, but it seems to be desirable to use double ones when the spread will exceed 18 to 20 in. If the standard hooks on double stirrups interfere with the placing of the main reinforcement, the hooks may be turned longitudinally, made sharper, the outer ones turned outward into the slab, or another set of inverted U-shaped ones added as in Fig. 4-17(*d*).

The Code requires that, where web reinforcement is needed, the following should be adhered to:



(*a*) Computed v_L Less Than $0.06 f'_c$

(*b*) Computed v_L Greater Than $0.06 f'_c$ But Not Over $0.08 f'_c$

FIG. 4-9. Maximum spacing of stirrups.

1. When v_L is less than $0.06f'_c$, at least one line of web reinforcement must cross every 45° line below the mid-depth of the beam, as shown in Fig. 4-9(*a*).
2. When v_L exceeds $0.06f'_c$, at least two lines of web reinforcement must cross the 45° lines, as pictured in Fig. 4-9(*b*).
3. When v_L exceeds $0.08f'_c$, vertical stirrups are not to be used alone as web reinforcement.

4-5. Inclined stirrups. Another type of web reinforcement is pictured in Fig. 4-10(*a*) which shows a reinforced-concrete beam with the stirrups placed in an inclined position. Let Fig. 4-10(*b*) represent the portion *ABCD* when it is cut out and fixed in position as shown. Obviously, the stirrups serve the same function as the vertical ones previously explained. The increment of tension in the longitudinal reinforcement is again equal to $(T' - T)$. This tensile force, the diagonal compression, and the tension in the stirrup constitute a system of three forces which are in equilibrium and which meet at a point. Their magnitudes must be proportional to the sides of a triangle, as shown in Fig. 4-10(*c*).

The triangle of forces in this case, assuming no help from the concrete, can be constructed by laying off the longitudinal force $(T' - T) = v_L bs$, drawing a line at 45° from one end parallel to the assumed diagonal com-

pression, and then drawing the closing line from the other end, making it parallel to the stirrups.

Examining Figs. 4-10(b) and (c) discloses some interesting facts. The stirrup EF withstands part of the horizontal force $(T' - T)$ as shown by RW. If RU represents the stress in a vertical stirrup, then the stress in the inclined stirrup is RT, which is less than RU. The diagonal compression can be assumed to be at an inclination of 45° from the vertical. If the stirrups are also placed so that the angle β equals 45°, then the diagonal compression TS is $0.5US$, or one-half of what it would be with vertical

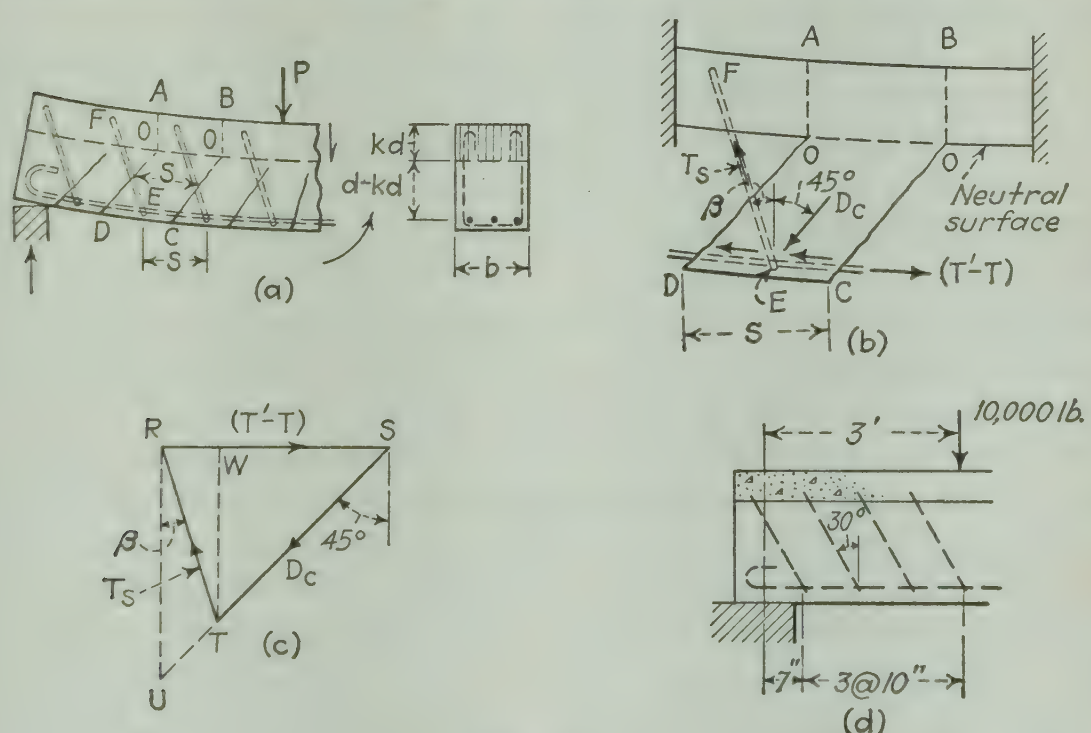


FIG. 4-10.

stirrups. The tension in the stirrup RT is $0.7RU$, which means that it is seven-tenths of what it would be if the stirrups were vertical. The value of RT for any value of the angle β can be found graphically or by using the following formula:

$$\frac{RT}{RS} = \frac{\sin 45°}{\sin (45° + \beta)}$$

$$T_s = A_i f_i = \frac{0.7(T' - T)}{\sin (45° + \beta)} = \frac{0.7 v_L bs}{\sin (45° + \beta)} \qquad (4\text{-}11)$$

$$A_i f_i = \frac{0.7 Vs}{jd \sin (45° + \beta)} \qquad (4\text{-}12)$$

where A_i = the area of the inclined stirrup, and f_i = the intensity of stress in it.

Allowing the concrete to take some share of the load, and remembering that v_L' represents the assumed safe longitudinal shearing stress in the

concrete alone and V_c is the safe vertical shear which it can withstand, then the preceding formulas become

$$A_i f_i = \frac{0.7(v_L - v'_L)bs}{\sin (45° + \beta)} \tag{4-13}$$

$$A_i f_i = \frac{0.7(V - V_c)s}{jd \sin (45° + \beta)} \tag{4-14}$$

Equations (4-11) and (4-13) will be used in this text in preference to Eqs. (4-12) and (4-14) because the former are expressed directly in terms of the longitudinal shearing stresses.

Although the inclined stirrups are more efficient that the vertical ones, there are practical considerations which offset this advantage. The stirrups should be mechanically fastened by welding them or otherwise connecting them to the longitudinal reinforcement in order to be sure that bond failure will not cause them to slip along the main rods. This work is expensive and troublesome. The stirrups must be held in place more firmly to prevent displacement of them during the depositing of the concrete. Furthermore, near the ends of beams, special short stirrups may have to be provided in order to get them in place at all.

4-6. Spacing of inclined stirrups. The spacing of inclined stirrups may be determined in the same general way as for vertical ones, using Eq. (4-13) instead of Eq. (4-10). Advantage may be taken of the increased length of embedment due to the slope of the stirrups when considering the length that is required to develop them through bond.

The maximum horizontal spacing of the inclined stirrups may be increased over that for vertical ones. A satisfactory limit may be set at $\frac{1}{2}d \sec \beta$, with a maximum of $\frac{3}{4}d$.

Example 4-3. Design U-shaped stirrups inclined at 30° from the vertical for the simply supported T beam shown in Fig. 4-13(g). It has a span of 15 ft, a uniform load of 800 plf including its own weight, also concentrated loads of 10,000 lb located 3 and 5 ft from each end as pictured in Fig. 4-13(a). Assume $n = 12$ and the allowable f_s, f_c, u, and $v'_L = 15{,}000$, 900, 175, and 75 psi, respectively.

By the methods of Chap. 2, using the transformed-section method and $d = 22$ in. to the extreme row of bars, k is found to be 0.28. Then $j = 0.91$ (approx).

The end shear equals $V = 800 \times 7.5 + 10{,}000 + 10{,}000 = 26{,}000$ lb. Therefore,

$$v_L = \frac{V}{b'jd} = \frac{26{,}000}{12 \times 0.91 \times 22} = 108 \text{ psi}$$

Under the first concentrated load, 3 ft from the support,

$$v_L = \frac{26{,}000 - 2{,}400 - 10{,}000}{12 \times 0.91 \times 22} = 57 \text{ psi}$$

Thus stirrups are needed only in the part of the beam from the end to this first concentration.

Notice that the use of $d = 20.5$ in. to the center of gravity of the tensile steel

would give large computed values for v_L. This would therefore be conservative, and it is the recommended procedure.

Assume No. 3 rods for the stirrups. Then, from Eq. (4-13),

$$2 \times 0.11 \times 15{,}000 = \frac{0.7(108 - 75)12s}{\sin(45° + 30°)}$$
$$s = 11.5 \text{ in.}$$

To place these stirrups, start with the bottom of the first one about 7 in. from the support as shown in Fig. 4-10(d). Then add three more at 10-in. spacing horizontally. This will bring the last one a little beyond the first concentration, which is ample. The difficulty of getting the first stirrups in place is obvious—unless the support is rather wide. Admittedly, this arrangement is chosen arbitrarily, but such decisions are very common in the design of reinforced concrete.

The necessary length to develop the required bond at the top of the stirrup is

$$L_s = \frac{A_i f_i}{ou} = \frac{0.11 \times 15{,}000}{1.18 \times 175} = 8 \text{ in.}$$

However, the stirrups should be welded to the main bars.

4-7. Bent-up rods. Inasmuch as the longitudinal reinforcement in a beam is determined by the maximum bending moment, it is obvious that all the rods are not needed where the bending moment is smaller. Furthermore, elimination of part of the rods will compel the remaining ones to withstand the tension at any given section if they can do so safely and if the bond is sufficient to transfer the stress to them. This fact can be utilized to provide web reinforcement in the beams by bending the rods upward and anchoring them in the concrete. Thus they will perform the function of stirrups.

To illustrate this stirrup action, let Figs. 4-11(a) and (b) represent a beam in which the longitudinal rods are bent up—two at a time. By cutting out the portion $ABCD$ as shown in Fig. 4-11(c), it is obvious that the bent-up rods serve the same function as the inclined stirrups which were considered in Art. 4-5, as far as their resistance to the breaking of the piece $OCDO$ is concerned.

However, these rods differ from the stirrups in a very important respect. When the rods EF are bent up at E, they already have a certain tensile load T_L represented by SX in Fig. 4-11(d), because they are continuous and the shearing strength of the concrete, together with the bond stresses, will compel all the rods to have practically the same deformation before they are bent and, therefore, practically the same unit stress.

The total tension in the longitudinal reinforcement near point E may be computed approximately by dividing the bending moment M' at the center of AB by jd. It is sufficient to assume that $j = 0.87$ in all cases regardless of the number of the rods remaining in the bottom of the beam if its actual magnitude is not known, because of the many unknown and secondary stresses at this point. Furthermore, T_L for the bent-up rods

at the point of bending must be the same proportion of the total tension in all the rods that the number of bent-up bars at E is to the total number of rods at E. For instance, if there are six rods, two of which are bent up, then T_L for these rods must equal one-third of the total tension in the entire set of rods.

This initial longitudinal tension should be combined with the stirrup action of the bent-up bars. Referring to Fig. 4-11(d), the increment of tension $(T' - T)$ in the space CD is represented by RS; T_L, by SX. These must be added together. Again, there are three forces which are in equilibrium, which meet at a point, and which must be proportional

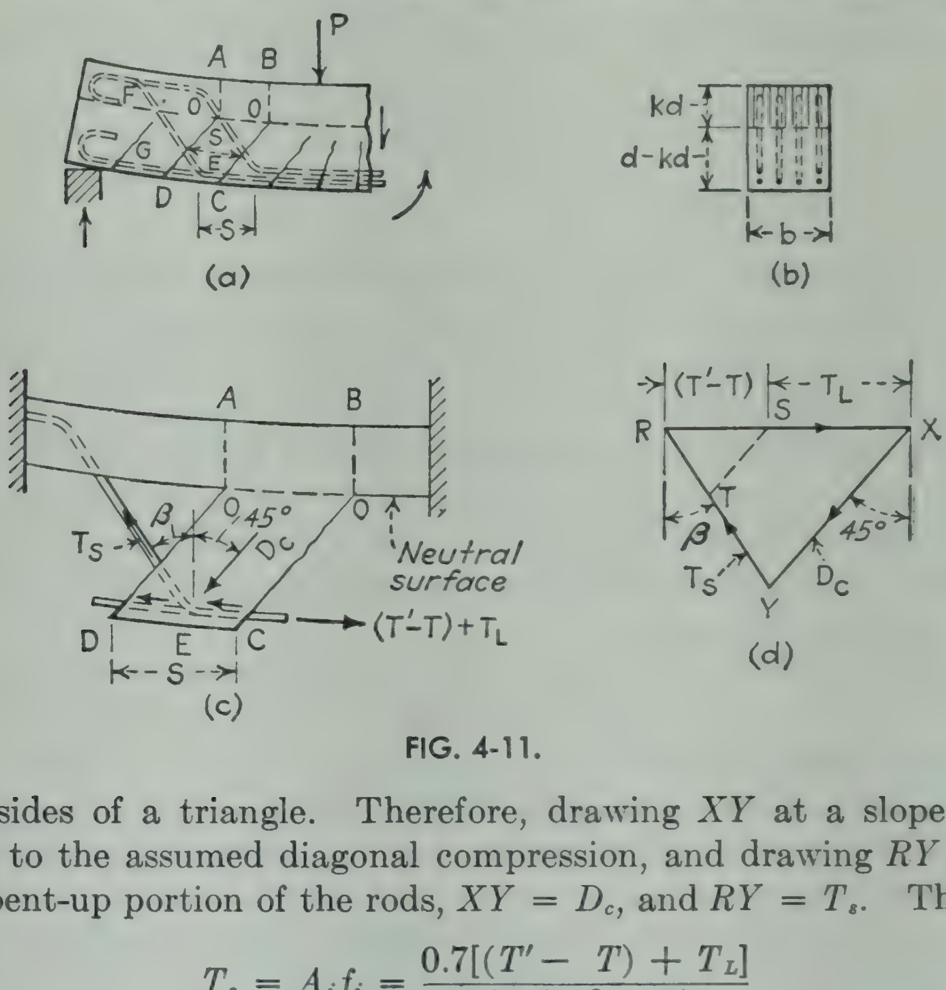


FIG. 4-11.

to the sides of a triangle. Therefore, drawing XY at a slope of 45°, parallel to the assumed diagonal compression, and drawing RY parallel to the bent-up portion of the rods, $XY = D_c$, and $RY = T_s$. Therefore,

$$T_s = A_i f_i = \frac{0.7[(T' - T) + T_L]}{\sin (45° + \beta)} \tag{4-15}$$

If N_b is the number[1] of rods that are bent up at E, and N_t is the total number of rods at E, then

$$T_L = \frac{M' N_b}{jd N_t} \tag{4-16}$$

Combining these values, Eq. (4-15) becomes

$$T_s = A_i f_i = \frac{0.7[v_L bs + (M' N_b / jd N_t)]}{\sin (45° + \beta)} \tag{4-17}$$

[1] Use relative areas if rods are not all the same size.

or, allowing for the longitudinal shear which can be taken by the concrete alone,

$$A_if_i = \frac{0.7[(v_L - v'_L)bs + (M'N_b/jdN_t)]}{\sin (45° + \beta)} \tag{4-18}$$

It must be remembered that T_s is the total stress in all the rods that are bent up at the given point. The stress thus computed is that which exists at or just above the bend at E. Naturally, through bond, this stress is gradually reduced and is transferred to the concrete.

The second term in the numerator of Eq. (4-18) often represents a very real force, and it should not be overlooked. The value of M' to be used in it can be scaled from the bending-moment diagram, or its magnitude can be quickly approximated by calculation.

The bent-up bars are usually so large that the effect of the first term in the numerator of Eq. (4-18) will not cause a serious increase in the tension in them. It should be remembered that the average stress in these rods due to bending must be considerably below the allowable value; otherwise they should not be bent up. However, in heavy construction with a large number of bars in tension, the bending of one or two may cause little reserve strength to be left for stirrup action. For example, if there are 16 bars in two rows and two are to be bent up as soon as possible, the 14 remaining will have to be able to resist the tension before the bent ones depart far from them. Before the bend point, therefore, if the bond does not fail, the average stress in the bars is approximately $\frac{14}{16}f_s$. If $f_s = 18{,}000$ psi, $\frac{7}{8}f_s = 15{,}750$ psi. Then the remaining strength in the two bent bars that is useful for web reinforcement is $2 \times 2{,}250 \times A_s$ lb. If A_if_i as computed from Eq. (4-13) exceeds this amount, the rods are theoretically insufficient. It is often convenient to analyze bent-up bars this way—as though this reserve strength is available in the form of inclined stirrups.

Since bent-up rods are usually relatively large in size, the bond stresses are likely to be excessive. In no case can such a rod pick up more longitudinal shear than the bond strength between faces OC and OD will permit. It is easy to see how large the local shearing and bond stresses really may be near such bends.

There are some practical considerations to bear in mind. The diagonal compression is not serious in the main portion of the concrete, but there is a high local pressure next to the bend in the rods. The radius used at this point should be large enough so that the bearing of the curved portion of the rod will not crush the concrete. Care must also be used to see that the bending of the rods at any point does not overstress the remaining longitudinal reinforcement which must withstand the bending moment in the beam. Another point of possible weakness is in the region

marked G in Fig. 4-11(a) where there may be no bent-up rods but stirrups may have to be added. Furthermore, there are frequently insufficient rods to permit an arrangement that will reinforce all the required portion of the beam against failure by diagonal tension because they should preferably be bent up in a symmetrical arrangement.

Two advantages of using bent-up rods for web reinforcement are the anchoring of the longitudinal rods themselves and the shifting of the bars from the bottom of the beam to the top where they may be needed to resist negative bending moments, as for continuous beams and frames. Rods b and c in Fig. 4-12(c) may almost be looked upon as cradled between anchorages at both ends of the beam.

From the standpoint of the tension in the bent-up steel due to the bending moment, it is best to avoid bending the rods at too great an angle because this causes very large local concentrations of stress at the bend. Ordinarily, a slope of 45° from the vertical is satisfactory.

4-8. Spacing of bent-up rods. As stated in Arts. 3-7 and 3-8, the bending-moment diagram is the first thing to investigate in order to determine points at which longitudinal rods in a beam may be bent up to act as web reinforcement. For instance, let Fig. 4-12(a) represent the bending-moment diagram for a simply supported beam when it has a concentrated load applied at D; let Sketch (d) be a similar diagram for uniform loading. Assume that the maximum moments are equal in both cases and that there are six rods in the bottom of the beam, as pictured at the left of Sketch (c). Because of the cracking of the beams, assume that the permissible bend points are determined by the dotted lines located $\frac{2}{3}d$ to the left of the boundaries of the respective bending-moment diagrams.

The shear diagram is the next thing to investigate. Therefore, compute the intensity of the longitudinal shear (the measure of the diagonal tension) from Eq. (4-7), and plot the diagrams for both loading conditions as in Sketches (b) and (e). If the concrete alone is relied upon to withstand a stress of v'_L, the remainder of the shearing stresses must be resisted by the bent-up rods.

The rest of the procedure is as follows:

1. Assume that the rods can be bent up at 45° and that these rods are to be bent up in pairs, for symmetry.

2. Assume that each rod withstands the same proportion of the bending moment, as shown by the horizontal dotted lines in (a) and (d).

3. Bend the first two rods c up a little to the left of the point C where they are no longer needed to resist the bending moment.

4. Project from point C downward to the other figure to locate the permissible points for the first bends.

5. Project down from B in a similar manner to find the points for the bending of the second pair of rods.

The distance between bends should not exceed about $\frac{1}{2}d$ times the secant of the angle of inclination of the bent rod measured from the vertical—with an arbitrary maximum limit of $\frac{3}{4}d$—unless other web

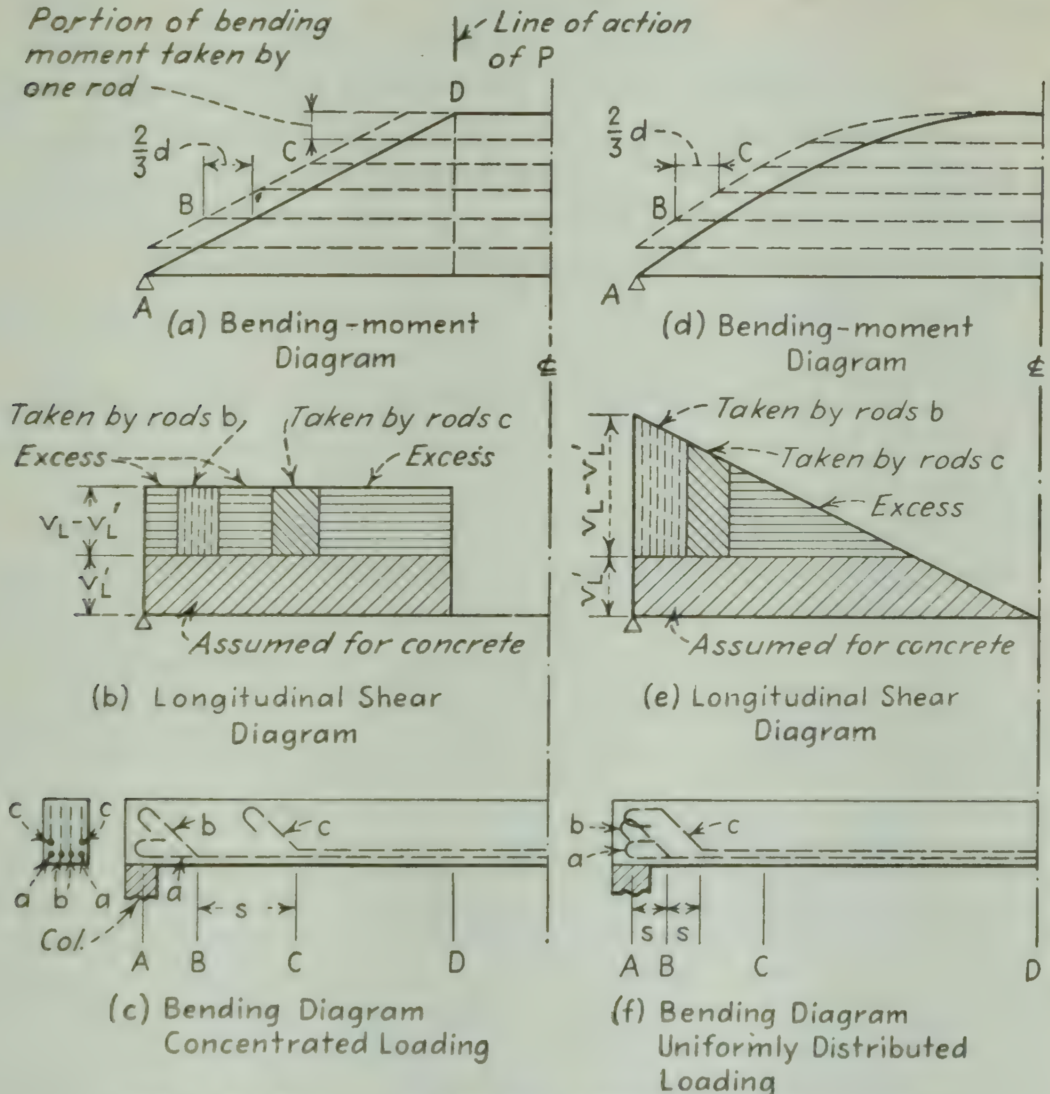


FIG. 4-12. Example of bending longitudinal reinforcement.

reinforcement is used or unless the rods are not needed as web reinforcement at all but are bent up in order to have them available for resisting negative bending moments.

Some of the difficulties of arranging bent-up rods in simply supported beams are shown in Fig. 4-12. The parts of the longitudinal-shear diagram that are labeled "excess" have no bent-up rods in this case to reinforce them. In Sketch (*c*) the rods are bent up as soon as the bending-moment diagram plus the extra $\frac{2}{3}d$ will permit. Since the spacing of bend points should not exceed $\frac{3}{4}d$, the distance s is too large. The parts of the shear diagram labeled "excess" in (*b*) are not taken care of; hence stirrups will be needed in these areas. On the other hand, Sketch

(f) shows the rods bent up close to the end of the beam. Rods b are bent $\frac{1}{2}s$ from the edge of the column; then rods c are bent at a distance s farther on, where $s = \frac{3}{4}d$. Again, the excess must be provided for by other means.

In Sketch (f), notice the hooks and over how short a distance the bent-up bars serve as web reinforcement. It is possible to bend the bars one at a time, in which case they could reinforce more of the beam's length. However, this is not customary in such a case as this because of the resultant lack of symmetry except for centrally located bars as shown in Fig. 4-13(f). Even if the rods are bent up one at a time, there will generally be the need for some stirrups. It may be better to use stirrups entirely and to stop the main rods when they are not needed.

The Code specifies certain practical rules regarding combined systems of web reinforcement. These are the following:

1. Stirrups placed perpendicular to the longitudinal reinforcement shall not be used alone as web reinforcement when the shearing unit stress v_L exceeds $0.08f'_c$.

2. Where more than one type of reinforcement is used to reinforce the same portion of the web, the total shearing resistance of this portion shall be assumed as the sum of the shearing resistances computed for each type separately. In such computations the shearing resistance v'_L of the concrete shall be included only once, and no one type of web reinforcement shall be assumed to resist more than two-thirds of the requirements of $v_L - v'_L$.

3. In no case should v_L exceed $0.12f'_c$.

Example 4-4. Bend up at 45°, for web reinforcement, the rods of the beam that is shown in Fig. 4-13(g) if the condition of loading is that which is pictured in Sketch (a). Assume the following allowable stresses: $f_s = 20{,}000$ psi, $u = 175$ psi, $f'_c = 2{,}500$ psi, and $v'_L = 0.03f'_c = 75$ psi. Let $n = 12$, $k = 0.28$, $j = 0.91$, and $d = 22$ in., the last being measured to the lowest row of bars in this case.

Most of the parts of Fig. 4-13 are self-explanatory. However, in (c), the bending moment is assumed to be resisted equally by all six rods, although the three upper ones are less effective than the lower row. The left portion of (c) is reproduced in (e) in order to represent it to a larger scale.

In planning this work, examine Figs. 4-13(f) and (g). Assume that rod a will be bent up first, then b, and finally both rods c, the last being bent up together for symmetry. Rods d should extend the full length of the beam as shown. The maximum spacing of the bends is $0.5d(\sec 45°) = 15.5$ in., but, in order to distribute them well, they will be spaced as shown in Fig. 4-13(f).

Figure 4-13(e) shows, by the horizontal lines, that the full length of bars d can be represented by the two bottom spaces between the horizontal dotted lines. It also shows that rods c are needed by the bending-moment diagram until they are about 2 in. to the left of point B. When $\frac{2}{3}d$ (14 in.) is added because of the cracking, the bending point will be approximately 6 in. from the center of the support. However, in order to bend them at all and have them do any good as web reinforcement, they will be bent 11 in. from the end even though this may be a little too soon. With the

spacing shown in Sketch (*f*), rods *a* and *b* will be found to be bent far enough beyond the point where they are needed for tensile resistance in the bottom of the beam.

Now check these bent-up bars to see if the stresses are safe. Try bar *a* first. It is probably the most critical one. By scaling from Fig. 4-13(*e*), M' at A = 820,000 in.-lb.

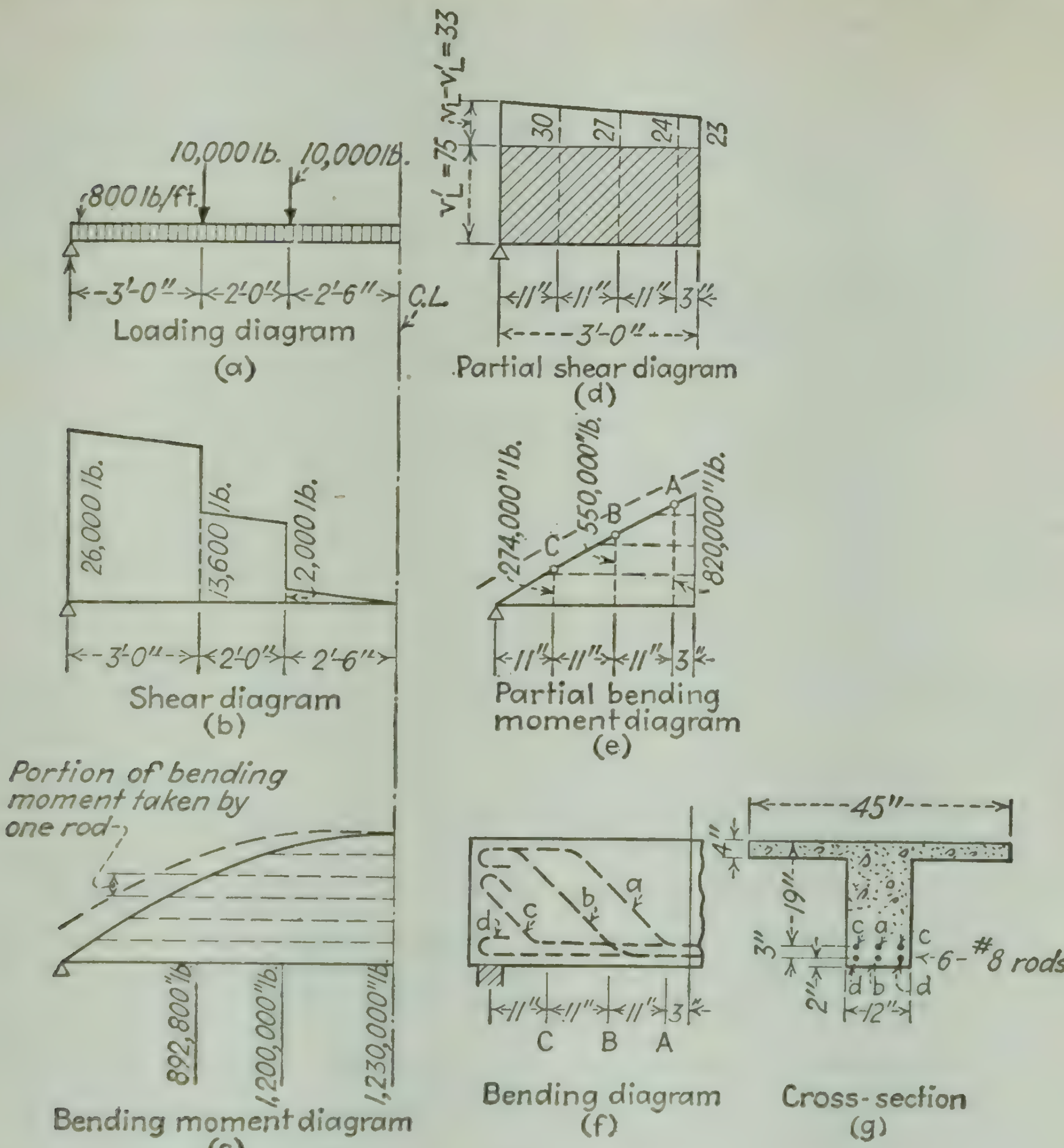


FIG. 4-13.

Similarly, from Fig. 4-13(*d*), $(v_L - v_L') = 24$ psi. Substituting these values in Fig. (4-18) gives

$$0.79 f_i = \frac{0.7[24 \times 12 \times 11 + (820{,}000 \times 1/0.91 \times 22 \times 6)]}{\sin\,(45° + 45°)}$$

$$f_i = \frac{0.7(3{,}170 + 6{,}820)}{0.79} = 8{,}850 \text{ psi}$$

This is the unit stress in rod *a*. If *j* had not been computed previously, a value of 0.87 would have been satisfactory.

Examine rod *a* of Fig. 4-13(*f*) further. If it is to develop the longitudinal resistance of 8,850 psi required, it must be anchored properly. The required length for this is

$$L_s = \frac{A_i f_i}{(\Sigma o)u} = \frac{0.79 \times 8{,}850}{3.14 \times 175} = 12.7 \text{ in.}$$

This length must be provided above a plane approximately $0.5d$ from the top of the beam. Obviously, the length shown is more than is needed.

Next, see if rod a can pick up the required increment represented by the force of 3,170 lb shown in the computation for f_i. Assume that the available length is $s = 11$ in. Then

$$u = \frac{3{,}170}{(\Sigma o)s} = \frac{3{,}170}{3.14 \times 11} = 92 \text{ psi} \qquad \text{(safe)}$$

Investigation of the transverse shearing stress in the concrete shows that

$$v_T = \frac{V}{bkd} = \frac{26{,}000}{12 \times 0.28 \times 22} = 350 \text{ psi} \qquad (\text{max allowable} = 0.2\, f'_c = 500 \text{ psi})$$

To find the approximate compression in the concrete under the bend of rod a, assuming the diameter inside the bend to be $8d$, use Eq. (3-2). Call the tension in the rod equal to that previously found, *viz.*, 8,850 psi. Then

$$T = A_s f_s = pr$$
$$0.79 \times 8{,}850 = p \times 4.5 \times 1 \qquad \text{or} \qquad p = 1{,}550 \text{ pli}$$

Therefore, $f_c = p/d = 1{,}550/1 = 1{,}550$ psi. This is high, but it can be accepted because it is localized and is in a mass of concrete where it will probably not cause splitting of the member.

The Code specifies that, when bent-up rods are used alone, when the radius of bend of the longitudinal bars is not more than three diameters of the bars, v_L should not exceed $0.06f'_c$. However, $0.01f'_c$ may be added to this limit for each increase of four diameters in the radius of bend up to the limiting shearing stress.

Figure 4-14 has been prepared to show more clearly the details of the web reinforcement near the end of a simply supported T beam and to compare stirrups vs. bent-up bars. Sketches (a) and (c) show three bottom rods a, the outer two of which will extend the full length of the beam. The other bar a and the two upper rods b will be stopped when they are not needed. Rods c at the top are small ones used as spacers to which stirrups d are wired. In Sketch (b) bars h extend the full length of the beam. In this case, as shown in (d), there are two layers of three bars each. Top rod e is bent up first; bottom rod f, next; and two top rods g last.

In Fig. 4-14(c), the reinforcement is shown supported by precast blocks r, although wire chairs like s can be substituted. The slab reinforcement is arranged so that alternate rods k are straight. Bars i and j are bent as indicated and then lapped past each other over the girder. This scheme provides the same amount of reinforcement at the bottom of the slab at mid-span and at the top over the beam. Bars m are ties.

In Fig. 4-14(d), the bottom bars are shown supported upon long narrow precast blocks t. Wire chairs are better for this purpose, and additional chairs are needed to hold the upper set of main bars. In the slab, the

top rods n are short and straight, extending each side at least one-fourth the clear span of the slab. Rods o are straight and may be full length or, for economy, arranged as alternate long ones for the full length and short ones that stop before reaching the beam. Bars p and q are ties and transverse reinforcement (or temperature steel) for the slab. This arrangement of slab reinforcement is much more simple than the use of bars with such small bends as those in Sketch (c), although the latter do not need chairs to support the top reinforcement.

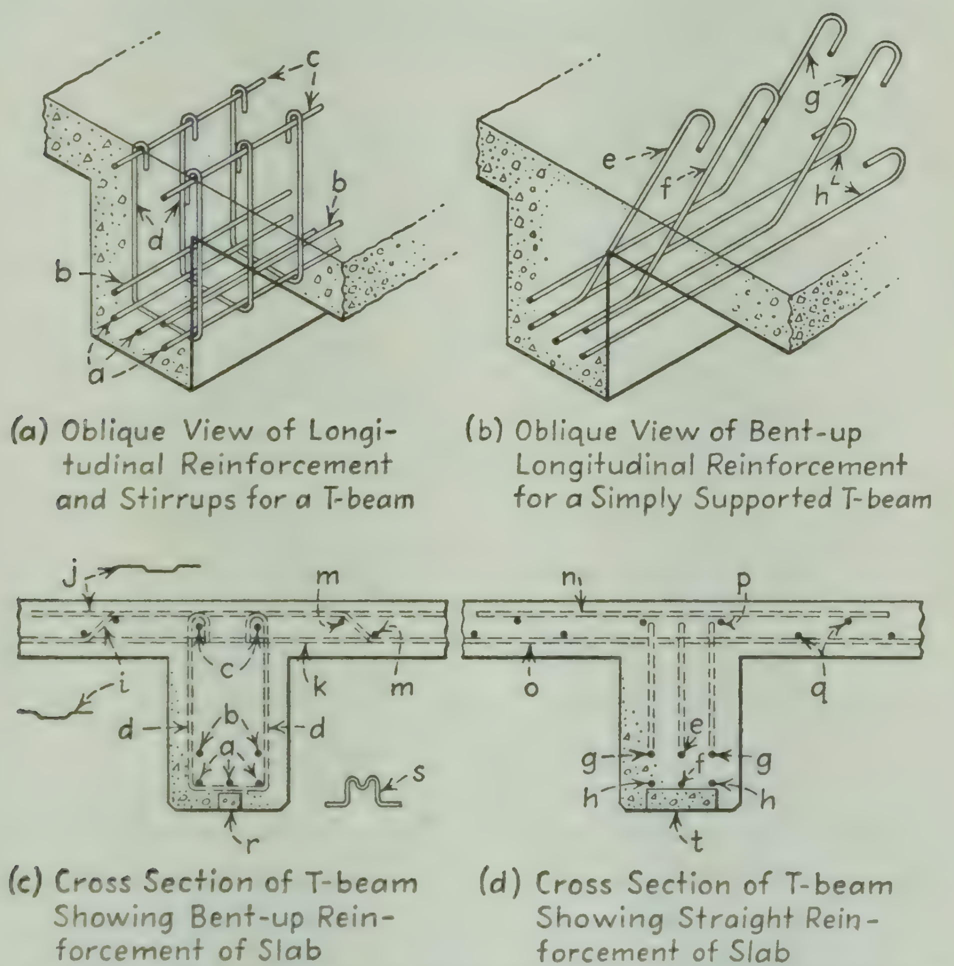


FIG. 4-14. Studies of arrangements of and supports for beam-and-slab reinforcement.

4-9. Web reinforcement at supports of continuous beams. A large portion of reinforced-concrete construction utilizes the advantage of continuity or restraint of members. This is partly in order to decrease the maximum bending moments but primarily because it is easier and more practicable to build concrete structures that way. The shears—both transverse and longitudinal—are affected by continuity to a much smaller extent than are the bending moments.

As an illustration of the effect of continuity and restraint, let Fig. 4-15(a) picture a simply supported beam with a concentrated load at its center. The bending-moment and shear diagrams are self-explanatory. Then let the ends of the beam be fixed as in Fig. 4-15(b). The bending-

moment and shear diagrams for this new condition are again easily understood. The maximum bending moment in the second case is only one-half that of the first one; also, the distribution of the bending moment is different. The rate of increase of bending moment in both cases is $(PL/4) \div (L/2)$. This is a constant. This fact is shown in the shear diagrams, which are also identical. Therefore, the required strength of the beam against shearing forces remains unchanged, although the ends are fixed in the second case. This means that the relative importance of the shear is greater than it is in a simply supported beam.

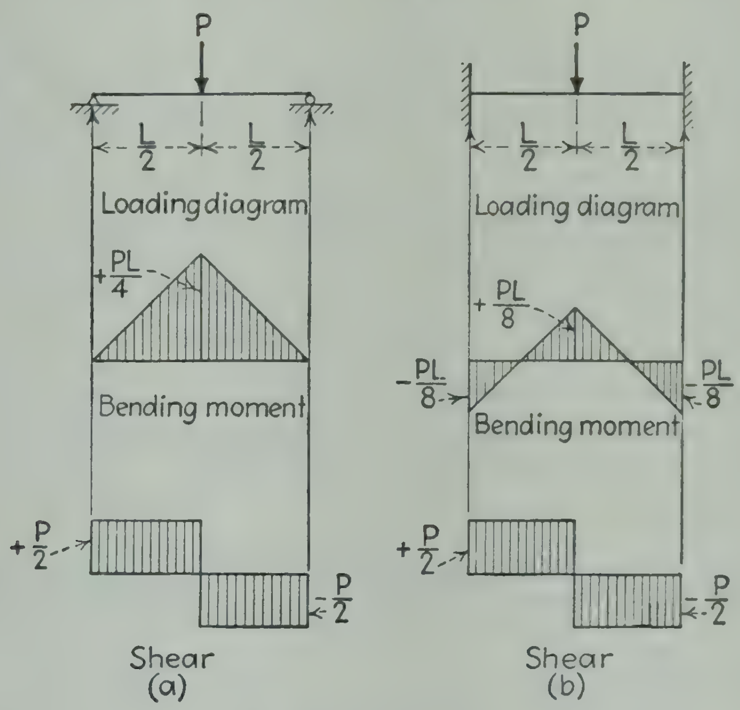


FIG. 4-15.

Now, examine the conditions at a point of support where the bending moment is negative. Figure 4-16(a) is an exaggerated picture of such a case, with vertical stirrups. The elongation of the longitudinal reinforcement causes cracking of the concrete in the same manner as for simply supported beams except that, in this case, the top fibers are elongated. If the piece $ABCD$ is cut out and gripped in rigid supports, as shown in Fig. 4-16(b), a brief examination of it shows that it is the same as Fig. 4-10(b) if the latter is rotated 180°. The tension in the top longitudinal reinforcement increases from D to the support S. The increment of stress which is picked up by the piece $OCDO$ must therefore, act toward the left. After $(T' - T) = v_L bs$ is computed, the diagonal compression and the stirrup tension can be found as before by constructing the force triangle of Fig. 4-16(c).

Another way to visualize the action in Fig. 4-16(b) is to imagine that the central portion of the beam tries to rotate counterclockwise in Sketch (a) in the vicinity of the point of inflection and to deflect down-

ward. This causes the stirrup to pull downward. Then the concrete and the main reinforcement must resist this action.

Vertical stirrups, inclined stirrups, and bent-up rods serve the same purpose whether they are in simply supported, continuous, fixed-end, or cantilevered beams. All of them can be designed by the same general method because the fundamental force to be considered is the longitudinal shear (or the diagonal tension) in the concrete. The diagrams showing the intensity of the longitudinal shear and the bending moment should be constructed; the excess of the longitudinal shear above that which is permissible in concrete beams without web reinforcement should be

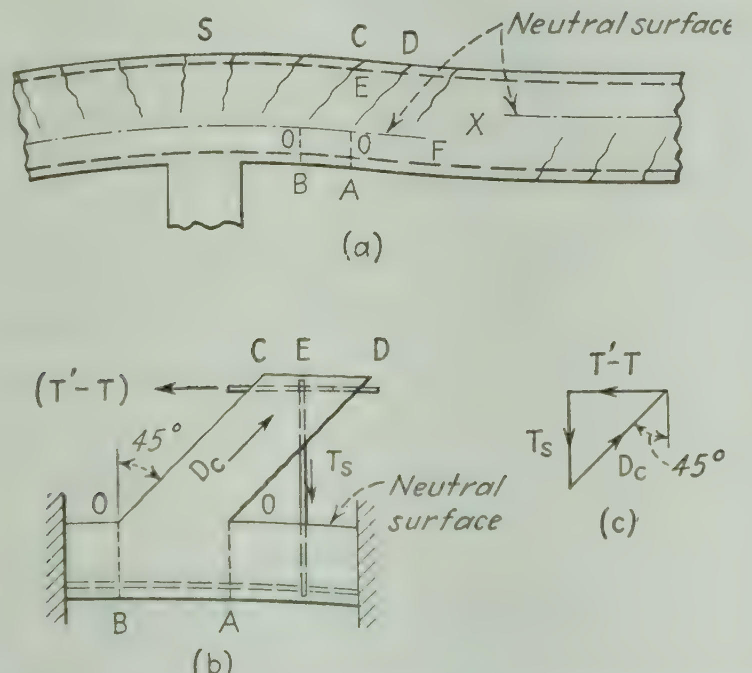


FIG. 4-16. Stirrups near support of continuous beam.

found; the tension in the longitudinal rods should be calculated when bent-up rods are to be used; then the size, spacing, and details of the stirrups and the bent-up rods should be determined just as they were in the previous cases.

It is important to study the right-hand portion of Fig. 4-16(*a*). As described in Art. 3-8, the neutral axis is usually below the middle of the depth of the beam at the support S because the tension is at the top. At the center of the span, the neutral axis is above the middle of the depth. Somewhere in the region of the point of contraflexure X, the neutral axis shifts from the one position to the other. If this portion of the beam is uncracked, the location of the neutral axis has little importance anyway. It is not difficult, however, to see how uncertain the magnitudes of the shearing stresses in continuous beams may be, especially when large live loads are applied and the point of inflection shifts with varying positions of these loads. However, because of the variation

of the pickup of tension in the top rods due to cracking, as described in connection with Fig. 3-17, it is desirable to extend web reinforcement to the vicinity of the point of contraflexure, even though the shear does not require it to go so far. Of course, the concrete close to this point may not be cracked when the loads are in fixed positions and of known magnitude so that the point of contraflexure will not be shifted for various loading conditions. In general, this point of inflection is about $0.2L$ to $0.25L$ from the supports.

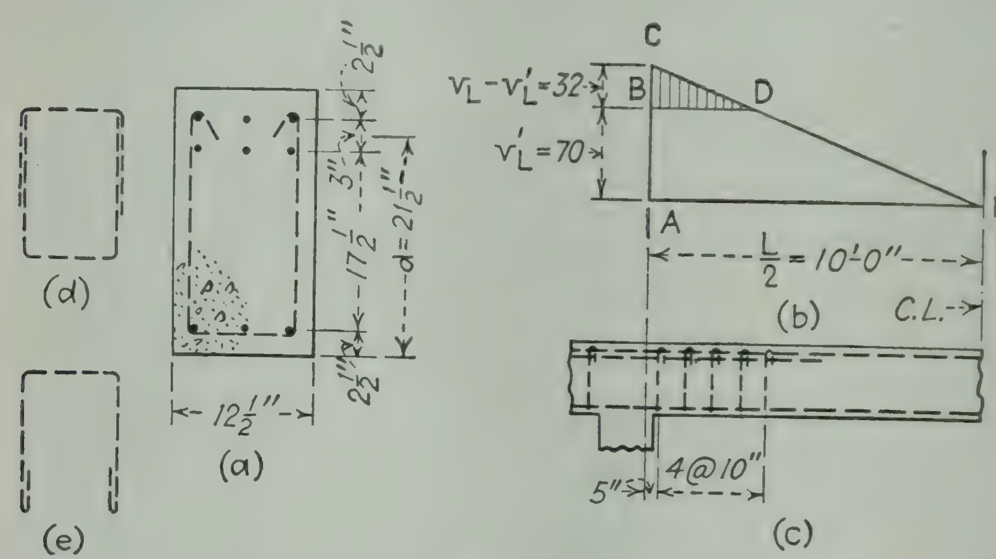


FIG. 4-17.

Example 4-5. The beam shown in Fig. 4-17(a) is continuous; it has a span of 20 ft; it carries a uniformly distributed load of 2,400 plf; its reinforcement at the support is pictured in (a); $n = 12$; $f_c' = 2{,}500$ psi; and the allowable f_v, u, and $v_L' = 18{,}000$, 175, and 70 psi, respectively. Design the required vertical stirrups for this beam, using U-shaped rods.

The value of $k = 0.37$ and $j = 0.88$.

$$v_L = \frac{V}{bjd} = \frac{2{,}400 \times 10}{12.5 \times 0.88 \times 21.5} = 102 \text{ psi}$$

$$v_L - v_L' = 102 - 70 = 32 \text{ psi}$$

$$BD \text{ in Sketch } (b) = \frac{10 \times 12 \times 32}{102} = 37.6 \text{ in., say 38 in.}$$

Assume No. 3 rods for the stirrups. Then, from Eq. (4-10),

$$0.11 \times 2 \times 18{,}000 = 32 \times 12.5 \times s \qquad \text{or} \qquad s = 9.9 \text{ in. at } A$$

Use a spacing of 10 in. with the arrangement that is shown in Fig. 4-17(c). The last stirrup at the right is added in order to extend the web reinforcement a little beyond the minimum required.

A check calculation for the transverse shearing stress gives, from Eq. (4-2),

$$v_T = \frac{2{,}400 \times 10}{12.5 \times 0.37 \times 21.5} = 242 \text{ psi}$$

The length of embedment required for the stirrups is

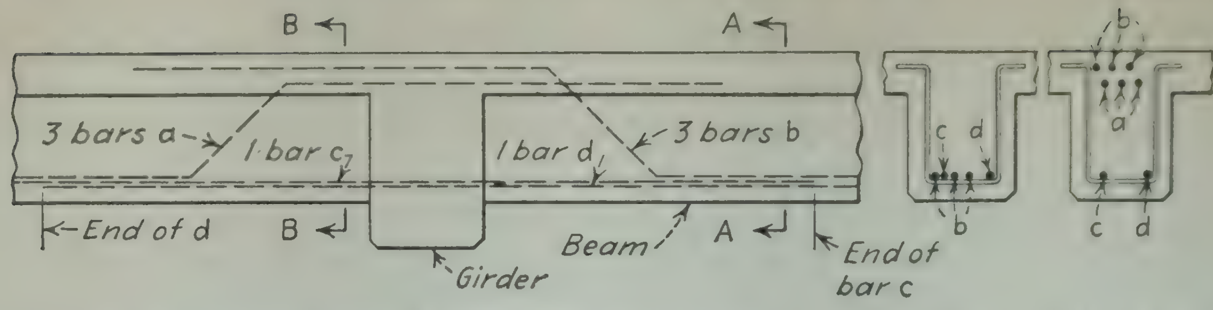
$$L_s = \frac{A_v f_v}{\Sigma o u} = \frac{0.11 \times 18{,}000}{1.18 \times 175} = 9.6 \text{ in.}$$

Difficulties are encountered in arranging the details of stirrups at points of negative bending moments in rectangular beams. A stirrup like that in Fig. 4-17(*a*) is not anchored thoroughly across the tension side of the beam; when it is inverted, it is difficult to get the hooks under the longitudinal rods if the stirrups are erected last or to place the main reinforcement if the stirrups are in position first. Sometimes two U-shaped rods without hooks are arranged as shown in Fig. 4-17(*d*), but the laps must be long enough to develop the stirrups, and it is wise to have as much of the lap as possible in the compression zone of the beam. Still another type is that which is pictured in Fig. 4-17(*e*). The hooks are parallel to the main reinforcement, permitting the stirrups to be slid down after the longitudinal rods are in place. However, the anchorage of these stirrups is poor because the entire hook is near the surface of the beam, a fact that may cause spalling of the concrete; the compressive reinforcement is not tied in, a precaution that should be taken in heavy members if the compressive stresses in the concrete at the support are large.

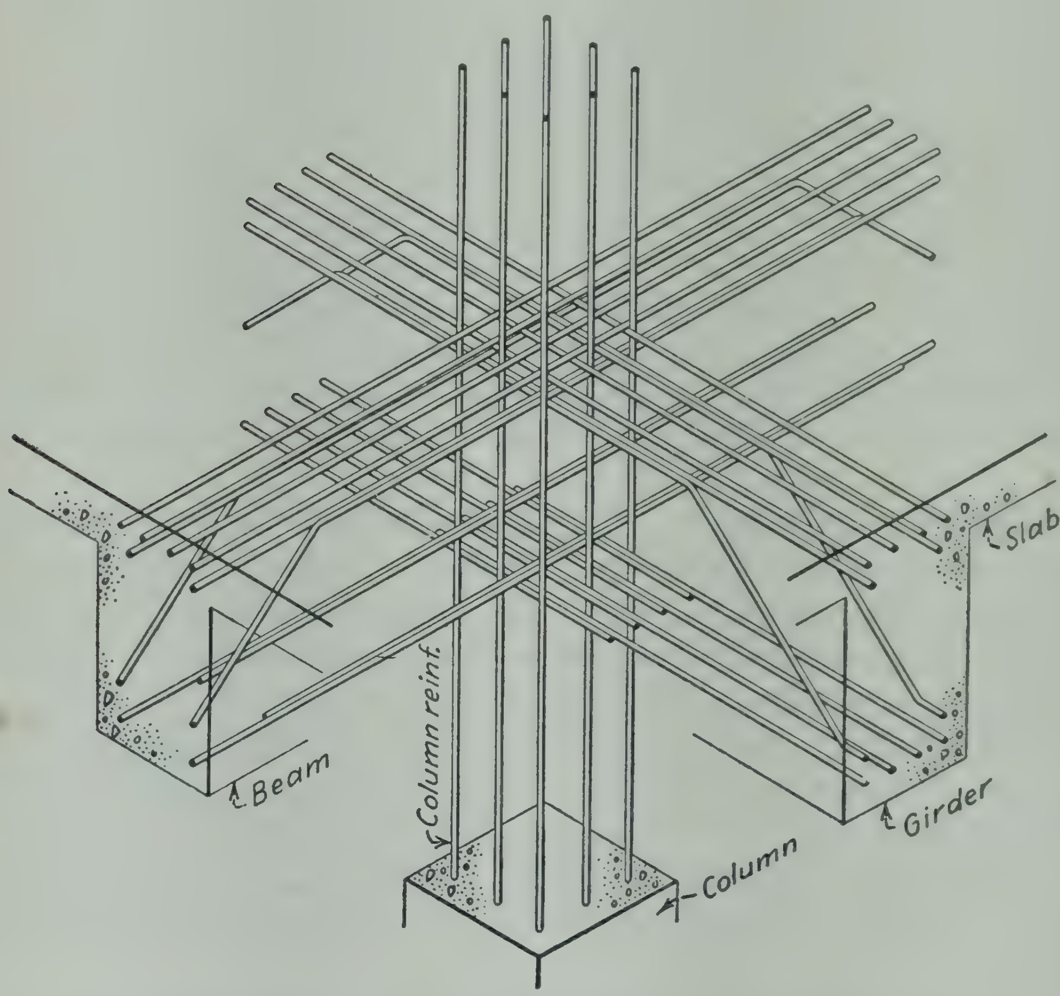
The stirrups in the stems of T beams may be detailed as shown in Fig. 4-17(*a*) because the concrete of the slab, and its "negative" reinforcement which crosses the top of the stem, will support the latter adequately.

4-10. Combination of stirrups and bent-up rods. As stated in Art. 4-8, and as shown in Fig. 4-12, it is often difficult to arrange the details of bent-up rods so that they will reinforce properly all parts of the web. The deficiency can be made up by adding stirrups in the otherwise unreinforced portions. It is not necessary to explain this in detail because both types of web reinforcement act in accordance with the same general principles. However, good judgment must be exercised in combining them.

Reinforcement pictured on a drawing may seem to be simple, but placing large bars in the forms in the field may be a different matter. Figure 4-18 shows the junction of a continuous beam and a girder. Three rods *a* and *b* are bent up from the bottom at each side to the top to resist negative bending. Obviously, they cannot pass through each other. If they are all six in the same plane, they will form a screen where they overlap, and it will be difficult to place the concrete through and around them. If they are offset to pass each other, then placed in two layers as pictured in Sketch (*c*), they still form an obstacle in the pouring of the concrete. Bar *c* in Sketch (*a*) extends across the span at the left and laps with the others at the right; bar *d* does the reverse. If they and the bent



(a) Section Showing Beam Reinforcement at Girder (stirrups and reinforcement of girder and slab not shown) (b) Section A-A (c) Section B-B



(d) Oblique Sketch Showing Reinforcement at Junction of Continuous Beam and Girder, Omitting Stirrups, Column Ties, and Bars in Floor Slab

FIG. 4-18. Illustration of difficulties in arranging reinforcement.

bars are in one horizontal plane, the overlap causes the screen effect pictured in (*b*). Generally, if the design is not carefully planned, as indicated in Sketch (*d*), the rods will have to be rearranged somewhat in the field, tilted a bit, raised or lowered to lap over and under each other, and otherwise made to fit as best they can. Doing this adjusting in the field may be a very troublesome job with stirrups, heavy longitudinal

bars in the beams, similar reinforcement in the girder, and all the slab bars that have to fit together in the same general region.

Perhaps this shows the advantages of simplicity and easy field work when straight bars and stirrups are used without bent bars. On the other hand, a nominal number of bent-up bars will often be desirable in tying together long and important structures. Whether or not to use them is largely a matter of judgment on the part of the individual engineer.

Theoretically, it seems that inclined stirrups are to be preferred when the number of bent-up bars is large, in order to have the tensile forces in the web reinforcement acting in the same direction. However, when used in combination with bent-up bars, they will cause very difficult work in the field if they are to be properly fastened to the longitudinal reinforcement. Therefore, vertical stirrups are more practical, but they will generally overlap some of the inclined portions of the main rods. Also, when only a few bars are bent up, it seems to be advisable to reinforce the web with vertical stirrups alone, neglecting the bent-up rods and letting their strength add to the factor of safety of the beam. Another advantage of this last arrangement is that, in continuous beams, the rods can be bent to meet the requirements of the bending moments alone rather than those of the longitudinal shear and the bending moments together.

Example 4-6. Assume a T beam that is continuous over a central column as pictured in Fig. 4-19(a). Use the allowable stresses given in the Code for 3,000-lb concrete and intermediate-grade steel. Design the reinforcement for this beam, using bent-up rods where possible for web reinforcement. The beam can be assumed to be fixed at B. The over-all depth is to be 30 in.

$$R_A = 3{,}100 \times 24 \times \tfrac{3}{8} = 27{,}900 \text{ lb}$$
$$R_B = 3{,}100 \times 24 \times \tfrac{5}{8} = 46{,}500 \text{ lb}$$

Because the outer columns are relatively limber the reaction is assumed to act at their centers. The middle column, with the opposite spans balanced, is assumed to be fixed and to have a uniform pressure across it. The shear diagram could therefore be drawn as shown in Fig. 4-19(b); but the tiny reduction across the column is not important, and the full value of 46,500 lb will be used for computing the shear.

However, the pressure across the column will be considered in computing the bending moment. Then

$$M_B = -\frac{3{,}100 \times 24^2}{8} + 46{,}500 \times \frac{4}{12} = -223{,}000 + 15{,}000$$
$$M_B = -208{,}000 \text{ ft-lb}$$
$$M_C = \tfrac{9}{128} \times 3{,}100 \times 24^2 = 125{,}000 \text{ ft-lb}$$

From these values and ones computed at intermediate points, plot the bending-moment diagram in Fig. 4-19(c).

Using a trial d of 26 in. and $j = 0.87$,

$$A_s \text{ at } B = \frac{208{,}000 \times 12}{20{,}000 \times 0.87 \times 26} = 5.5 \text{ in.}^2$$

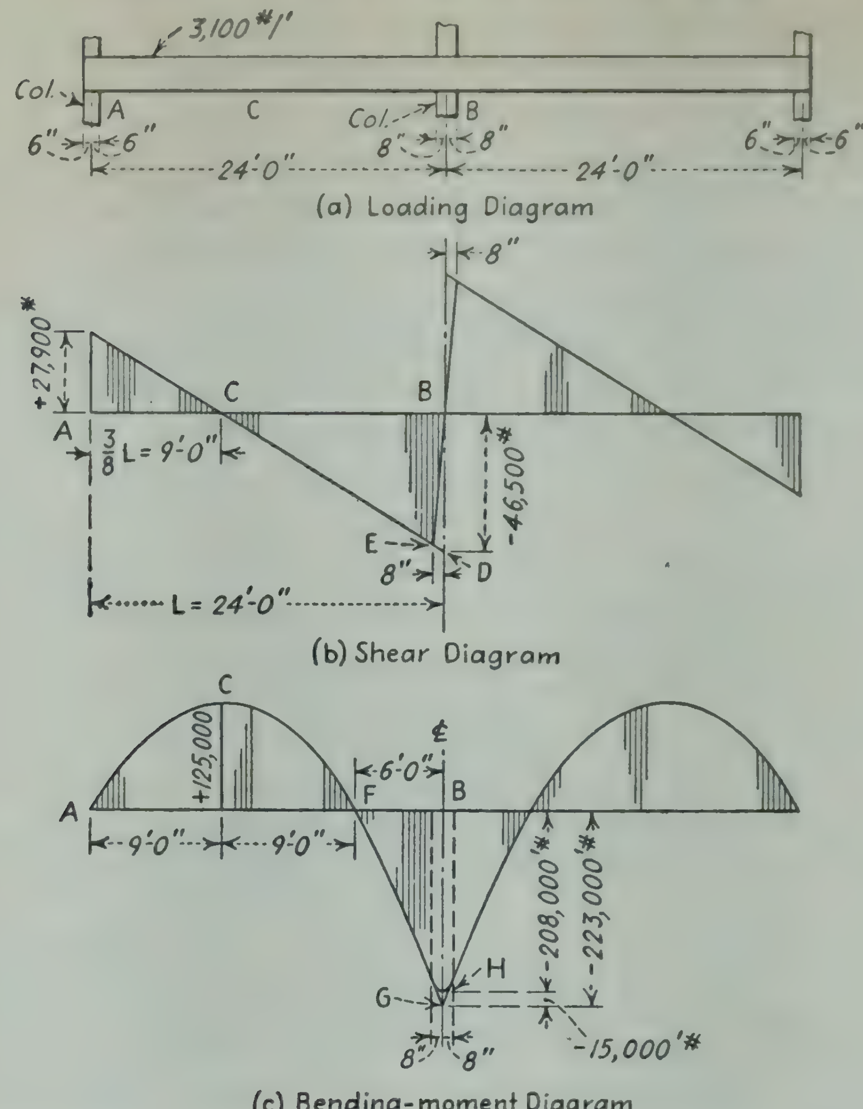


FIG. 4-19. A heavy beam continuous across a central column.

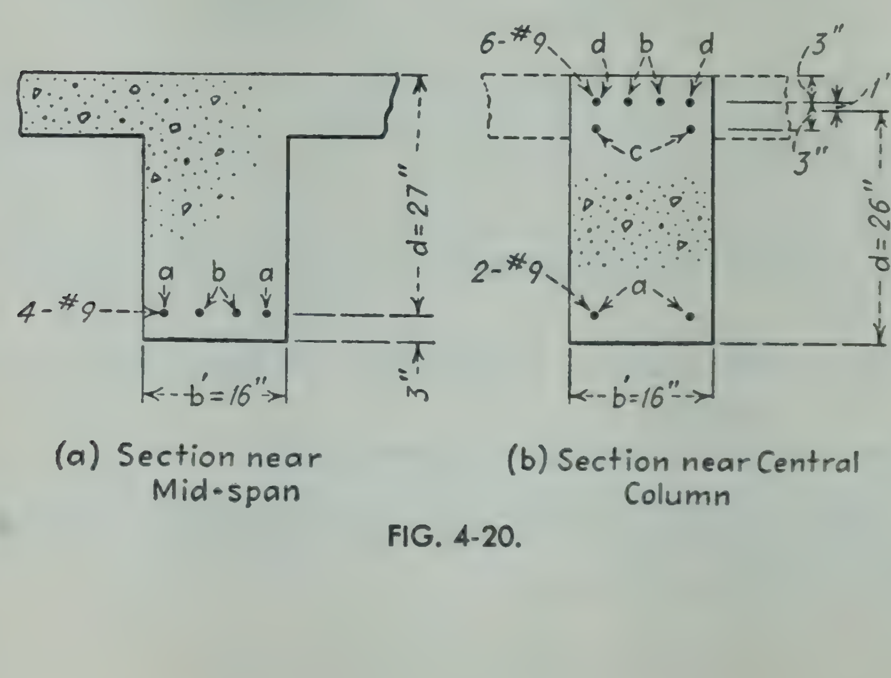


FIG. 4-20.

Try six No. 9 bars.

$$A_s \text{ at } C = \frac{125{,}000 \times 12}{20{,}000 \times 0.87 \times 26} = 3.2 \text{ in.}^2$$

Try four No. 9 bars. Therefore, the sections shown in Fig. 4-20 are proposed and are to be tested further. Using Fig. 6 in the Appendix as an approximation, with

$$\frac{nA_s}{b} = \frac{10 \times 6}{16} = 3.75$$

and

$$\frac{(n-1)A_s'}{b} = \frac{9 \times 2}{16} = 1.1 \qquad \text{(called 1.0)}$$

find $S_c/b = 130$. Therefore,

$$f_c = \frac{208{,}000 \times 12}{130 \times 16} = 1{,}200 \text{ psi}$$

(satisfactory, but the diagram is too small for exact answers). Notice that d at $B = 26$ in., whereas, in the section from A to F, it is 27 in. Use it accordingly. Assume $j = 0.87$ throughout, which is close enough for practical purposes.

The bond stress near B will be the critical one.

$$u_B = \frac{46{,}500}{21.3 \times 0.87 \times 26} = 97 \text{ psi} \qquad \text{(very safe)}$$

Also, assuming only two bars at A,

$$u_A = \frac{27{,}900}{7.09 \times 0.87 \times 27} = 168 \text{ psi} \qquad \text{(safe)}$$

$$v_L \text{ at } A = \frac{27{,}900}{16 \times 0.87 \times 27} = 77 \text{ psi}$$

$$v_L \text{ at } B = \frac{46{,}500}{16 \times 0.87 \times 26} = 128 \text{ psi}$$

These are plotted in Fig. 4-21(a), using straight lines from L and D to point C and neglecting the difference in the values for d. No web reinforcement is needed near A but some must be provided for the excess shear represented by the triangle OND. By similar triangles,

$$\frac{ON}{CB} = \frac{DN}{DB} \qquad \text{or} \qquad ON = \frac{15 \times 12 \times 38}{128} = 53.5 \text{ in.}$$

The bending-moment diagram is redrawn in Fig. 4-21(b), and it is divided by the dotted lines to represent the portions resisted by the individual rods. The dotted lines labeled 1, 2, and 3 show the extensions required to reach the permissible bend and cutoff points for the bars, as described in Art. 3-8. Assume that $\frac{2}{3}d = 18$ in.

The reinforcement is pictured in Fig. 4-21(c). The bars can be identified by referring to Fig. 4-20. The details are obtained in the following manner:

1. With Eq. (3-7), find the length of bars a needed beyond the center of point A:

$$L_s = \frac{0.75V}{(\Sigma o)u} = \frac{0.75 \times 27{,}900}{7.09 \times 210} = 14 \text{ in.}$$

This is provided by hooking both these rods, since $C + D$ in Table 10 of the Appendix equals 15 in. Since the length would be too long for easy handling, these bars will be lapped at B.

2. The top rods d are short straight ones. The minimum cutoff is found at T by projecting down from the top of dotted line 3 beyond F in Sketch (c); then these bars are made somewhat over 1 ft longer at each end for 50 per cent development.

3. As seen in (b), point R is the one at which the second pair of bars can be bent down. Bars c are selected for this. They are extended approximately 1 ft, as shown,

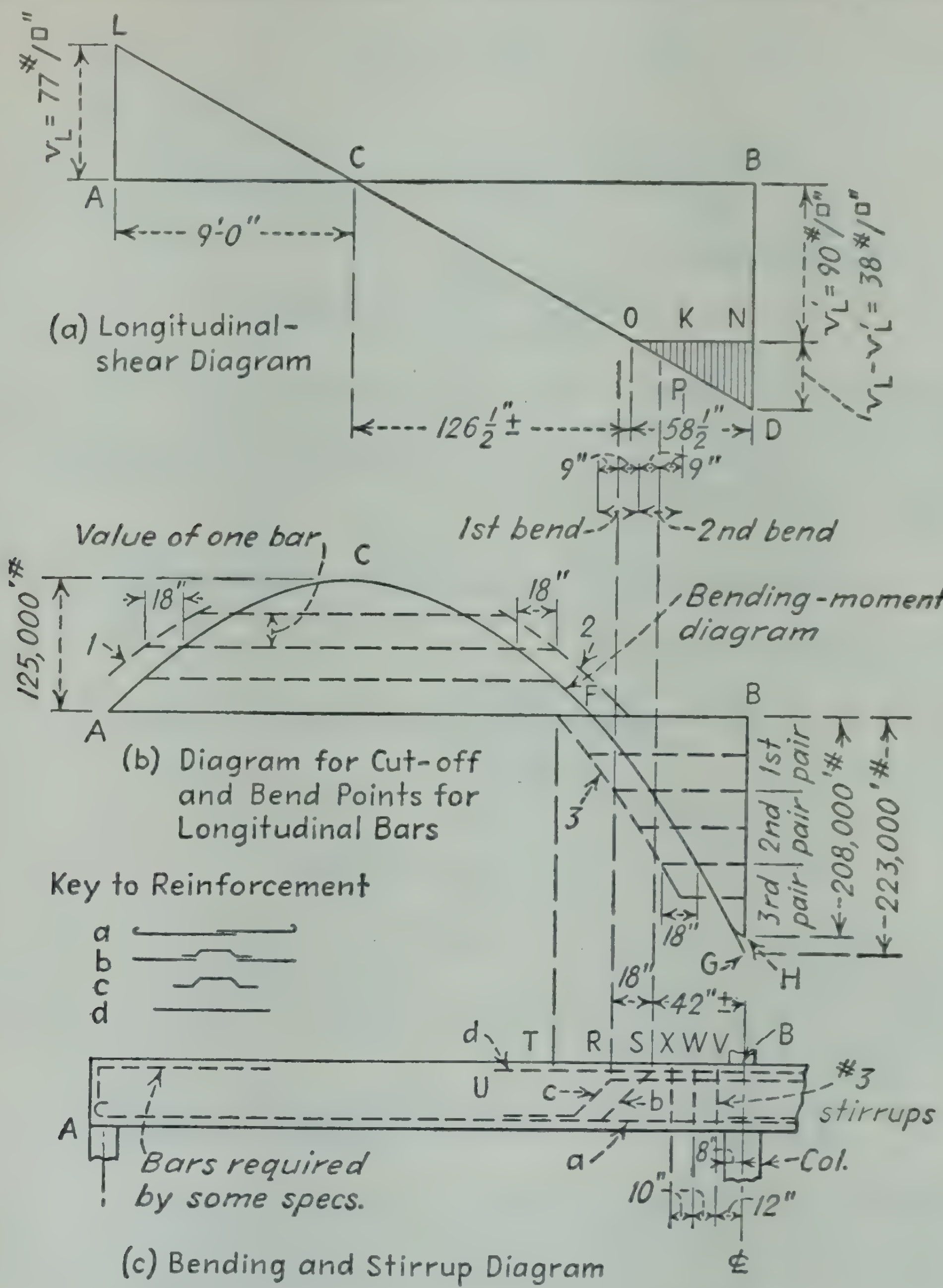


FIG. 4-21. Combined bent-up bars and stirrups.

to obtain anchorage at both ends. Since the excess shear is very little for the 18-in. width taken care of by these bars in Fig. 4-21(a), their stress from stirrup action need not be computed.

4. The next pair of rods is assumed to be 18 in. (less than $\frac{3}{4}d$) from point R. This point S is projected to Sketch (b) and is found to be beyond the intersection of curve 3 and the line representing the third pair of bars; hence it is satisfactory. If it cut inside line 3, the assumed 18 in. should be reduced as required by the bending limit. The top central bars b are therefore bent down at S. Since a full-length bar will be too long, they should be lapped by other short straight bars in the bottom, the splice

being near the region of small bending moment at the left of F. These bars are also obviously very safe, but they will be checked for purposes of illustration. Treat these bars b as inclined stirrups.

a. Available strength of these two bars at S, when only four of the six available are needed, is $\frac{2}{6} \times 20{,}000 = 6{,}700$ psi.

b. Strength T_s available for two bars $= 2 \times 6{,}700 = 13{,}400$ lb.

c. From Eq. (4-13), with the average $v_L - v'_L$ scaled as 10 psi from Fig. 4-21(*a*) and $s = 18$ in., find the required T_s.

$$T_s = A_i f_i = \frac{0.7 \times 10 \times 16 \times 18}{1} = 2{,}030 \text{ lb} \qquad \text{(very safe)}$$

5. Since the space between S and B of Fig. 4-21(*c*) cannot be reinforced by bent-up rods, use No. 4 U-shaped stirrups. At B, neglecting the effect of the width of the column, find the spacing s.

$$A_v f_v = (v_L - v'_L)bs$$

or

$$s = \frac{18{,}000 \times 0.4}{38 \times 16} = 11.8 \text{ in.} \qquad \text{Call it 10 in. (conservative)}$$

Now allowing 8 in. for one-half the width of the column at B, place the stirrup 8 in. plus a little less than $\frac{1}{2}s$ from the center. Call it 12 in. Therefore, use the spacing shown in Fig. 4-21(*c*), giving spaces of 10 in. for SX, XW, and WV.

The arrangement shown for all this reinforcement in Fig. 4-21(*c*) will be called satisfactory. However, it is obvious that two or three more stirrups without bent-up bars would be a more simple design, and it would be satisfactory and probably cheaper in this case.

Example 4-7. Figure 4-22(*a*) shows an end span of some of the continuous-girder viaduct construction of the New Jersey approach of the Lincoln Tunnel at Weehawken. The cross section of the girder is shown in Sketch (*b*); the shear diagram in (*c*). Design vertical stirrups for this girder, assuming $f'_c = 2{,}500$ psi, $v'_L = 75$ psi, $u = 175$ psi, and the allowable stress in stirrups $f_v = 16{,}000$ psi. Since the girder is a very deep one, assume No. 5 stirrups in order to have them strong enough as columns to support the top longitudinal rods and to secure their reasonable spacing. Because of the negative moments at the supports, use inverted U stirrups in the portion DE as shown in Fig. 4-22(*b*) and for the reasons that have been explained in Art. 4-9.

1. *Section AB.* The critical section is at B where $d = 72$ in. It is not necessary to compute j for all the varying depths in such a member as this one. The fact that it is a T beam generally indicates that k will be less than 0.38. Therefore, if j is assumed to be equal to 0.87 instead of a larger value, the fact that it occurs in the denominator of Eq. (4-8) indicates that its use will yield conservative results.

$$v_L = \frac{V}{b'jd} = \frac{111{,}000}{27 \times 0.87 \times 72} = 66 \text{ psi}$$

which is less than v'_L; hence no stirrups are required theoretically.

2. *Section BD.* The shear is less than it is in section AB, so that no stirrups are necessary for resisting the shearing forces.

3. *Section DE.* The critical section is at D.

$$v_L = \frac{V}{b'jd} = \frac{214{,}000}{27 \times 0.87 \times 72} = 127 \text{ psi}$$

and, from Eq. (4-10),

$$s = \frac{A_v f_v}{(v_L - v'_L)b'} = \frac{2 \times 0.31 \times 16{,}000}{(127 - 75)27} = 7.1 \text{ in.}$$

Therefore, use stirrups at 7 in. c.c.

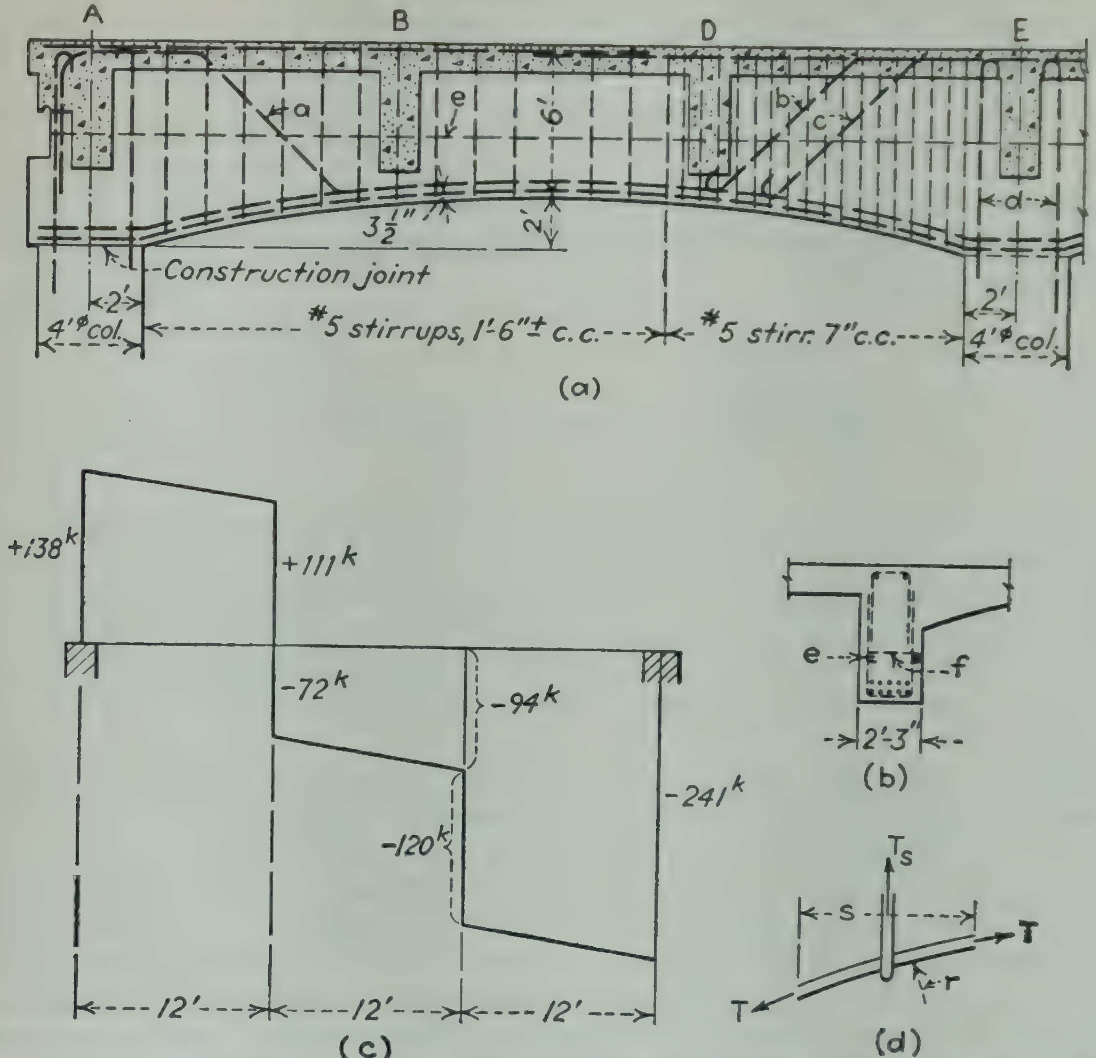


FIG. 4-22. Girder used in New Jersey approach to Lincoln Tunnel, New York City.

The top and bottom portions of the stirrups should be overlapped a distance

$$L_s = \frac{A_v f_v}{(\Sigma o)u} = \frac{0.31 \times 16{,}000}{1.96 \times 175} = 14.5 \text{ in., or, say, 16 in.}$$

4. *Practical Details.* The following practical details should be noticed in Figs. 4-22(*a*) and (*b*):

a. The bottom longitudinal rods, except those near *E*, will be in tension. As shown in Sketch (*d*) they will tend to straighten out and to spall off the concrete below them. Therefore, stirrups must be added to hold them back. (A similar situation occurs if compressive rods are bent around a corner when the stresses tend to buckle them outward.)

The stresses in these stirrups may be approximated by using Eq. (3-2), $T = pr$. In this case, $r = c^2/8m = 32^2/8 \times 2 = 64$ ft (approx).[1]

Approximately mid-ordinate $m = c^2/8r.$,

Assuming the maximum tension in all the bottom rods to be 330 kips, and using the typical No. 5 stirrups at 18 in. c.c.

$$p = \frac{T}{r} = \frac{330{,}000}{64} = 5{,}150 \text{ plf}$$

$$T_s \text{ in Fig. 4-22}(d) = ps = 5{,}150 \times 1.5 = 7{,}700 \text{ lb}$$

$$f_v = \frac{T_s}{A_v} = \frac{7{,}700}{2 \times 0.31} = 12{,}400 \text{ psi}$$

These stirrups must be strong enough, as a sort of combined tie and beam, to carry the radial pull of the rods near the center of the stem [see Sketch (*b*)]. This is mostly a matter of judgment; but if the width of the member is such that any of the main rods are more than 10 or 12 in. from the vertical part of a stirrup, additional supports should be provided.

Therefore, these stirrups will be used throughout the girder from *A* to *D*. As a practical matter, a few stirrups are often desirable in similar straight girders, too, in order to tie the member together thoroughly and to hold the main reinforcement in position during the pouring of the concrete.

b. Rods *a* are bent up, and then they are curved down at the end so as to reinforce the top corner.

c. Rods *b* and *c* are bent down to anchor them and to use them as extra web reinforcement—an arbitrary but conservative procedure.

d. The splicing of the bottom longitudinal rods is made at the top of the column at *E*; a few of the top rods are spliced near *D* and extended to *A*.

e. Rods *d* represent the column reinforcement which is extended up into the girder in order to reinforce the joint for continuous frame action.

f. The longitudinal rods *e* and the hooked ones *f* of Sketch (*b*) are used to tie all the stirrups together in both directions.

4-11. Web reinforcement in beams carrying moving loads. The floors of bridges and warehouses, longitudinal girders under railroad tracks, crane girders, and similar structures usually carry moving live loads of large magnitude. Therefore, these structures must be designed to withstand the maximum possible combination of these loads. Such conditions often cause severe stresses in the web reinforcement. The bending moment and the rate of increase of the bending moment—and therefore the longitudinal shearing stresses—differ with various positions of the loads. This means that, for the design of each section, the critical positions of the loads must be ascertained and the resultant stresses must be determined to see if the structure is safe. The reader should review Art. 3-9 because some of the principles stated there are applicable for shearing strength as well as for bond.

The use of influence lines will greatly facilitate the design of members that carry a series of moving live loads. A discussion of the theory of these diagrams and their construction will not be included here.

When more than one moving concentrated load is used, the maximum shear can be found at enough points to enable one to plot a curve showing the maximum longitudinal shear at all points in one-half of the beam.

The excess of these values over the assumed allowable stress v'_L for the unreinforced web of the beam can then be used in the design of the web reinforcement.

Because of the rapid changes in the magnitudes of the web stresses as the live loads pass over such beams, it is conservative to have web reinforcement throughout the length of the member, using nominal sizes in the central portions. Vertical stirrups are generally most suitable for this purpose. It also seems advisable to proportion the stirrups to carry all the shearing stresses but to use a theoretical unit stress of about 25,000 psi in them. However, the stirrups should never be weaker than those required by the use of Eq. (4-10). It is difficult to predict the shearing strengths of the portions of such beams that have tension in their tops under one position of the loads whereas there is tension in their bottoms for other positions of the same loads almost immediately thereafter, and vice versa. At such points, bent-up rods (if well anchored) are especially beneficial in preventing disintegration of the beams and in providing steel "hangers" to knit them together.

This shifting of the point of contraflexure in continuous beams with moving live loads is of great practical importance. It spreads the region of possible large bond stresses described in Art. 3-8; it shifts the theoretical bend and cutoff points; it casts doubt upon the resistance of this area to transverse shearing; it tends to disintegrate the beam if the changes are sufficiently violent; and it requires that one be very conservative in his design when planning web reinforcement and discontinuance of the longitudinal bars. He should provide against the most extreme conditions that will affect either the top or the bottom reinforcement.

Example 4-8. Assume that Fig. 4-23 is part of a small simply supported highway bridge. The loading diagram in (*a*) is to represent one set of wheels of a very short heavy truck. The figures include the allowance for impact. Design vertical stirrups for this beam, using the allowable stresses f_v, u, and v'_L = 18,000, 250, and 75 psi, respectively, in order to be very conservative with the last one. Accept the values of f'_c = 3,750 psi, $n = 8$, $k = 0.227$, and $j = 0.92$.

Then, if $x = 0$ in Fig. 4-23(*a*), the shear at A equals 39.38 + 12.6, or 51.98 kips. This figure includes the dead load of 900 plf. Also, if x = 14 ft, the shear at the left of C is 17.5 kips.

Therefore, using $j = 0.92$,

$$v_L \text{ at } A = \frac{V}{bjd} = \frac{51{,}980}{14 \times 0.92 \times 30} = 135 \text{ psi}$$

and

$$v_L \text{ at } C = \frac{17{,}500}{14 \times 0.92 \times 30} = 45 \text{ psi}$$

These values are used to construct Fig. 4-23(*c*). It is obvious that DE is a straight line. Also, it can be seen readily that the maximum longitudinal shear at any point between A and C is the ordinate from the line AC to DE. Therefore, allowing 75 psi

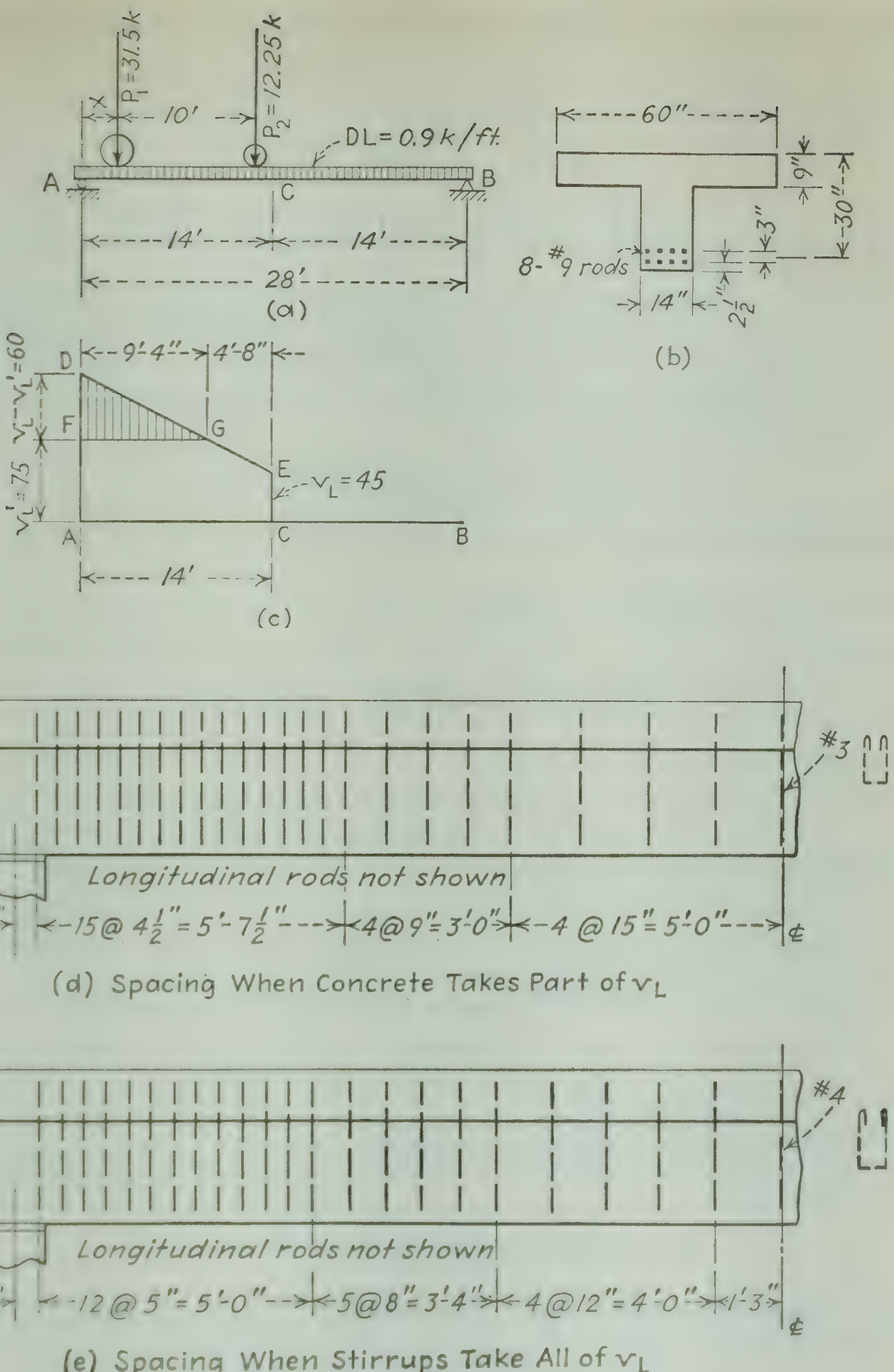

P1=31.5 k
P2=12.25 k
10′
DL=0.9 k/ft.
A
B
C
14′
14′
28′
(a)
60″
9″
30″
3″
8-#9 rods
14″
2½″
(b)
9′-4″
4′-8″
D
F
G
E
$v_L - v'_L = 60$
$v'_L = 75$
$v_L = 45$
A
C
B
14′
(c)
#3
Longitudinal rods not shown
4½″
15 @ 4½″ = 5′-7½″
4 @ 9″ = 3′-0″
4 @ 15″ = 5′-0″
℄
(d) Spacing When Concrete Takes Part of v_L
#4
Longitudinal rods not shown
5″
12 @ 5″ = 5′-0″
5 @ 8″ = 3′-4″
4 @ 12″ = 4′-0″
1′-3″
℄
(e) Spacing When Stirrups Take All of v_L

FIG. 4-23.

for v'_L, the longitudinal shear to be taken by the stirrups is represented by the triangle *DFG*.

Assume U-shaped stirrups made of No. 3 rods. Then, from $A_v f_v = (v_L - v'_L)bs$, the maximum spacing of these stirrups at A is

$$s = \frac{2 \times 0.11 \times 18{,}000}{60 \times 14} = 4.7 \text{ in.}$$

The spacing halfway between D and G may be 9.4 in. The maximum value of s is assumed to be $0.5d = 15$ in.

Furthermore, although stirrups are not theoretically needed from G to E, they will be used at the maximum spacing of 15 in. because the concrete may be cracked in this region owing to the deformation of the longitudinal steel.

Now check the critical spacing if the stirrups are to withstand all the shear at a unit stress of 25,000 psi. Then

$$s = \frac{A_v f_v}{v_L b} = \frac{2 \times 0.11 \times 25{,}000}{135 \times 14} = 2.9 \text{ in.}$$

This is too close a spacing. If No. 4 bars are used, then

$$s = \frac{2.9 \times 0.2}{0.11} = 5.3 \text{ in.}$$

At point E,

$$s = \frac{2 \times 0.2 \times 25{,}000}{45 \times 14} = 15.9 \text{ in.} \qquad \text{(more than } \tfrac{1}{2}d\text{)}$$

Halfway between A and C, $s = 10.6$ in.

If the allowance for v'_L is acceptable, No. 3 stirrups can be used at $4\frac{1}{2}$ in. c.c. for the end 5 or 6 ft of the beam, at 9 in. c.c. for the next 3 ft and at 15 in. c.c. for the central portion, as pictured in Fig. 4-23(*d*). However, the plan shown in Fig. 4-23(*e*) for No. 4 stirrups is preferred, and the extra cost is very little compared with the value received in greater reliability. If the beam were continuous, with alternating compression and tension in the vicinity of the region from $0.2L$ to $0.3L$, the use of an arrangement like Fig. 4-23(*e*) would be even more desirable.

The greatest transverse shear in the beam will occur at A. Its magnitude is

$$v_T = \frac{V}{bkd} = \frac{51{,}980}{14 \times 0.227 \times 30} = 545 \text{ psi}$$

or less than $0.2 \times 3{,}750 = 750$ psi. Nevertheless, this is rather large.

The No. 4 stirrups at 25,000 psi require good anchorage. The necessary length above mid-depth is

$$L_s = \frac{A_v f_v}{(\Sigma o)u} = \frac{0.2 \times 25{,}000}{1.57 \times 250} = 12.7 \text{ in.}$$

This is available but hooks should be used anyway because these stirrups are so important.

4-12. General considerations. The allowance that may be made for the strength of the concrete alone (v'_L) in Eqs. (4-10), (4-13), and (4-18) is somewhat indefinite, and certainly it is arbitrarily established. Sometimes it is specified; sometimes it must be based upon judgment. At best, it is merely an empirical method of allowing for the strength of

the beam without web reinforcement. The designer must decide whether or not the importance of the structure justifies assuming that the concrete alone may be relied upon to develop a safe longitudinal shearing stress of not over $0.03f'_c$ when the longitudinal reinforcement is anchored adequately.

It is very instructive for a student to design reinforced-concrete beams and then to test them to failure so that he can observe their behavior. However, this is not always possible.

Refer to Fig. 4-5 again. It shows some beams with various types of web reinforcement which were tested to failure. The arrangement of the cracks should be studied very carefully in order to give the student a clear idea of how these beams acted under load. For instance, bearing in mind the previous discussion, notice the following:

1. The sketches show the tendency of the hair cracks to be vertical between the loads where the shear is zero but to be inclined at about 45° outside this central region.

2. The T beams, No. 2210.1, etc., show a large number of cracks which extend to or above the bottom of the flange. This cracking is more severe than for rectangular beams, and it clearly indicates the rise of the neutral axis toward the top because of the greater area of concrete which can resist the compression.

3. Beams 221.1, 222.1, and 2210.1 indicate the progressive failure of the concrete due to diagonal tension.

4. The beams that have heavy closely spaced web reinforcement seem to have finer cracks which are more uniformly spaced than those of the other members.

5. The stripping of the unanchored longitudinal rods away from the concrete is shown at the left end of beam 221.1.

Such considerations as these will help one to visualize more clearly the probable actions of the structures that he designs. This ability to look upon the members of a structure as parts of an almost living whole, each part acting and deforming in accordance with known laws, is one of the attributes of the expert designer.

Practice Problems

4-1. Assume a rectangular beam for which $b = 16$ in., $d = 27$ in., $k = 0.36$, $j = 0.88$, and the maximum shear $V = 35{,}000$ lb. Compute v_L and v_T. Is the beam safe without web reinforcement if $v'_L = 75$ psi?

Discussion. Remember that, as used here, v_L = intensity of longitudinal shear, v_T = intensity of transverse shear when confined to the depth kd, and v'_L = allowable intensity of longitudinal shear when no web reinforcement is used.

4-2. Compute the spacing of No. 3 vertical U stirrups at the end of the beam of Prob. 4-1 if the stress in the stirrups f_v is 16,000 psi and if they are to withstand the excess shear over v'_L. *Ans.* 12.9 in. Use 12 in.

4-3. Compute v_L and v_T for a rectangular beam for which $b = 18$ in., $d = 32$ in., $k = 0.33$, $j = 0.89$, and $V = 40{,}000$ lb. If $v_L' = 60$ psi, find the tensile stress in No. 3 vertical U stirrups spaced 8 in. c.c. to withstand the excess shear over v_L'.

4-4. Assume that a simply supported beam is 20 ft long and that it carries a total uniformly distributed load of 2,000 plf. Assume that $b = 12$ in., $f_c' = 3{,}000$ psi, $j = 0.88$, and $v_L' = 0.03f_c'$. Determine the depth of the beam that will be safe with no web reinforcement.

Discussion. Find the end reaction; then substitute v_L' for v_L in Eq. (4-4), and solve for d. Add $2\frac{1}{2}$ or 3 in. for cover at the bottom.

4-5. Assume a simply supported T beam with a span of 25 ft and a uniformly distributed load of 4,500 plf. Let $d = 26$ in., $b' = 16$ in., $f_v = 18{,}000$ psi, and $j = 0.88$. Draw the shear diagram and design No. 3 vertical U-shaped stirrups to act as web reinforcement. Assume $v_L' = 90$ psi.

4-6. Assume a simply supported T beam with a span of 24 ft. The moving live loads and the dead load cause a shear at the end equal to 30,000 lb; that at the center of the span equals 10,000 lb. Assume that this shear varies uniformly between these points. If $b' = 15$ in., $d = 27$ in., $j = 0.92$, and $v_L' = 60$ psi, determine the required spacing of No. 3 vertical U-shaped stirrups if the permissible tension in the steel = 15,000 psi. *Ans.* $s = 10.7$ in. Use 10 in.

4-7. Assume that the beam of Prob. 4-5 is to have the No. 3 vertical U-shaped stirrups designed to take all the shear, using a unit stress of 25,000 psi in them. Make a detailed layout of the stirrups for this case. What is the maximum spacing near the center regardless of the stress?

4-8. Assume a T beam which is continuous (call it "fixed") over a series of supports. Let one interior span be 30 ft. If the total load is 3,500 plf (uniformly distributed), detail and space No. 4 vertical U-shaped stirrups to provide for the shear in excess of $v_L' = 60$ psi, using $f_v = 16{,}000$ psi, $b' = 15$ in., $d = 27$ in., and $j = 0.88$.

Discussion. Draw the shear and moment diagrams. In the regions where tensile stresses exist at the top of the beam, use lapped U stirrups as shown in Fig. 4-17; elsewhere, use standard U-shaped ones and assume $u = 150$ psi allowable for computing the lap of the stirrup rods. Throughout the central portion, use stirrups at a spacing not to exceed $\frac{1}{2}d$ c.c.

4-9. Design No. 4 U-shaped stirrups which are inclined 30° from the vertical for Prob. 4-8.

4-10. Design the web reinforcement for a simply supported T beam, using bent-up rods where they are feasible, and using No. 3 U-shaped vertical stirrups elsewhere. Assume the following data: $b' = 12$ in.; $d = 30$ in.; $j = 0.94$; the tensile reinforcement = six No. 9 rods in two rows 3 in. apart; the total load = 3,500 plf; span = 24 ft; the two outer rods of the bottom row are to be straight throughout; $v_L' = 90$ psi; $f_v = 18{,}000$ psi; the seat under the beam is assumed to be 1 ft long.

Discussion. Draw the shear and bending-moment diagrams as in Fig. 4-13; determine the possible bend points of the bars; find where the bent bars do not reinforce the web (if any such parts exist), and add No. 3 vertical stirrups; make a detailed sketch of the reinforcement.

4-11. Design and detail the web reinforcement for the T beam of Fig. 2-27, using bent-up rods and additional No. 3 vertical U-shaped stirrups if they are needed. Assume $j = 0.9$; $f_v = 18{,}000$ psi; the top steel = six No. 8 rods with four in the top row and two in the second row 3 in. farther down; the span = 30 ft; the total load is 2,100 plf, uniformly distributed; the ends may be called fixed; four No. 8 rods are used in the bottom at the center, two being straight for their full length; $v_L' = 60$ psi; and the support under the beam is 18 in. long.

4-12. Redesign the web reinforcement of the beam of Prob. 4-11 if the total loading is changed to 1,000 plf uniformly distributed and two concentrated loads of 15,000 lb each at points 10 ft from each support. (See Fig. 1 in the Appendix.) Use the same rods as before and bend them. Add vertical U-shaped stirrups where required.

4-13. Redesign the web reinforcement of the beam of Prob. 4-11 with the loading of Prob. 4-12, but use No. 3 vertical U-shaped stirrups (or double ones), stopping the longitudinal rods or bending them as desired but not relying upon the bent rods as web reinforcement. Let $f_v = 16{,}000$ psi and $v'_L = 50$ psi. Detail the reinforcement.

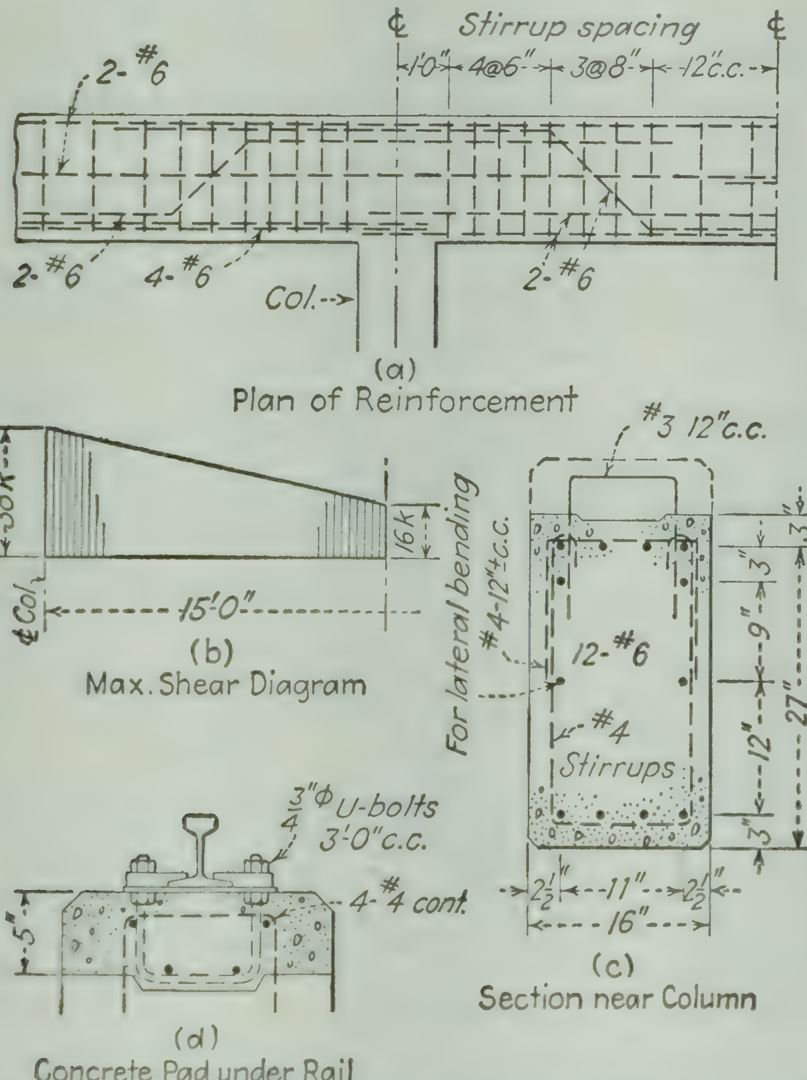


FIG. 4-24. Reinforced-concrete crane girder used in the concentrator of the low-grade ore project. (Courtesy of Cananea Consolidated Copper Company, Cananea, Sonora, Mexico.)

4-14. Figure 2-30 shows a tapered, cantilevered, rectangular beam. The proposed vertical stirrups are No. 4 at 6 in. c.c. throughout the beam. Consider the weight of the concrete, and assume the column to be 18 in. wide. Test the deep and the shallow portions, using $d = 26$ in. at the column and $d = 16$ in. at the end near the concentrated load. Assume $j = 0.88$ in both cases. (*a*) Is the beam safe if the concrete is to resist 60 psi and the allowable $f_v = 16{,}000$ psi? (*b*) Is it safe if the stirrups are designed to carry all the shear with $f_v = 30{,}000$ psi? (*c*) What should be done to remedy the case if the beam were otherwise satisfactory?

Ans. (*a*) f_v near column = 12,200 psi, satisfactory; f_v near end = 26,600 psi, no good; (*b*) f_v near column = 26,600 psi, satisfactory; f_v near end = 41,000 psi, no good;

(*c*) bend down two lower bars of tensile reinforcement on general principles, then use closer stirrup spacing toward end.

4-15. Figure 4-24 shows a continuous reinforced-concrete crane girder used in the concentrator of the Cananea Consolidated Copper Co., Cananea, Sonora, Mexico. Are the stirrups safe if they are to carry all the shear, assuming that the allowable $f_v = 25{,}000$ psi and $j = 0.88$?

Discussion. The 5-in. cap shown in Figs. 4-24(*c*) and (*d*) is for the purpose of aligning, leveling, and grouting the rail after the girders have been completed and their forms removed. Otherwise it would be difficult to set the rail and its attachments accurately. The total depth of the beam is assumed to be 30 in. Incidentally, such heavy elevated concrete construction requires strong, and perhaps expensive, formwork.

Ans. Yes; $f_v = 24{,}000$ psi max.

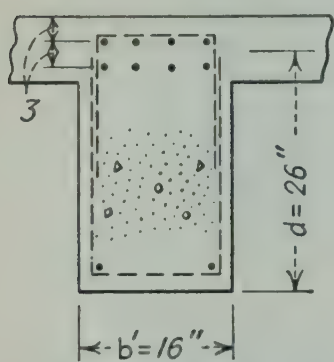
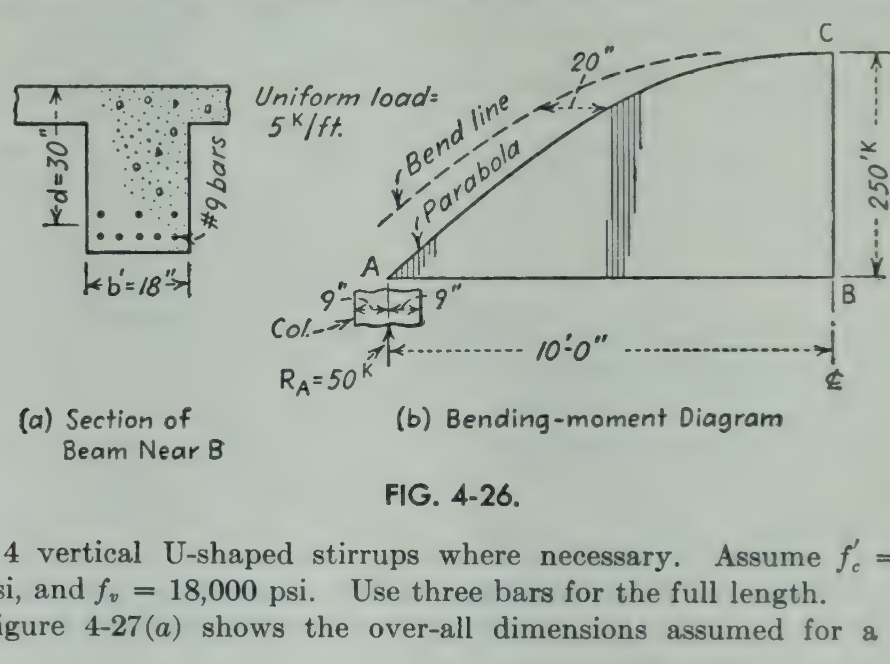


FIG. 4-25.

4-16. Figure 4-25 shows the cross section of a continuous beam just outside a column. The shear $V = 62{,}000$ lb. Assume 3,000-lb concrete and $j = 0.88$. By using the Code stresses, with $u = 0.07f'_c$, design vertical U-shaped stirrups for 6-in. spacing near the column. Use the type shown in Fig. 4-17(*d*), and compute the overlap.

Ans. Use No. 4. Overlap = 12 in.

4-17. Figure 4-26 shows a simply supported T beam with a span of 20 ft. Using a scale of ½ in. = 1 ft 0 in., draw a side elevation for bending up the longitudinal reinforcement shown in (*a*).

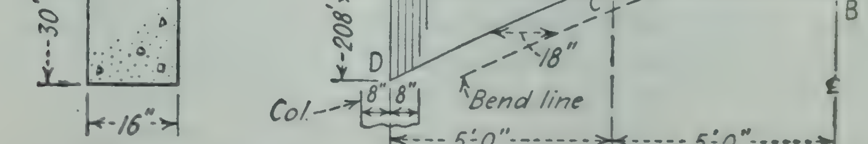


(*a*) Section of Beam Near B (*b*) Bending-moment Diagram

FIG. 4-26.

Add No. 4 vertical U-shaped stirrups where necessary. Assume $f'_c = 2{,}500$ psi, $v'_L = 75$ psi, and $f_v = 18{,}000$ psi. Use three bars for the full length.

4-18. Figure 4-27(*a*) shows the over-all dimensions assumed for a continuous



(*a*) Overall Dimensions of Beam (*b*) Bending-moment Diagram

FIG. 4-27.

T beam with a span of 20 ft. It supports a total uniform load of 4,000 plf plus a concentrated load of 30,000 lb at its center. Assume the bending-moment diagram shown in (*b*). Design the longitudinal reinforcement at *A* and at *B*. Show a scheme for bending the rods for the purpose of resisting bending moment only. Design vertical stirrups for web reinforcement. Use 3,000-lb concrete, intermediate grade steel, and Code unit stresses.

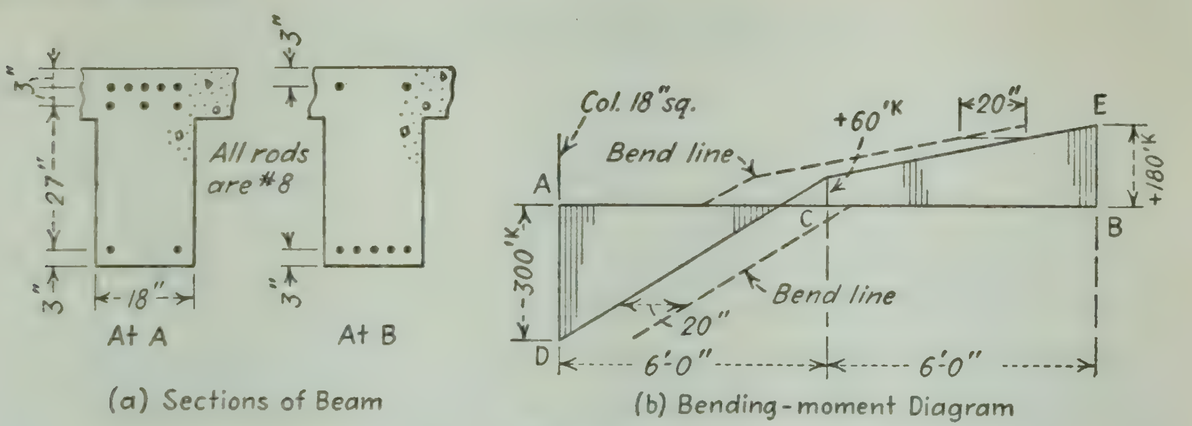


FIG. 4-28.

4-19. Assume that a continuous T beam is to have the bending moment and the cross sections shown in Fig. 4-28. The span is 24 ft. The end shear is 60 kips and the shear will be assumed to be constant from *A* to *C*. Design the web reinforcement as stirrups only. Draw a side elevation showing where the top and bottom rods may be stopped and what the spacing of stirrups is to be. Assume 2,500-lb concrete, structural-grade steel, and unit stresses allowed in the Code. Assume $j = 0.88$. Is the bond satisfactory?

5

COMPOSITE BEAMS

5-1. Introduction. Sometimes it becomes necessary to analyze or to design a structure that is composed of steel I beams, girders, or light trusses that are encased in concrete. The concrete may be used as a protection against fire and corrosion, or it may be designed to act as a load-carrying element of the structure. The latter case will be illustrated. The properties of the structural-steel sections can be found in suitable handbooks.

In highway bridge construction it is becoming accepted practice to attach reinforced-concrete slabs to the top flanges of bare steel members and to design the structure so that the concrete will help the steel in supporting loads.

The term *composite beam* is used to denote the foregoing cases; *i.e.*, the two materials are to act together in resisting the bending moment. Such a beam is differentiated from an ordinary reinforced-concrete one, primarily because the steel is a large rolled or fabricated unit which usually has great strength in itself. The concrete adds to the strength and stiffness of the steel, but it is the weaker of the two materials.

5-2. I-beam and thin-slab construction. A type of construction that is common in steel-framed buildings is shown in Fig. 5-1(*a*). It consists of a rather thin concrete slab which is supported on steel I beams. Ordinarily, the slab is poured monolithically with the encasement of the beams. When a load is placed upon such a floor, it is obvious that both the concrete and the steel will be affected.

Frequently, in such a case, the steel beam is designed to act alone in carrying the entire load. If its top flange is thoroughly embedded in the concrete of the slab—$\frac{1}{2}$ in. or more above the bottom of the latter—the beam is usually considered to have adequate lateral support because the concrete will not let it bend sideways. Under such conditions, some specifications permit the use of a higher unit stress in the beam than would be allowed for one that is not encased. This is done in order to allow indirectly for the benefit from the action of the concrete in assisting or augmenting the load-carrying capacity of the steel.

This again calls attention to the very important fact that, when steelwork is encased in concrete, a material of high ductility is covered by another one with very little ability to stretch but with great compressive strength. Therefore, when a load is applied, the two materials try to act in accordance with their own particular characteristics. The top flange of the I beam in Fig. 5-1(a) tends to compress inside the concrete; the bottom flange tries to elongate; both flanges endeavor to deform about a neutral axis which is at the center of the web. Simultaneously, the concrete tries to act as a T beam with its neutral axis close to the bottom of the slab. These two different actions must be in conflict because of the bond between the steel and the concrete. Unless this

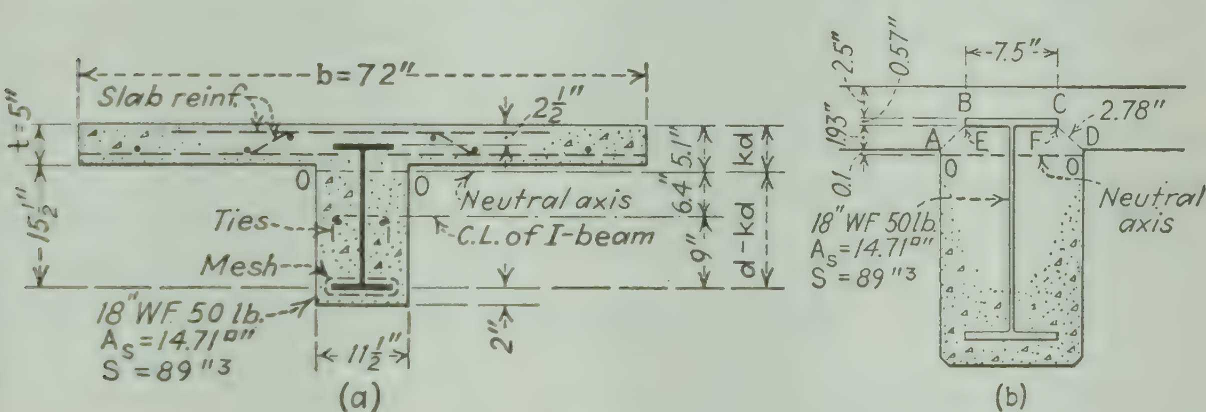


FIG. 5-1.

bond is broken, the steel cannot deform one way while the concrete deforms another way. Therefore, the action is that of a composite unit.

Assuming that the bond does not fail, the beam can be analyzed by the transformed-section method. To illustrate this, analyze the beam of Fig. 5-1(a). Let $n = 10$ and the allowable f_s and f_c = 20,000 and 1,000 psi, respectively. Neglect the small tie rods in the slab.

The transformed section of the I beam will be assumed to equal

$$nA_s = 10 \times 14.71 = 147.1 \text{ in.}^2$$

Theoretically, the portion of the I beam above the neutral axis of the composite beam should have its area multiplied by $(n - 1)$, but although the flange area of the steel equals several square inches, it is too small compared with the concrete to warrant such refinement.

Taking the static moment about the neutral axis of the composite section, neglecting any of the stem that may be in compression,

$$bt\left(kd - \frac{t}{2}\right) = nA_s(11.5 - kd)$$

$$72 \times 5(kd - 2.5) = 147.1(11.5 - kd)$$

$$kd = 5.1 \text{ in.} \qquad d - kd = 15.4 \text{ in.} \qquad \text{(near enough)}$$

Assume that the moment of inertia of the transformed section of the I beam about its own center of gravity is nI.

$$nI = 10 \times 800.6 = 8{,}006 \text{ in.}^4 \qquad \text{(see any steel handbook)}$$

$$I_c = \frac{bt^3}{12} + bt\left(kd - \frac{t}{2}\right)^2 + nI + nA_s(11.5 - kd)^2$$

$$I_c = \frac{72 \times 5^3}{12} + 72 \times 5 \times 2.6^2 + 8{,}006 + 147.1 \times 6.4^2 = 17{,}200 \text{ in.}^4$$

$$S_c = \frac{17{,}200}{5.1} = 3{,}370 \text{ in.}^3$$

$$S_s = \frac{17{,}200}{10 \times 15.4} = 112 \text{ in.}^3$$

Then

$$\max M_c = f_cS_c = 1{,}000 \times 3{,}370 = 3{,}370{,}000 \text{ in.-lb}$$
$$\max M_s = f_sS_s = 20{,}000 \times 112 = 2{,}240{,}000 \text{ in.-lb}$$

Therefore, the stress in the concrete, assuming composite-beam action, is determined by the strength of the steel. When the stress in the steel is 20,000 psi,

$$f_c = \frac{M_s}{S_c} = \frac{2{,}240{,}000}{3{,}370} = 665 \text{ psi}$$

Considering the I beam alone with $f_s = 20{,}000$ psi, its safe resisting moment, when laterally supported, is

$$M = Sf_s = 89 \times 20{,}000 = 1{,}780{,}000 \text{ in.-lb}$$

Therefore, the permissible increase in the safe bending moment for the composite beam over that for the plain steel beam is

$$\frac{100(M_s - M)}{M} = \frac{100(2{,}240{,}000 - 1{,}780{,}000)}{1{,}780{,}000} = 25.8 \text{ per cent}$$

The transverse shearing forces are not important in this case because they are resisted by both the web of the I beam and the concrete. The former is usually capable of carrying the entire load.

Also, the longitudinal shear cannot cause failure of the same character as that which was discussed in the preceding chapter. However, it may produce excessive bond stresses or high local shearing stresses in the concrete.

To investigate this problem, let Fig. 5-1(b) represent an enlargement of a portion of Fig. 5-1(a). The beam will be assumed to have a span of 25 ft and a uniform load of 2,200 plf, including the dead load. The longitudinal shearing stress upon any plane of any section of the beam per unit length may be assumed as

$$S_L = \frac{VQ}{I_c}$$

where S_L equals the total longitudinal shear in pounds per linear inch of the beam, V equals the transverse shear at the given section in pounds, Q is the static moment of the part beyond the plane being considered (in this case *above AEBCFD* in the figure) computed about the neutral axis of the composite section, and I_c is the moment of inertia of the composite beam about its own center of gravity (or neutral axis). Q and I_c must be expressed in terms of inch units. They must also be computed upon the basis of the transformed section in terms of concrete. Therefore, at the end of the beam,

$$S_L = \frac{(12.5 \times 2{,}200)(72 \times 5 \times 2.6 - 7.5 \times 2.5 \times 1.35 - 1.93 \times 2 \times 0.74)}{17{,}200}$$

$$S_L = 1{,}450 \text{ pli}$$

$$v_L = \frac{S_L}{AEBCFD} = \frac{1{,}450}{2 \times 2.78 + 2 \times 0.57 + 7.5} = \frac{1{,}450}{14.1} = 103 \text{ psi}$$

The stress on the surfaces AE and FD is shear in the concrete, whereas that on $EBCF$ is bond stress. The magnitude of the latter seems to be rather large to be relied upon for such wide flat surfaces of steel. Furthermore, since the bond stress is really a shearing stress, it is not reasonable to assume that the shear on the sections of concrete at AE and FD can have a value that is much different from the bond stress on the steel, because all the material must deform together until the bond fails or the concrete shears off. It is therefore advisable to use some kind of anchorage between the steel and the concrete, as illustrated in the next problem.

Example 5-1. A modification of the problem that has previously been illustrated is shown in Fig. 5-2(a). Using the same data, find the bond stress at the top of the I beam.

Although this arrangement appears strange, such conditions often occur in building construction because of practical considerations. They should be guarded against because a crack is likely to form along the plane $ABCD$.

Proceeding as before, an analysis of this beam gives the following values:

$$kd = 6.7 \text{ in.} \qquad d - kd = 19.3 \text{ in.} \qquad I_c = 30{,}700 \text{ in.}^4$$
$$S_c = 4{,}580 \text{ in.}^3 \qquad S_s = 159 \text{ in.}^3$$
$$M_c = 4{,}580{,}000 \text{ in.-lb} \qquad \text{(greater than before)}$$
$$M_s = 3{,}180{,}000 \text{ in.-lb} \qquad \text{(greater than before)}$$
$$\text{Computed } f_c = \frac{wL^2 \times 12}{8 \times S_c} = \frac{2{,}200 \times 25^2 \times 12}{8 \times 4{,}580} = \frac{2{,}060{,}000}{4{,}580} = 450 \text{ psi}$$

when the real external bending moment is carried by the composite beam.

The static moment Q of the portion *below* AD about the neutral axis O-O is

$$Q = nA_s(8 + 9 - kd) = 10 \times 14.71 \times 10.3 = 1{,}515 \text{ in.}^3$$

Therefore, using a transverse shear V equal to the end reaction of the beam,

$$S_L = \frac{VQ}{I_c} = \frac{(12.5 \times 2{,}200)(1{,}515)}{30{,}700} = 1{,}360 \text{ pli}$$

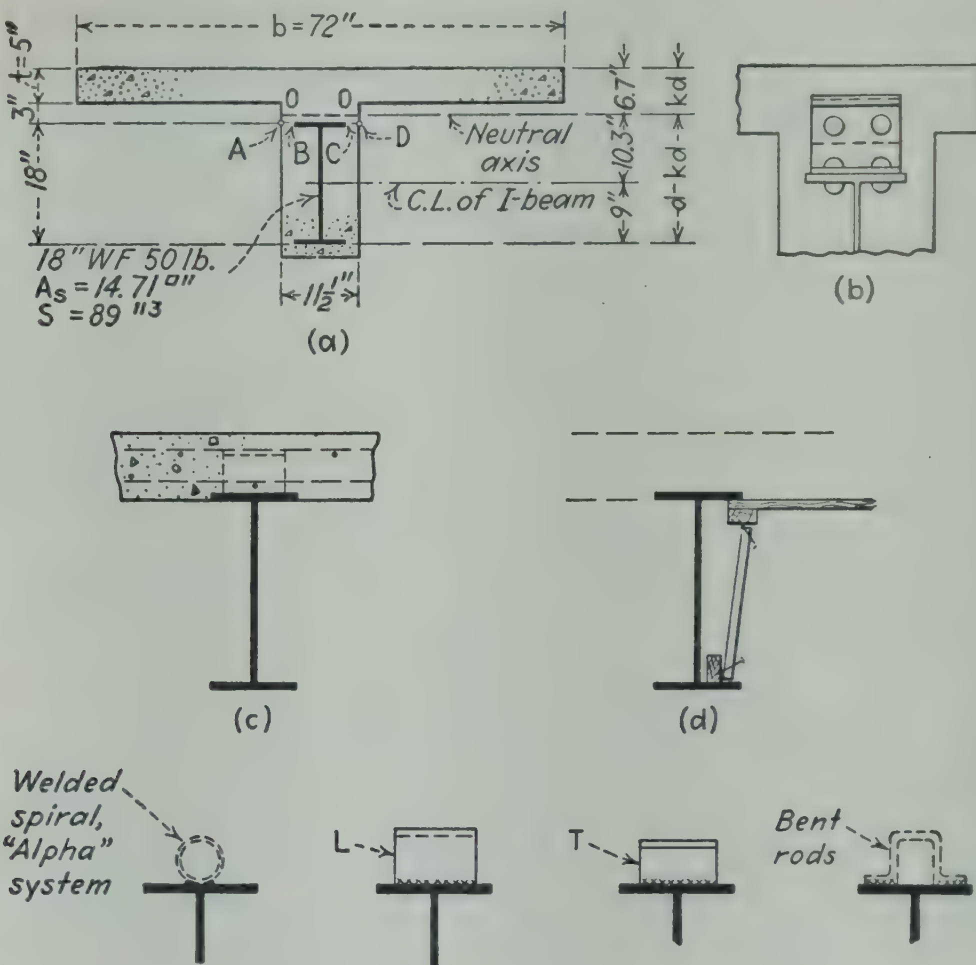
or

$$v_L = \frac{S_L}{AD} = \frac{1{,}360}{11.5} = 118 \text{ psi}$$

These figures show that a crack may form along the plane AD. One possible method of preventing this is by riveting double angles on the top flange as shown by Fig. 5-2(b), forming Z-shaped lugs which provide a mechanical bond. These angles can be spaced as required to withstand the longitudinal shear, being closer together near the ends of the beam where the shearing forces are the greatest. These lugs provide a mechanical bond which helps to resist the longitudinal shear and to

enable the beam to act as a composite section. Incidentally, these angles are useful in tying the slab and the steel together so that continuity of the slab, or any other action, will not cause the slab to be pulled away from the I beam.

The patented "Alpha" system utilizes spiral rods which are welded to the top flange of the beam to resist the longitudinal shearing forces. In



(e) Some Welded Shear Lugs

FIG. 5-2.

some of the structures that have been built by The Port of New York Authority, the beams are set so that the reinforcing trusses in the slab can be welded directly to the top flange, thus forming a lug on the beam.

A very common type of construction for building work and bridges, where the steel beams are exposed, is shown in Fig. 5-2(c). The bottom of the slab is flush with the lower side of the top flange. This facilitates the formwork, as pictured in (d). The slab also provides lateral support for the beam. The reinforcement of the slab may be laid directly on the top flange of the beam when the latter is thick enough. A rod may be used to raise the reinforcement in order to obtain adequate cover when the flange is thin, as pictured in (c).

Some other types of shear lock that can be welded directly to the steel beams are pictured in Fig. 5-2(*e*). Notice that they provide mechanical means for holding the slab down to the beam as well as for resisting longitudinal shear. This is especially desirable in bridge floors that may tend to vibrate and loosen the slab. The welded angles or tees with their flat tops serve as steps on which workmen can walk more easily during construction than they can upon narrow edges. Although experience has shown that some slabs poured on bare steel beams have bonded the steel sufficiently to provide composite action, it is advisable to use some type of mechanical locking device.

Some specifications state a limit of 15 or 20 per cent for the increase of bending resistance allowed for the composite beam over that of the steel beam alone.

If structural-steel members—I beams or trusses—are used to support the forms during the placing of the concrete, the stresses resulting from this dead load must be computed and deducted from the allowable stress in the steel, the remainder being the stress that is available for the steel as reinforcement in the concrete. For instance, if such dead loads caused a tension of 9,000 psi in the I beam of this problem, for which the allowable stress is 20,000 psi, then 11,000 psi would be the maximum that could be used in the composite section. However, the dead load that is already carried by the I beam should not be included again in computing the loads that cause bending moments and shears in the composite beam.

When composite beams frame into steel girders or columns, special care should be taken to avoid cracking along the top flanges because of deflection of the beams. Where feasible, the steel beams should be made continuous with tension plates and bottom thrust angles; otherwise use rods in the slab to carry tension across the girder or past the column. The whole framework should be looked over to find the places where deflections under live loads will be likely to cause cracks; then, if it is not practicable to make the steelwork continuous, it may be advisable to make definite joints or cuts in the concrete so that the cracking will occur at such predetermined locations.

To illustrate the combination of a concrete slab on a bare steel stringer in a bridge consider Fig. 5-3. This structure is built with a number of continuous stringers 6 ft c.c. The bridge has two 50-ft spans. The maximum tensile stress is to be 18,000 psi. Let $n = 10$. The shear at the simply supported end is to be 50 kips. Assume that the maximum positive bending moment is 283 ft-k for live loads and 111 ft-k for dead load, the latter being supported by the steel stringer alone when the concrete is poured.

The bottom flange is reinforced with a welded cover plate that extends over most of the length where tensile stresses will exist. This is done to provide more strength here so as to utilize the heavy slab to better advantage.

As a first try, assume that the stringer is adequately supported laterally so that the

full stress of 18,000 psi is allowable. Then the required section modulus for resisting the bending produced by the dead load is

$$S_s \text{ for D.L.} = \frac{111{,}000 \times 12}{18{,}000} = 74 \text{ in.}^3$$

Next assume that the steel alone resists about 75 or 80 per cent of the live load moment. Therefore,

$$S_s \text{ for L.L.} = \frac{0.75 \times 283{,}000 \times 12}{18{,}000} = 142 \text{ in.}^3$$

Together, these amount to $74 + 142 = 216$ in.^{3} required.

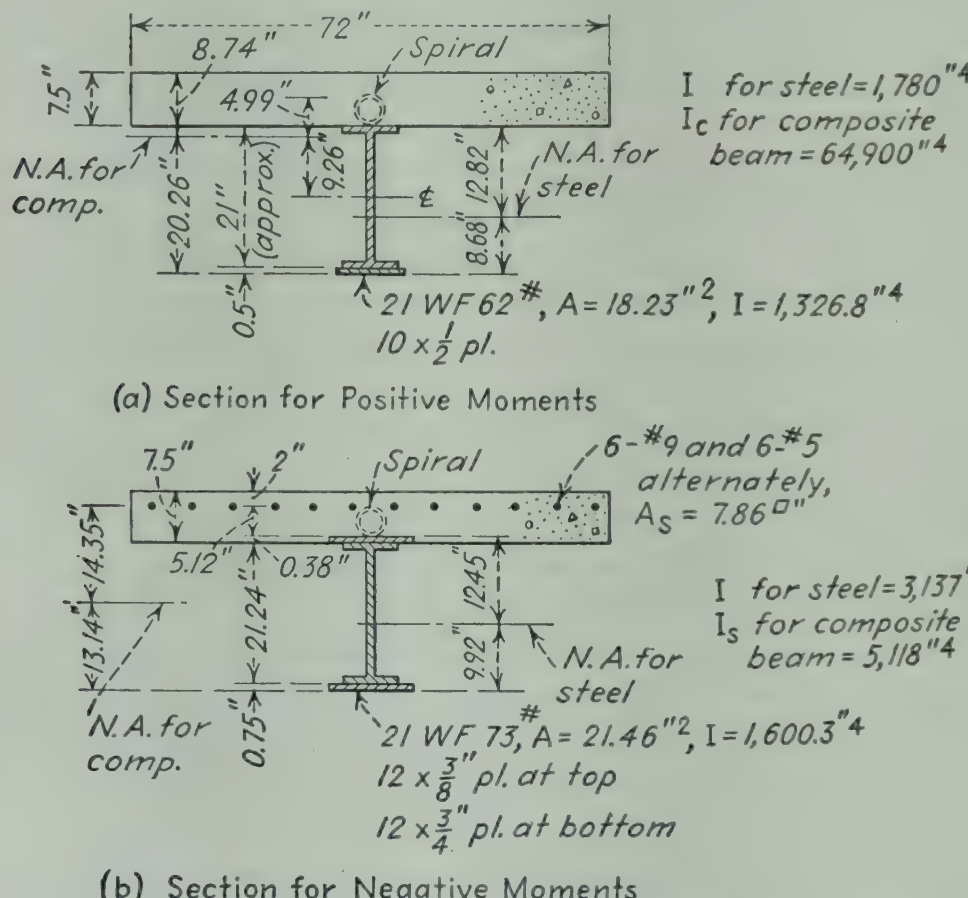


FIG. 5-3. Analysis of the Alpha composite construction used in a two-span continuous highway bridge. *(Adapted from data furnished by the Porete Manufacturing Co., North Arlington, N.J.)*

The section modulus of a 21*WF*62 alone is 126.4 in.^{3}, leaving about 90 in.^{3} for cover plates. Assuming the equivalent of equal cover plates top and bottom, each with an area A and a trial thickness of ½ in., their moment of inertia will be

$$2 \times A \times 10.75^2 = 231\ A$$

Their section modulus is

$$S_s \text{ for covers} = \frac{231A}{11} = 21A$$

If this equals the required amount of 90 in.^{3}

$$A = 90/21 = 4.28 \text{ in.}^2$$

Therefore, use a 10- by ½-in. cover plate on the bottom but let the concrete slab reinforce the top.

For the steel beam and the cover plate alone, the neutral axis is located as shown in Fig. 5-3(a), and the moment of inertia is $I = 1{,}780$ in.^{4}. For the composite section assume that the entire slab between centers of stringer spacings is available since it is so thick and well reinforced. Taking moments about the center of the I beam, the following are found:

Material	*Area*	*Lever arm*	*Moment*
Concrete = 72 × 7.5	= 540	× 14.25	= 7,695
Beam = nA_s = 10 × 18.23	= 182.3	× 0	= 0
Cover plate = nA_s = 10 × 5	= 50.	× (−10.75)	= −538
	772.3 in.^{2}		7,157

$$\text{Center of gravity} = \frac{7{,}157}{772.3} = 9.26 \text{ in. above center of beam}$$

This is shown in Fig. 5-3(a).

The moment of inertia of the composite beam about this center of gravity is

$$\begin{aligned} I_c &= \frac{72 \times 7.5^3}{12} + 72 \times 7.5(8.74 - 3.75)^2 \text{ for concrete} \\ &\quad + 10 \times 1{,}326.8 + 10 \times 18.23 \times 9.26^2 \text{ for beam} \\ &\quad + 10 \times 5 \times 20.01^2 \text{ for cover plate} \\ I_c &= 64{,}900 \text{ in.}^4 \end{aligned}$$

Using these data as shown in Fig. 5-3(a), the stresses are found to be the following:
D.L. for beam alone:

$$f_s \text{ in top} = \frac{111{,}000 \times 12 \times 12.82}{1{,}780} = 9{,}600 \text{ psi}$$

$$f_s \text{ in bottom} = \frac{111{,}000 \times 12 \times 8.68}{1{,}780} = 6{,}500 \text{ psi}$$

L.L. for composite beam:

$$f_s \text{ in top} = \frac{283{,}000 \times 12 \times 8.74}{64{,}900} = 458 \text{ psi}$$

$$f_s \text{ in bottom} = \frac{M(d - kd)n}{I_c} = \frac{283{,}000 \times 12 \times 20.26 \times 10}{64{,}900} = 10{,}600 \text{ psi}$$

TABLE 5-1. Allowable Load per Pitch for Alpha Composite Construction*

Bar No.	Safe load, kips
4	7.06
5	11.04
6	15.09

* Based on 18 ksi in reinforcement, and upon recommendations of Porete Mfg. Co.

Totals: f_s in top of beam is

$$\begin{aligned} f_s &= 9{,}600 + nf_c\left(\frac{8.74 - 7.5}{8.74}\right) = 9{,}600 + 10 \times 458 \times \frac{1.24}{8.74} = 9{,}600 + 650 \\ &= 10{,}250 \text{ psi compression} \end{aligned}$$

$$f_c \text{ in concrete} = 458 \text{ psi}$$

$$f_s \text{ in bottom of beam} = 6{,}500 + 10{,}600 = 17{,}100 \text{ psi tension}$$

These will be accepted.

With an end shear of 50 kips,

$$S_L = \frac{VQ}{I_c}$$

where Q is the static moment of the slab above the beam flange about the neutral axis. Thus

$$Q = 72 \times 7.5(8.74 - 3.75) = 2{,}690 \text{ in.}^3$$
$$S_L = \frac{50{,}000 \times 2{,}690}{64{,}900} = 2{,}070 \text{ pli}$$

If a No. 5 bar in a 4½-in.-diameter spiral is to be welded on the top of the beam, and if each welded point is safe for a shear of 11,000 lb, the spacing S of welds, or pitch of spiral, should be

$$S = \frac{11{,}000}{S_L} = \frac{11{,}000}{2{,}070} = 5.32 \text{ in.}$$

Use 5 in. near the end; then increase the spacing near mid-span if desired, being limited by the requirements for shear.

Over and near the center pier, the tension will be in the top of the composite beam. In such a case, extra reinforcement may be added in the slab as shown in Fig. 5-3(b) where the No. 5 typical longitudinal bars at 12 in. c.c. have No. 9 bars placed between them. Top cover plates can be added to strengthen the beam, or both plates and bars can be used as shown in the sketch. The transformed section can then be treated as though it were steel only, with the area of the bars added in its proper position in excess of the cover plates and the beam. The data shown in Fig. 5-3(b) have been taken directly from the Porete Manufacturing Company's computations.

For a dead-load moment of 234 ft-k taken by the steel beam and cover plates alone,

$$f_s \text{ at top } = \frac{234{,}000 \times 12 \times 12.45}{3{,}137} = 11{,}160 \text{ psi}$$
$$f_s \text{ at bottom } = \frac{234{,}000 \times 12 \times 9.92}{3{,}137} = 8.890 \text{ psi}$$

For a live-load moment of 288 ft-k taken by the composite member,

$$f_s \text{ at top of beam } = \frac{288{,}000 \times 12 \times 9.23}{5{,}118} = 6{,}240 \text{ psi}$$
$$f_s \text{ in top bars } = \frac{288{,}000 \times 12 \times 14.35}{5{,}118} = 9{,}700 \text{ psi}$$
$$f_s \text{ at bottom of beam } = \frac{288{,}000 \times 12 \times 13.14}{5{,}118} = 8{,}900 \text{ psi}$$

Therefore, the maximum stresses in the structural steel are

$$f_s \text{ at top } = 11{,}160 + 6{,}240 = 17{,}400 \text{ psi tension}$$
$$f_s \text{ at bottom } = 8{,}890 + 8{,}900 = 17{,}790 \text{ psi compression}$$

For a shear of $V = 57{,}000$ lb at the interior support, the longitudinal shear

$$S_L = \frac{VQ}{I} = \frac{57{,}000 \times 7.86 \times 14.35}{5{,}118} = 1{,}255 \text{ pli}$$

where Q = the area of the bars times their lever arm from the neutral axis of the composite member. Therefore,

$$S = \frac{11{,}000}{1{,}255} = 8.77 \text{ in.}$$

Use $8\frac{1}{2}$ in. for the pitch of the spirals near this support.

The spacing of other kinds of shear locks can be computed similarly.

5-3. I beams completely encased in thick slabs. In some cases, small I beams may be completely encased in thick concrete slabs as shown in Fig. 5-4. In such cases, it is reasonable to assume that, with proper inspection in the field, the bond and shearing stresses will be

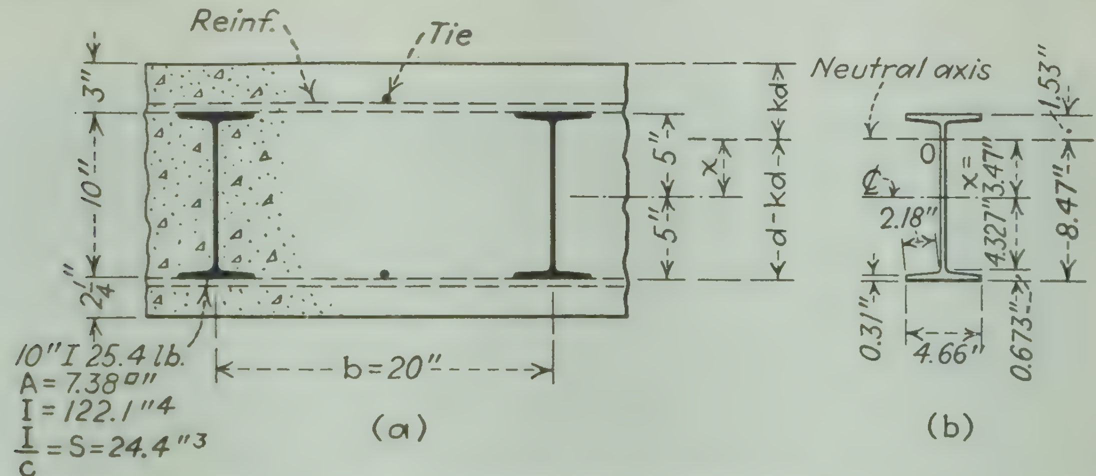


FIG. 5-4. Floor of Lincoln Tunnel, New York City.

sufficient to make the structure act as a composite unit. The sketch shows the floor of the Lincoln Tunnel which is of this type of construction (see Fig. 5-5).

The slab of Fig. 5-4(a) will now be analyzed, using $n = 8$ and f_s, f_c, and u = 20,000, 1,200, and 150 psi, respectively. The tie rods will be neglected.

$$nA_s = 8 \times 7.38 = 59.04$$

Assuming b = 20 in., and taking the static moments about the neutral axis,

$$\frac{20(kd)^2}{2} = 59.04(8 - kd)$$

$$kd = 4.53 \text{ in.} \qquad \text{and} \qquad d - kd = 8.47 \text{ in.}$$

If I equals the moment of inertia of the I beam and x = the distance from its center to the neutral axis of the composite section, then

$$I_c = \frac{b(kd)^3}{3} + nI + nA_s(x)^2$$

$$I_c = \frac{20 \times 4.53^3}{3} + 8 \times 122.1 + 59.04(8 - 4.53)^2 = 2{,}307 \text{ in.}^4$$

$$S_c = \frac{2{,}307}{4.53} = 509 \text{ in.}^3 \qquad S_s = \frac{2{,}307}{8 \times 8.47} = 34 \text{ in.}^3$$

$$M_c = 509 \times 1{,}200 = 611{,}000 \text{ in.-lb for a 20-in. strip}$$

$$M_s = 34 \times 20{,}000 = 680{,}000 \text{ in.-lb for a 20-in. strip}$$

In this case, the concrete determines the safe resisting moment of the slab.

If the I beams are assumed to carry the load by themselves without the help of the concrete, then, using the ordinary flexure formula,

$$M = \frac{sI}{c} = 20{,}000 \times 24.4 = 488{,}000 \text{ in.-lb for a 20-in. strip}$$

The concrete thus enables the slab to develop a resisting moment which is

$$\frac{100(M_c - M)}{M} = \frac{100(611{,}000 - 488{,}000)}{488{,}000} = 25.2 \text{ per cent}$$

greater than for the I beams alone.

The composite action of the type of construction shown here is much more reliable than that of the type illustrated in the previous article.

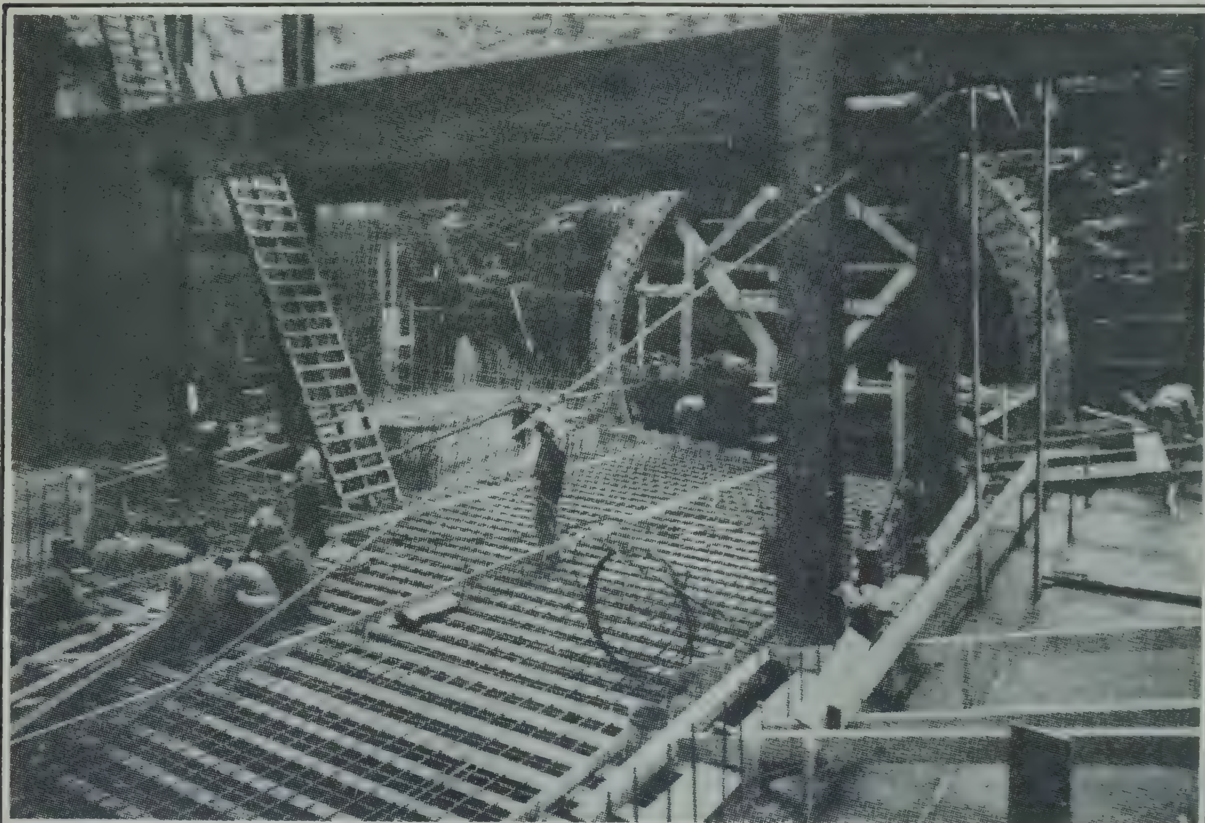


FIG. 5-5. Construction of the roadway slab of the Lincoln Tunnel at the New Jersey shaft.

The horizontal and vertical forces are resisted by both the concrete and the I beams, although the latter will usually be found to be capable of withstanding the entire shearing force if necessary. Also, because of the thorough embedment of the steel, the bond stresses are generally unimportant. They cannot be tested accurately by the formula

$$u = \frac{V}{(\Sigma o)jd}$$

because of the continuity of the webs of the I beams and their participation in the composite action.

When the importance of a structure requires at least an *approximate* analysis of the bond stresses, their maximum value may be found by assuming that all the increments in the steel stresses come through the action of the bond between the concrete and the I beams in the same manner that they do in the case of ordinary reinforcing rods.

Therefore, referring to the beam of Fig. 5-4, if dM represents the increment of the bending moment per linear inch of beam, the increment of the stress in the steel df_s will be

$$df_s = \frac{dM}{S_s} = \frac{dM}{34} \quad \text{for this example}$$

Next, find the section modulus of the I beam alone about an axis at the center of gravity of the composite beam [point O, Fig. 5-4(b)]. For this particular case, it is

$$S = \frac{I}{c} = \frac{I_{CL} + Ax^2}{c} = \frac{122.1 + 7.38 \times 3.47^2}{8.47} = 25 \text{ in.}^3$$

where I_{CL} is the moment of inertia of the I beam about its own center of gravity. Then find the section modulus of the entire surface area of the I beam about the same axis O, using a length of beam of 1 in. This gives

$$S \text{ for surface } = \frac{I'_{CL} + A'x^2}{c} = \frac{547 \text{ (approx)} + 36.59 \times 3.47^2}{8.47} = 117 \text{ in.}^3$$

For $\Sigma M = 0$, the increment of moment of the steel stresses, per unit of length of the beam, about the axis O must equal the moment of the bond stresses, per unit of length of the beam, about this same axis, or

$$df_s \text{ (section modulus of the steel section)} = u\text{(section modulus of surface)}$$

or

$$df_s(25) = u(117) \qquad \text{for this special problem}$$

This can be solved when the maximum increment of moment is known.

A disadvantage of the construction that is shown in Fig. 5-4 is the tendency of the I beams to isolate the concrete into separate units. This may result in the destruction of the bond if the structure carries heavy moving loads and if the beams are relatively flexible so that the lateral distribution of the loads is poor. Furthermore, the wide, flat surfaces of the top flanges of the I beams tend to cause cracks in the concrete above them due to weathering, if the structure is exposed.

5-4. Fabricated trusses in concrete slabs. Members that contain fabricated trusses as reinforcement for the concrete are classed here as composite beams because these trusses are shop-fabricated units which serve the same general purpose as the I beams of the previous article but which are designed to overcome the disadvantages of the latter. Such trusses are made in various types. Some are merely welded reinforcing rods; some are made with chords which are composed of small structural shapes; still others are really expanded I beams. In some cases, these

trusses are made strong enough to act as supports for the forms and the wet concrete.

These trusses, as pictured in Fig. 5-6, have many practical advantages as reinforcement in concrete slabs and beams. Being fabricated units, they are easily handled in the field, especially when the members are continuous and when they require reinforcement for both positive and negative bending moments. The open-web systems eliminate the isolating effect of the webs of the I beams, and they permit the use of effective transverse rods which thoroughly tie the structure together; the web members provide a certain amount of mechanical bond for the longitudinal reinforcement; and the diagonal members serve as an effective—partially, at least—system of web reinforcement.

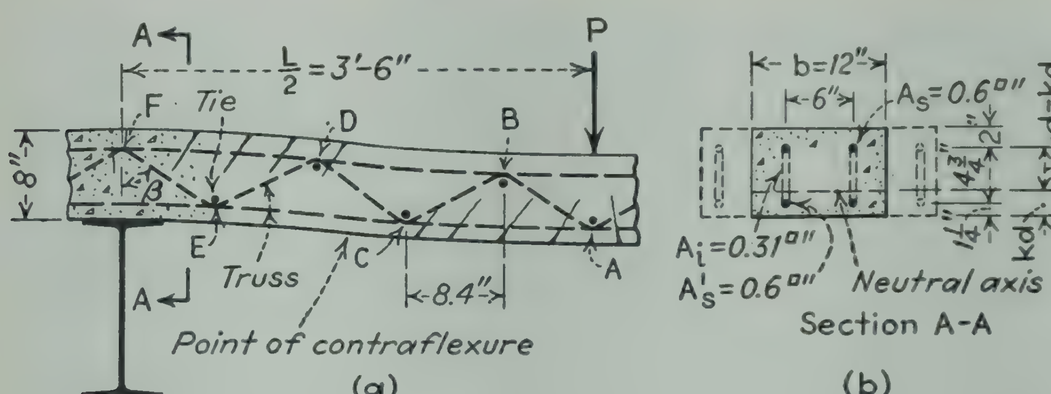


FIG. 5-6.

This last feature may be clarified by an examination of Fig. 5-6(a). The diagonal members AB, CD, and EF function as inclined stirrups. However, BC and DE are not effective as anchors to withstand the longitudinal shearing forces which tend to break open the concrete below B and above E. They have some value as steel compression members which assist directly in carrying the vertical shear, but they should not be relied upon too much. The trouble can usually be remedied by inverting alternate trusses so that the directions of the web members of adjacent trusses will be reversed. By assuming that the trusses function in pairs, the web reinforcement generally will prove to be adequate even though the spacing or panel length exceeds that generally allowed for inclined stirrups—$\frac{3}{4}d$. Naturally, trusses with a double web system (X type) need not be alternated in this way.

Example 5-2. Assume that a strip of the slab of Fig. 5-6 1 ft wide carries a load P equal to 10,000 lb at its center plus its own weight of 100 psf.

For this problem, assume that the bending moment at the support is

$$M = -\frac{PL}{8} - \frac{wL^2}{12}$$

If n equals 8, and the details of the slab and the trusses are as shown in the illustration, find f_s, f_c, u and the stress in the diagonal EF, if alternate trusses are shifted 8.4 in. and

one diagonal rod is assumed to carry the entire stress for the 1-ft strip with no allowance for the strength of the concrete in resisting longitudinal shear ($v'_L = 0$). Also, neglect any action of the truss as an independent member, because it is merely reinforcement in the concrete.

$$M = \left(-\frac{10{,}000 \times 7}{8} - \frac{100 \times 7^2}{12}\right) 12 = 110{,}000 \text{ in.-lb}$$

$$nA_s = 8 \times 0.6 \times 2 = 9.6 \qquad (n-1)A'_s = 7 \times 1.2 = 8.4$$

$$\frac{12(kd)^2}{2} + 8.4(kd - 1.25) = 9.6(6 - kd)$$

$$kd = 2.19 \text{ in.} \qquad \text{and} \qquad d - kd = 3.81 \text{ in.}$$

$$k = 0.36 \qquad \text{and} \qquad j = 0.88(\pm)$$

$$I_c = \frac{12 \times 2.19^3}{3} + 8.4 \times 0.94^2 + 9.6 \times 3.81^2 = 188.8 \text{ in.}^4$$

$$S_c = \frac{188.8}{2.19} = 86 \text{ in.}^3 \qquad \text{and} \qquad S_s = \frac{188.8}{8 \times 3.81} = 6.2 \text{ in.}^3$$

$$f_c = \frac{M}{S_c} = \frac{110{,}000}{86} = 1{,}280 \text{ psi}$$

$$f_s = \frac{M}{S_s} = \frac{110{,}000}{6.2} = 17{,}700 \text{ psi}$$

$$v_L = \frac{V}{bjd} = \frac{5{,}000 + 350}{12 \times 0.88 \times 6} = 84.4 \text{ psi}$$

The tangent of β, the angle of inclination of EF, is

$$\tan \beta = \frac{8.4}{4.75} = 1.77 \qquad \text{or} \qquad \beta = 60°32'$$

Therefore, the stress in this one inclined rod for the 12-in. strip is

$$A_i f_i = \frac{0.7 v_L b s}{\sin (45° + \beta)} = \frac{0.7 \times 84.4 \times 12 \times 8.4}{0.963}$$

or

$$f_i = \frac{0.7 \times 84.4 \times 12 \times 8.4}{0.31 \times 0.963} = 20{,}000 \text{ psi}$$

The intensity of the transverse shear is

$$v_T = \frac{V}{bkd} = \frac{5{,}350}{12 \times 0.36 \times 6} = 206 \text{ psi}$$

The maximum bond stress at the support, which is the point where it is the greatest for the longitudinal reinforcement, is, assuming the rods to be No. 7,

$$u = \frac{v_L b}{\Sigma o} = \frac{84.4 \times 12}{2 \times 2.75} = 184 \text{ psi}$$

This is safe for deformed rods but a little too high for plain bars. Although the chords of the trusses may be smooth, the mechanical bond of the web members where they are rigidly fastened to the longitudinal rods should permit the use of this maximum value.

As a matter of detail, these trusses can be spliced by locating the joint over a support, setting the trusses end to end or side by side. The web

members need not be spliced in such a case, but separate rods can be wired to the chords so as to make up the necessary area and to allow the proper length for bond to develop the splice bars. In other cases, the trusses may be lapped at a floor beam.

Trusses will probably increase in use as their cost becomes less and as engineers become more accustomed to using them in their designs. The

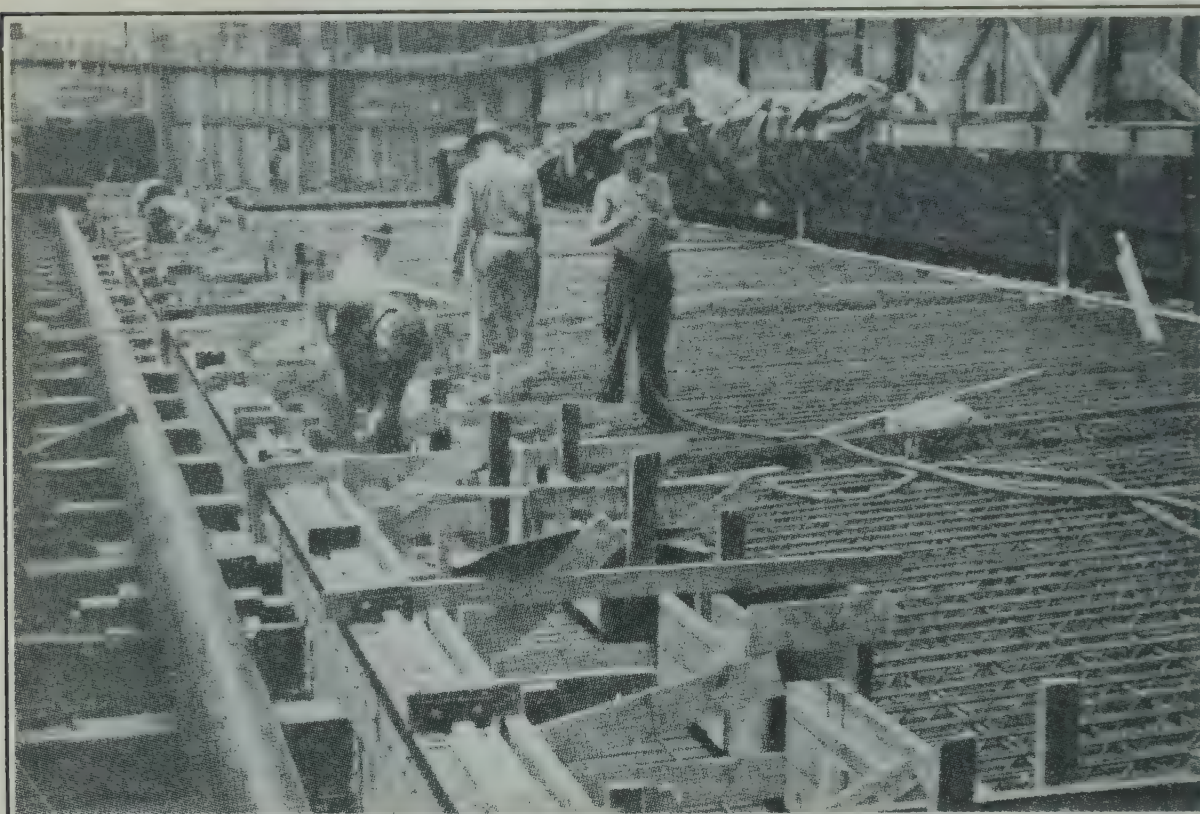


FIG. 5-7. Trussed reinforcement in the roadway of the West Thirty-seventh St. Bridge over the New York approach to the Lincoln Tunnel.

use of structural-steel supporting members in this illustration is taken from bridgework such as that of Fig. 5-7. However, trusses are just as advantageous in the case of similar structures that are made entirely of concrete.

Practice Problems

5-1. Assume that a composite beam similar to Fig. 5-1(a) is to be analyzed. The flange width of the T beam $b = 66$ in.; its thickness $t = 8$ in.; the steel beam is 24 in. deep, weighs 80 lb per ft, the flange width $= 9$ in., $I = 2{,}230$ in.^{4}, $A = 23.54$ in.^{2}, the section modulus $= 185.8$ in.^{3}; the width of the stem (encasement) $= 13$ in.; and the top of the beam is 3 in. below the top of the concrete slab. Find the section moduli S_c and S_s of the composite beam if $n = 10$. Find the safe resisting moment if the allowable $f_c = 900$ psi and $f_s = 18{,}000$ psi.

Ans. $S_c = 6{,}060$ in.^{3}; $S_s = 228$ in.^{3}; $M_s = 342{,}000$ ft-lb.

5-2. Determine the critical longitudinal shearing stress along the top of the beam of Prob. 5-2 if it is simply supported, has a span of 28 ft, and carries a total uniformly distributed load of 3,000 plf, including the dead load.

Discussion. Find the end reaction, the static moment of the concrete outside the probable planes of failure (similar to Fig. 5-1(b), the longitudinal shear per inch of length ($S_L = VQ/I_c$), and the longitudinal shearing stress per square inch of surface.

5-3. Assume that the composite beam of Prob. 5-1 has a span of 27 ft and that it is simply supported. Assume that the forms are supported from the steelwork so that the dead load is carried by the steel beam alone. If the allowable f_s in the steel is 18,000 psi, calculate the uniformly distributed live load per linear foot that the composite beam will support.

Discussion. Find f_s for the dead-load bending moment for the steel beam alone; subtract it from 18,000 psi; use the difference times S_s to find the available resisting moment for live loads; and then compute the uniform load that will cause this moment.

Ans. L.L. = 2,630 plf.

5-4. Recompute Prob. 5-1 if $b = 75$ in., $t = 8$ in., and $n = 12$.

5-5. Compute the maximum longitudinal shearing stress at the top of the beam of Prob. 5-4 if the beam is simply supported, has a span of 25 ft, and carries a total uniformly distributed load of 3,600 plf.

5-6. Recompute the composite beam of Fig. 5-4 as in Art. 5-3, using exactly the same steel beams but a weaker concrete for which $n = 12$, $f_c = 900$ psi, and $f_s = 18{,}000$ psi. Find the safe resisting moment.

5-7. Determine the safe negative bending moment for a floor slab that is reinforced as shown in Fig. 5-6, if $n = 12$. Assume that the allowable $f_c = 900$ psi and $f_s = 18{,}000$ psi.

Discussion. Use the same dimensions, steel, etc., as in Fig. 5-6. Follow the procedure of Example 5-2.

5-8. Find the safe positive bending moment for a floor slab like that of Fig. 5-6 if the total depth is 9 in. and the distance center to center of truss chords = 5.75 in. Otherwise, use all the dimensions and the data that are given in the pictures. Let $n = 10$, the allowable $f_c = 1{,}000$ psi, and $f_s = 20{,}000$ psi.

Ans. $M_c = 135{,}000$ in.-lb; $M_s = 159{,}000$ in.-lb.

5-9. A composite beam similar to that of Fig. 5-3(a) is composed of a 24*WF*76 beam, a bottom cover plate $12 \times \frac{3}{4}$, and a deck slab 8 in. thick and 78 in. wide. The span is 45 ft, and the bridge is simply supported. Compute the safe uniform live load per square foot if the dead load is held by the steel alone and the live load is supported by the composite section. The allowable stress in the steel is 20,000 psi. Let $n = 8$.

5-10. Recompute Prob. 5-9 with a slab 8 in. thick and 72 in. wide, and $n = 10$.

6

COLUMNS

6-1. Introduction. Members carrying direct axial loads which cause compressive stresses of such magnitude that these stresses largely control the design of the members may be included in the general classification called *columns*. They may also be divided into two types, *viz.*, "short" columns, the lengths of which are less than ten times their least lateral dimension; and "long" columns, the relative lengths of which exceed this limit. Columns may also be subjected to bending moments as well as to axial loads. Therefore, the foregoing definition is given as a general one to differentiate between a column that resists bending and a beam that carries a direct compressive load.

Because of the nature of the material, concrete columns are generally of the short-column type. Longitudinal steel rods are usually added to assist in carrying the direct loads; also, hoops and spirals serve the same general purpose; and sometimes structural-steel sections are considered to be a sort of glorified reinforcement.

Columns need not be vertical, but, to avoid confusion, it will be advisable to consider them so, using the term "strut" to describe inclined or horizontal compression members.

6-2. General discussion of reinforced-concrete columns. A simple square concrete column with eight vertical reinforcing rods is pictured in Fig. 6-1(*a*). Under the action of the direct compression, the concrete bulges out laterally, as shown in exaggerated manner in Fig. 6-1(*b*). From a consideration of Poisson's ratio, this is to be expected when a material is placed under compression. It is also obvious that the rods themselves are somewhat like very slender columns. Naturally, they tend to buckle, but they cannot bend inward against the concrete. Therefore, they will buckle in the line of least resistance, *viz.*, away from the column's axis. This action causes tension in the outside shell of the concrete which, if the pressure becomes sufficient, will crack open somewhat as shown, the failure usually being sudden.

The way to overcome this trouble seems to be obvious. If the column is a square one, as pictured in Fig. 6-1(*c*), the apparent remedy is the

placing of small rods around the longitudinal reinforcement, forming a series of bands or ties which are wired to the main rods and which are supposed to keep the latter from buckling, as well as to restrain the bulging action of the concrete. However, if the upper view of this figure

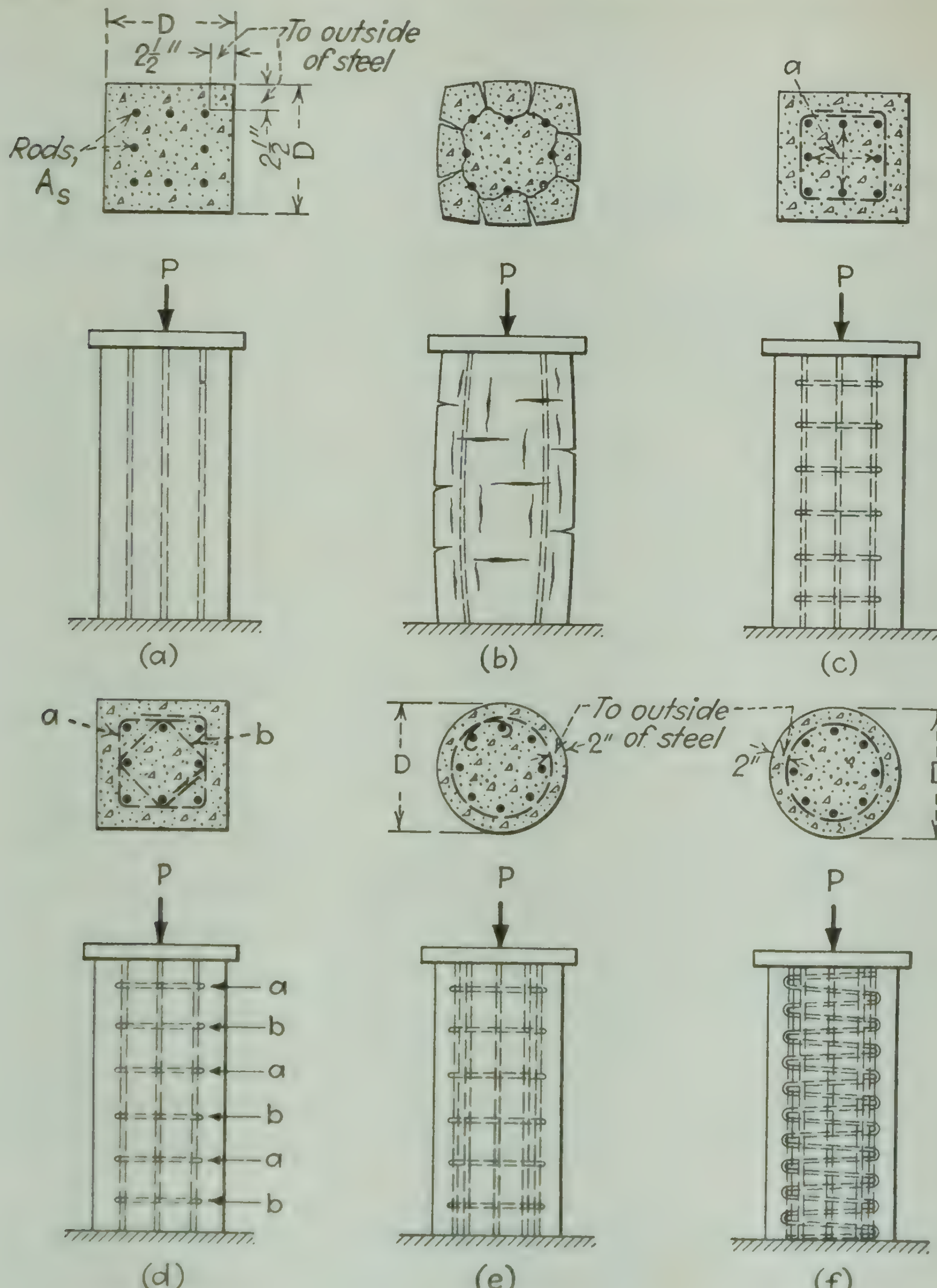


FIG. 6-1.

is examined carefully, it is seen that these bands are in the form of hollow squares. Therefore, when rods *a* try to bend sideways, they exert a lateral force which is normal to the straight sides of the bands. The bulging of the concrete does likewise. However, the bands are not effective in withstanding such beam action without bending outward so much that the concrete may crack. It is therefore advisable to use two kinds

of ties, placing them alternately so that one holds the corner rods while the other supports the intermediate ones, as shown in Fig. 6-1(*d*). This arrangement results in troublesome details, and it handicaps the "rodding," or compacting, of the concrete.

The next logical improvement seems to be the placing of the longitudinal rods in a circle, with hoops placed outside and wired to them as pictured in Fig. 6-1(*e*). The buckling tendency of the rods and the bulging of the inner portion of the concrete merely cause tensile stresses in these hoops, which means that they are really effective in restraining this action. However, the single hoops have to be spliced by lapping or by bending their ends around some of the main rods. This again is troublesome when a large number of hoops must be used.

The best way to support the longitudinal rods and the concrete is by means of spiral reinforcement as illustrated in Fig. 6-1(*f*). These spirals are merely long rods of small diameter which are bent around the main rods, forming a helix. In this way, small pieces are eliminated, the field work is decreased, and the waste of material in splices is avoided.

The arrangement of the main rods in a circular pattern with spiral reinforcement (or hoops) is advisable even when the cross section of the finished column must be square rather than round. Of course, the protective coating over the rods is needed to guard the steel against fire and corrosion, but the exterior surface can be shaped to suit the architectural requirements as long as a minimum cover of $1\frac{1}{2}$ in. or $1\frac{1}{2}$ times the maximum size of aggregate is maintained over the outer surfaces of the rods.

It was the general practice in the past to neglect the strength of the concrete covering that is outside the hoops or spirals—the 2-in. layer shown in Figs. 6-1(*e*) and (*f*). This was done because this covering was assumed to be for protection against possible fire and because, in that event, it might spall off. Also, this covering is outside the portion that can be restrained by the hoops or spiral reinforcement. The present tendency, however, seems to be to assume that the entire section of the concrete will participate in resisting the loads, or at least part of it can be relied upon.

This assumption seems to be entirely logical because all the material must be shortened when the member is compressed. Then all of it must also resist this deformation, especially within the range of ordinary working loads. There are differences of opinion regarding how much concrete outside of the hoops or spirals may be relied upon in design. Obviously, some practical limits must be set up. Since the Code states that, for columns which are poured monolithically with walls or piers the effective cross section shall be assumed as that within $1\frac{1}{2}$ in. of the outsides of the spirals, this definition will be adopted here, even though the thickness of the cover outside the spirals, hoops, or ties may exceed $1\frac{1}{2}$ in.

The arrangement of the reinforcement divides columns into two general classes, *viz.*, "tied" columns which have longitudinal rods with intermittent hoops or ties; and "spirally reinforced" columns, which have longitudinal rods that are enclosed within steel spirals. The advisability of using rods of large diameter which will be stiff and strong as "little" columns is self-evident.

Some of the requirements of the Code that are applicable to columns are the following:

1. Principal columns in buildings shall have a minimum diameter of 12 in. when circular, or a minimum thickness of 10 in. and a gross area of at least 120 in.^{2} when rectangular.

2. The unsupported length of a column shall be the clear distance between the floor at the bottom and the lower extremity of the following at the top:

a. Capital, drop panel, or slab, whichever is least, for flat-slab construction.

b. Underside of the deeper beam framing into the column in each direction.

c. The clear distance between consecutive struts framing into the column in each vertical plane, provided that two such struts shall meet the column at approximately the same level and provided that vertical planes through the struts do not vary more than 15° from a right angle. Of course, these struts are to be adequate and well anchored to prevent lateral deflection of the column.

d. When beams or struts have brackets equal to their width and at least one-half as wide as the column, the clear distance is to be measured from the floor to the bottom of the bracket.

3. The length of rectangular columns shall be considered as that which produces the greatest ratio of length to depth of column.

4. Lapped splices of column reinforcement are to be at least the following:

a. Twenty diameters for deformed bars when $f'_c = 3{,}000$ psi or more and the rods are intermediate- or hard-grade steel. For bars with higher yield point, add one diameter for each 1,000 psi that the allowable stress exceeds 20,000 psi. For weaker concretes, increase the lap by one-third that previously specified.

b. The minimum lap for plain bars shall be twice that specified for deformed bars.

c. When successive columns differ in size so that the longitudinal bars are offset at a splice, the outer bars shall be sloped at an inclination not exceeding 1:6 with respect to the axis of the column, and the portions of the bars above and below the offset shall be parallel to the axis of the column. Metal ties, spirals, or the floor construction itself shall provide

adequate horizontal support at the bend points. The ties and spirals should never be more than eight bar diameters from these bend points. For design, assume that the horizontal thrust at a bend point is 1½ times the horizontal component of the nominal stress in the inclined portion of the bar. Such offset bars shall be bent before placing, not when in the forms or when partially embedded in concrete.

5. Welded butt splices may be used for longitudinal bars if the splices will develop in tension at least the yield-point stress of the bars. If any bars exceed 1½ in. in diameter, the splices shall preferably be welded.

6. The c.c. spacing of longitudinal bars shall not be less than 2½ times the diameter of round bars, three times the side dimension of square bars, 1½ in., or 1½ times the maximum size of the coarse aggregate used. These spacings also apply to adjacent pairs of bars at a lapped splice, although two bars that are spliced by lapping may be in contact.[1]

At the bottoms of reinforced-concrete columns the depth of the footing, or of the footing plus a pedestal on it, should be sufficient to enable dowels to develop the compressive stress in the column reinforcement through bond if the concrete at the bottom of the column is not to be overloaded. This required length of embedment of dowels is ordinarily about one-half of that required to develop the tensile strength of the bars. A pedestal should be large enough for the concrete to resist the column load without the aid of reinforcement like that in the column.

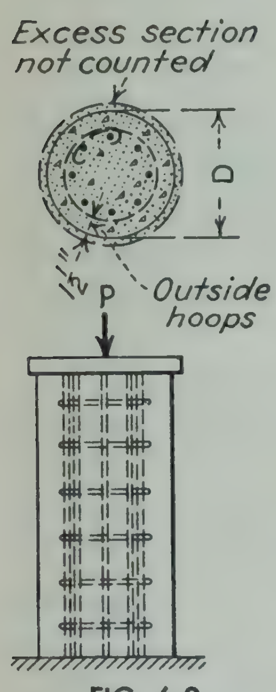


FIG. 6-2.

6-3. Design of tied columns. The design of short reinforced-concrete columns may be based upon the theory of elasticity of the materials. If the load P is applied to the column of Fig. 6-1(a), it causes a corresponding shortening of the member. Both the concrete and the steel are squeezed down equally. If the stresses in them are below the elastic limit of the materials, the relation between the values of the unit stresses in the steel and in the concrete f_s and f_c would appear to be

$$f_s = \frac{E_s f_c}{E_c} = nf_c$$

Therefore, the total resistance provided by the column would seem to be the area of the concrete times f_c plus the area of the steel times f_s.

As an example, refer to Fig. 6-2. Let A_g equal the gross effective area of the column in square inches and p_g equal the ratio of the total area of

[1] See Table 11 of the Appendix.

the longitudinal rods to this gross effective area. Then the area of the steel is

$$A_s = p_g A_g \tag{6-1}$$

The net area of the concrete $= A_g - A_s = A_g - p_g A_g = A_g(1 - p_g)$. Therefore,

$$\begin{aligned} P &= A_s f_s + (A_g - A_s) f_c = p_g A_g f_s + A_g(1 - p_g) f_c \\ P &= p_g A_g n f_c + A_g(1 - p_g) f_c \\ P &= A_g f_c [1 + (n - 1) p_g] \end{aligned} \tag{6-2}$$

which gives the load in terms of the transformed section and of the stress in the concrete. This formula is based upon the elastic theory. In reality, however, it is not in accord with recent practice.

Experience has shown that the action of a reinforced-concrete column under load seems to be rather peculiar. At first, the conditions are practically those given by Eq. (6-2); but if the load is large and if it is continued for a long time, plastic flow of the concrete (a tendency of the concrete to get out from under the load when the latter is continued for a long period) seems to take place, resulting in a decrease of the unit stress in the concrete but an increase of the stress in the steel because of the latter's inability to "get out from under" the load also. Thus more of the burden of supporting the load tends to be shifted from the concrete into the rods. This action may continue until the stress in the steel reaches the yield point,[1] whereupon the rods deform appreciably without taking much further increase in stress, even though they cannot get rid of the stress already in them. Then, at this stage, the shifting of the load from the concrete to the steel must practically cease because the latter is so much more compressible than the former. If the load on the column is then increased, the steel will continue to have a stress which is at or a little above the yield point, but the concrete, by itself, must carry the increase of the load until it fails.

According to these principles, a formula for the ultimate strength of a reinforced-concrete column might be

$$\begin{aligned} P' &= (A_g - A_s) f'_c + A_s f'_s \\ P' &= A_g[(1 - p_g) f'_c + p_g f'_s] \end{aligned} \tag{6-3}$$

In Eq. (6-3), P' = the ultimate load on the column and f'_s = the stress in the steel when at its yield point. Applying a suitable safety factor to these formulas will give a formula for the safe load P of a tied column. However, according to the Code, the formulas for P, given in two forms, are

[1] Unless it is relieved by "bond creep" at the splices; see J. R. Shank, Bond Creep and Shrinkage Effects in Reinforced Concrete, *J. ACI*, November, 1938.

$$
\begin{aligned}
P &= 0.18f'_cA_g + 0.8A_sf_s \\
P &= A_g(0.18f'_c + 0.8f_sp_g)
\end{aligned}
\tag{6-4}
$$

where $f_s = 16{,}000$ psi for intermediate-grade steel and 20,000 psi for hard-grade and rail steel. Notice that the area of the steel A_s should be deducted from A_g according to Eq. (6-3), but this would have a minor effect. It is evidently considered in determining the coefficients of f'_c and f_s in Eq. (6-4), which are empirical but based upon tests, experience, and judgment.

Equation (6-4) will be adopted for the computation of the safe load of any tied column. However, it merely sets a maximum safe load for a column that has a given size and one specific make-up. It does not give the unit stresses in the steel and concrete when the column is not fully loaded. On the other hand, as long as the column is admittedly safe, the lesser unit stresses in it for partial loading are of little interest or importance.

The first form of Eq. (6-4) is convenient when a given column is to be analyzed—when A_g and A_s are known. The second form is more convenient when one is trying to determine the size of a tied column to hold a specified load.

The ratio of the longitudinal reinforcement p_g for a tied column should not be less than 0.01 or more than 0.04. At least four bars should be used, and the minimum size specified in the Code is No. 5. This is because the rods should be stiff enough for proper action and handling.

Of course, the ties in any column should be adequate to brace the rods. According to the Code, such ties must be at least ¼ in. in diameter. They must not be spaced over sixteen times the diameter of the longitudinal rods, forty-eight times the diameter of the ties, or the least dimension of the column. When there are more than four longitudinal bars, additional ties should be provided so that every one of the longitudinal bars is held firmly in its proper position and has a lateral support that is equivalent to that provided by a 90° corner of a tie. In very large columns that are heavily reinforced it is not always practicable to meet this requirement completely.

The size of bars to be used as ties is a matter to be settled by good judgment. If they are too small, they will bend out of shape very easily. If they are too large, they will be difficult to bend, they may be wasteful of steel, and the bends will not be so sharp as desired. A suggested rule for their size is the following:

1. One-fourth inch for small members where the ties are to be bent by hand in the field.
2. No. 3 (⅜ in.) for columns having a least width of 10 to 20 in.
3. No. 4 (½ in.) for columns having a least width of 20 to 30 in.

4. No. 5 ($\frac{5}{8}$ in.) or larger for columns of larger size, and for piers where workmen may climb on the ties.

If, for architectural or other reasons, a tied column is made larger than required for loading, a reduced effective area A_g as needed but not less than one-half of the total area may be used when computing the required area of longitudinal reinforcement.

The steps in the design of a tied column are as follows: (1) the determination of the quality of the concrete to be used and the allowable stress in the steel; (2) the assumption of a size and shape; (3) the assumption of p_g (or the number and size of rods); and, finally, (4) the test of the safe load for the column by the use of Eq. (6-4).

Example 6-1. Find the safe load for the short tied column shown in Fig. 6-3(a). Assume $f'_c = 2{,}500$ psi and $f_s = 16{,}000$ psi.

Use the first form of Eq. (6-4). Since A_g is assumed to include only $1\frac{1}{2}$ in. of concrete outside of the ties whereas the actual cover is 2 in., the size of the column will be the equivalent of 15 in. square. Then

$$P = 0.18 \times 2{,}500 \times 15^2 + 0.8 \times 6.32 \times 16{,}000$$
$$P = 101{,}000 + 81{,}000 = 182{,}000 \text{ lb}$$

The corners of the column in Fig. 6-3(a) are shown with chamfers in order to avoid sharp edges that may not be filled properly and that may chip off easily. Any reduction of area that such chamfers may have on A_g is customarily neglected.

The Code specifications for No. 3 ($\frac{3}{8}$-in.) ties will now be applied. Thus

$16 \times 1 = 16$ in. based upon the longitudinal rods
$48 \times \frac{3}{8} = 18$ in. based upon the size of ties used
16 in. = min least actual dimension (not that for computing A_g)

Therefore, the 16-in. spacing is satisfactory.

Notice how the ties in Fig. 6-3(a) are arranged. One is square, with the ends hooked around the corner bar at a. A second tie is also square and bent around the middle rod at b. These No. 3 rods are too strong to be bent conveniently by hand in the field. However, when bent in advance, they are difficult to place. They generally have to be slid down the longitudinal bars or stacked at the bottom and pulled up after inserting the vertical reinforcement in the loops and corners. Both ties are placed in contact and wired to the main bars, and the two together constitute what is called one tie. They can also be alternated at one-half the spacing, as shown in Fig. 6-1(d).

A different arrangement of ties is pictured in Fig. 6-3(b). Two U-shaped bars are placed across the main reinforcement from the sides, lapped vertically, and wired in place. Two other U-shaped bars with bent ends about 3 or 4 in. long are laid across the center with the bent ends horizontal and wired on top of the other ties, as shown by c, or with the bent ends vertical and crossing over the other ties, as pictured by d.

As many crossbars are used as seems to be necessary. However, it is easy to see that it is too extreme to have every vertical bar tied directly when the columns are large and have many bars at 4 or 5 in. c.c. It is suggested that, for such cases, the crossties may be spaced so as to form rectangular openings 6 or 8 in. in size so that an "elephant's trunk" (a pipe or hose on the bottom of a movable hopper) may be inserted for depositing the concrete.

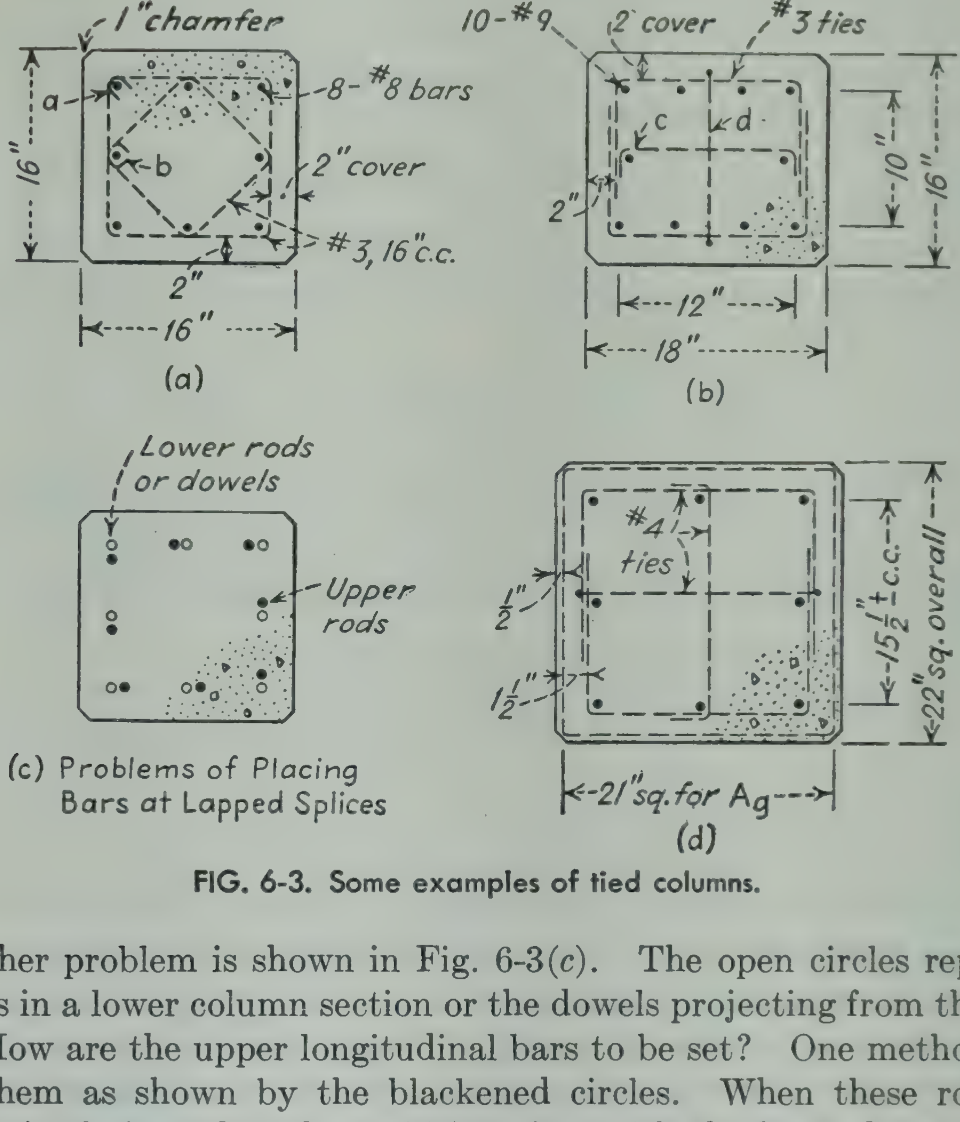


FIG. 6-3. Some examples of tied columns.

Another problem is shown in Fig. 6-3(*c*). The open circles represent the bars in a lower column section or the dowels projecting from the footing. How are the upper longitudinal bars to be set? One method is to place them as shown by the blackened circles. When these rods are large, it is obvious that they will interfere with the hooked ties of Fig. 6-3(*a*). The corner rods will not really fit in the corners unless they are pulled over as soon as possible, which is what usually happens in the field.

Example 6-2. Compute the safe load for a short tied column having the cross section pictured in Fig. 6-3(*b*). Determine the spacing of ties if they are of the size and type shown on the drawing. Assume $f'_c = 3{,}500$ psi, and $f_s = 16{,}000$ psi.

Assuming the bars to be $1\frac{1}{8}$ in. in diameter, the approximate size of the effective area A_g is $1\frac{1}{8} + 2 \times \frac{3}{8} + 2 \times 1\frac{1}{2} = 4\frac{7}{8}$ in. larger than the c.c. spacing of corner bars. Call it 5 in. larger. Then $A_g = 15 \times 17$ in.². Therefore, from Eq. (6-4),

$$P = 0.18 \times 3{,}500 \times 15 \times 17 + 0.8 \times 10 \times 1 \times 16{,}000$$
$$P = 161{,}000 + 128{,}000 = 289{,}000 \text{ lb}$$

From the specified rules, the ties should be spaced

$$16 \times 1\tfrac{1}{8} = 18 \text{ in.}$$
$$48 \times \tfrac{3}{8} = 18 \text{ in.}$$
$$\text{or } 16 \text{ in.}$$

Therefore, the 16-in. spacing based on the width of the column controls.

Example 6-3. Design a square short tied column to support a load of 400 kips, using 3,000-lb concrete and $f_s = 16{,}000$ psi.

One way to solve such a problem is to guess a size and test it. Another way is to assume a percentage or ratio of reinforcement p_g somewhere between the prescribed ratio of 0.01 and 0.04, then use this in the second form of Eq. (6-4) and solve for A_g. Unless the columns are to be made as small as it is practicable to have them, a value of $p_g = 0.02$ to 0.03 is generally satisfactory.

Using $p_g = 0.03$ in this case,

$$P = A_g(0.18f'_c + 0.8f_sp_g)$$
$$400{,}000 = A_g(0.18 \times 3{,}000 + 0.8 \times 16{,}000 \times 0.03)$$
$$A_g = 433 \text{ in.}^2$$

If the column is 21 in. square, then $A_g = 441$ in.^{2}, which is satisfactory. Then

$$A_s = 0.03A_g = 0.03 \times 433 = 13 \text{ in.}^2$$

Preferably, the number of rods used should be in multiples of four in order to have a symmetrical arrangement. Therefore, from Table 3 of the Appendix, the following can be assumed to be close enough for practical use: 20 No. 7 bars = 12.0 in.^{2}; 16 No. 8 bars = 12.6 in.^{2}; or 8 No. 11 bars = 12.5 in.^{2}. Before choosing one of these, look at Fig. 6-3*(d)*. With a cover of $1\tfrac{1}{2}$ in., the size of the core (out to out of ties) will be 18 in. Next, look at Table 11 in the Appendix. For square tied columns with a core dimension of 18 in., it is found that any one of these three arrangements can be used as far as spacing is concerned. The choice will depend upon which size is available, which size is to be used in all columns if so ordered, and which scheme will make the most practicable construction. The eight No. 11 bars will provide the most simple arrangement for the ties, which are pictured as No. 4 rods in Fig. 6-3(*d*). The cover is increased to 2 in. for fire protection, thus making the over-all size of the column 22 in. square.

The No. 4 ties can be used at the following spacing: $16 \times 1\tfrac{7}{16} = 23$ in.; $48 \times \tfrac{1}{2} = 24$ in.; or minimum width = 22 in. They will be spaced at 20 in. c.c. in order to be a bit conservative.

Figure 13.4 in the Appendix has been prepared to enable one to check or estimate column sizes quickly. To check the safe load for this column, use the central graph for 3,000-lb concrete. From the top, for a side of a square equal to 21 in., project down to the diagonal line for $p_g = 0.03$, then project to the left. P is found to be slightly above the 400 kips required. In order to obtain a size for this column use the central graph for 3,000-lb concrete and a load of 400 kips, project horizontally to the diagonal line for $p_g = 0.03$, then project to the top where the side dimension of a square column is found to be approximately 21 in.

6-4. Design of spirally reinforced columns. When a column has reinforcement of the type that is pictured in Fig. 6-4(*a*), it is said to be

"spirally reinforced." As previously stated, the concrete in such a member is supported more adequately than it is in the case of a tied column. Therefore, although the fundamental action of the materials is the same as that which was described in the preceding article, the concrete of a spirally reinforced column will have a much greater ultimate resistance to failure than that of the tied one. Its safe working load can be larger also.

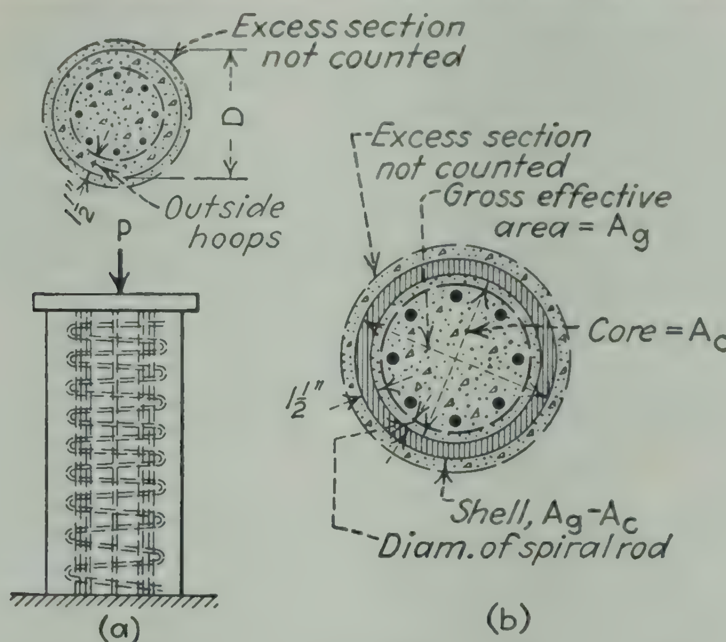


FIG. 6-4.

The formula given by the Code for the safe load of spirally reinforced columns is

$$\begin{aligned} P &= 0.225f'_cA_g + A_sf_s \\ P &= A_g(0.225f'_c + p_gf_s) \end{aligned} \tag{6-5}$$

where all symbols have the same meaning and value as for Eq. (6-4) and the limiting safe loads will be 25 per cent larger than for tied columns having similar sections. Figure 13*B* in the Appendix is useful in the practical design and analysis of spirally reinforced columns.

The Code specifies that p_g shall be not less than 0.01 or more than 0.08. The minimum number of bars shall be six; the minimum size No. 5. Generally, when p_g exceeds about 0.05 or 0.06 the sizes and numbers of bars become unduly great for ordinary columns. It then tends to make what looks like a cage of steel with concrete filling.

Of course, the increase of the safe load of the column as given in Eq. (6-5) over that in Eq. (6-4) is due to the additional strength of the concrete because of the supporting power of the spiral reinforcement. Therefore, the latter must be subjected to considerable stress. An expression for the design of such spirals, as given by the Code, is

$$p' = 0.45(R - 1)\frac{f'_c}{f'_s} \tag{6-6}$$

where p' = volume of the spiral ÷ volume of the concrete core (out to out of spiral); f_s' = "useful limit stress" = 40,000 psi for hot-rolled rods of intermediate grade, 50,000 psi for hard grade, and 60,000 psi for cold-drawn wire; and R = gross area of the effective section ÷ area of the core = A_g/A_c. Equation (6-6) will be adopted here. An approximate derivation of it might be made, but it is largely empirical.

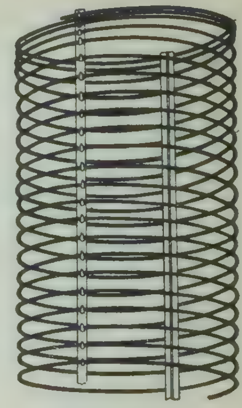


FIG. 6-5.

The spirals should be made continuous and with even spacing except for at least 1½ extra turns at the ends. They should be held in place firmly and kept in line by vertical spacers, as pictured in Fig. 6-5. At least two spacers should be used for spirals 20 in. or less in diameter, three spacers for sizes from 20 to 30 in., and four spacers for larger sizes and for spiral rods ⅝ in. or larger in size. The spirals are generally prefabricated. They should therefore be stiff and of a size and length that can be handled satisfactorily in the field.

The minimum size of spiral rod should be ¼ in. for rolled bars or No. 4 W. & M. gage for drawn wire. Splicing of them shall be by welding or by lapping 1½ turns.

The spacing of spiral rods shall not exceed one-sixth of the core diameter or 3 in. c.c. The minimum spacing specified by the Code is 1⅜ in. or 1½ times the maximum size of coarse aggregate used.

The spiral shall extend from the floor level in any story or the top of the footing or pedestal at the bottom to the lowest horizontal reinforcement in slab, drop panel, or beam at the top. If a conical capital is used, the top of the spiral shall extend at least to a level where the diameter of the capital is twice that of the column.

Still other Code requirements are the following:

1. If two or more interlocking spirals are used in an isolated column, the effective section shall be assumed to be a rectangle the sides of which are the out-to-out dimensions of the spirals plus 1½ in. of cover or 1½ times the maximum size of the coarse aggregate.

2. If a spirally reinforced column is built monolithically with a wall, the outer boundary of the section may be taken as either a circle 1½ in. outside the spiral or as a square or rectangle having its sides 1½ in. outside the spiral or spirals.

3. When a spiral column is built with a square or other shaped section, with the same least lateral dimensions, for architectural or other reasons, the effective gross area shall be assumed to be that of the circular column.

Figure 14 in the Appendix has been prepared to give a graphical solution for the spiral rods of particular sizes in accordance with Eq. (6-6). Furthermore, Table 9 of the Appendix shows one size and spacing of

spiral for various sizes of columns and strengths of concrete. These two are generally sufficient for checking and designing spirals.

Reinforced-concrete columns are made frequently with light longitudinal rods but with strong spiral reinforcement. Within reasonable limits, this is good construction, provided enough longitudinal rods are used to hold the spiral sufficiently to prevent it from collapsing downward during the placing of the concrete, and provided that the column is not subjected to severe bending moments.

Example 6-4. By means of Eqs. (6-5) and (6-6), design a short circular column to support a centrally applied load of 300,000 lb, including its own weight, using f'_c, f_s, and f'_s equal to 3,000, 16,000, and 40,000 psi, respectively.

For the purposes of illustration, design this column first with $p_g = 0.06$ and then with $p_g = 0.015$.

First Solution ($p_g = 0.06$).

Let D = the outside diameter of the column, and use the second form of Eq. (6-5).

$$P = A_g(0.225f'_c + p_g f_s)$$
$$300{,}000 = \frac{\pi D^2}{4}(0.225 \times 3{,}000 + 0.06 \times 16{,}000)$$

$D = 15.3$ in. (Say 16 in. in order to have column dimensions in full inches.)

$$A_g = \frac{\pi \times 16^2}{4} = 201 \text{ in.}^2$$
$$A_s = p_g A_g = 0.06 \times 201 = 12.1 \text{ in.}^2$$

Let d_1 = diameter of the spiral $= D - 3 = 13$ in. Then the core A_c is

$$A_c = \frac{\pi d_1^2}{4} = \frac{\pi \times 13^2}{4} = 133 \text{ in.}^2$$

Allowing $1\frac{1}{2}$ in. for the thicknesses of spirals and main reinforcement, the circumference of the circle in which the longitudinal rods will be set inside the spiral $= \pi(d_1 - 1.5) = 3.14(13 - 1.5) = 36$ in. With 10 rods, the spacing will be $36/10 = 3.6$ in. The area of one rod must be $A_s/10 = 12.1/10 = 1.21$ in.². This requires No. 10 bars, giving $A_s = 12.7$ in.². As the reader will realize, this reinforcing is very heavy, and the splicing of the rods will be difficult.

From Table 11 in the Appendix, with a core diameter of 13 in., only seven No. 10 bars are recommended. Six No. 11 bars, as also recommended, provide only 9.36 in.² of steel. Then the desired minimum spacing will have to be violated or the column made a bit larger.

The Code requirement of $2\frac{1}{2}$ times the bar diameter gives $2\frac{1}{2} \times 1\frac{1}{4} = 3\frac{1}{8}$ in. compared with the 3.6 in. available. If the bars are lapped alongside, somewhat as pictured in Fig. 6-3(c), the addition of one more bar diameter gives a required spacing of $3\frac{1}{8} + 1\frac{1}{4} = 4\frac{3}{8}$ in. This is not available. However, the lower rods can be pulled inward at the top so that the bars lap in a radial pattern. However, with a reduced bar circle having a diameter of $13 - 1\frac{1}{2} - 2 \times 1\frac{1}{4} = 9$ in., the average spacing of 10 bars is about 2.8 in. compared with the required minimum of $3\frac{1}{8}$ in. This meets neither the "letter of the law nor its spirit." However, it does show some of the troubles encountered when p_g is too large. Therefore, redesign the column with $p_g = 0.04$.

$$300{,}000 = \frac{\pi D^2}{4}(0.225 \times 3{,}000 + 0.04 \times 16{,}000)$$
$$D = 17 \text{ in.}$$
$$A_g = \frac{\pi \times 17^2}{4} = 227 \text{ in.}^2$$
$$A_s = 0.04 \times 227 = 9.1 \text{ in.}^2$$
$$d_1 \text{ for spiral} = 17 - 3 = 14 \text{ in.}$$
$$A_c = \frac{\pi \times 14^2}{4} = 154 \text{ in.}^2$$

From Table 11 of the Appendix, eight No. 10 bars can be used in a circular column with a core of 14 in. These provide 10.2 in.^{2}. The table also shows the nine No. 9 bars, having an area of 9 in.^{2}, can also be used. This is close enough to the 9.1 in.^{2} required and will be adopted.

Now continue with the assumption of a 14-in. core, and design the spiral. From Eq. (6-6),

$$p' = 0.45(R - 1)\frac{f'_c}{f'_s}$$
$$p' = 0.45\left(\frac{227}{154} - 1\right)\frac{3{,}000}{40{,}000} = 0.016$$

The required volume of the spiral in 1 ft of column $= 12A_cp'$ in.^{3}.

$$12 \times 154 \times 0.016 = 29.6 \text{ in.}^3$$

Assuming a pitch of 1¾ in., the length of spiral in 1 ft of column is

$$\frac{\pi \times 14 \times 12}{1.75} = 300 \text{ in.}$$

The required cross section of the spiral rod $= 29.6/300 = 0.099$ in.^{2}. Use No. 3 rods with about two extra turns at each end.

These computations for the sizes of spirals seldom need to be made in practice. Using Fig. 14 of the Appendix with $p' = 0.016$, a core diameter of 14 in., and No. 3 rods, a satisfactory spacing is found to lie between the diagonal lines for 2- and 1¾-in. spacing of No. 3 bars. The smaller will be adopted, and it checks the preceding calculations. Furthermore, Table 9 of the Appendix, using 3,000-lb concrete and a core diameter of 14 in., shows the same answer.

Now check the two preceding solutions by means of the central diagram of Fig. 13*B* of the Appendix. With $p_g = 0.04$ and $P = 300$ kips, the diameter is found to be approximately 17 in. This is the same answer.

Second Solution ($p_g = 0.015$).

$$300{,}000 = \frac{\pi D^2}{4}(0.225 \times 3{,}000 + 0.015 \times 16{,}000)$$
$$D = 20.4 \text{ in.} \qquad \text{Use 21 in.}$$
$$A_g = \frac{\pi \times 21^2}{4} = 346 \text{ in.}^2$$
$$A_s = 346 \times 0.015 = 5.2 \text{ in.}^2$$
$$d_1 = 21 - 3 = 18 \text{ in.} \qquad \text{and} \qquad A_c = \frac{\pi \times 18^2}{4} = 254 \text{ in.}^2$$
$$\pi(d_1 - 1.5) = \pi(18 - 1.5) = 52 \text{ in. circumference}$$

Try nine rods at $^{52}\!/_9 = 5.8$ in. c.c. (approx). Area of one rod $= 5.2/9 = 0.58$ in.^{2}. Therefore, use nine No. 7 rods, giving $A_s = 5.4$ in.^{2}.

Table 9 in the Appendix shows that No. 3 spiral rods with a 2-in. pitch will be satisfactory. Checking again by means of the central diagram of Fig. 13*B* of the Appendix, with $p_g = 0.015$, the required diameter is found to be about 21 in.

This second solution seems to result in a column that is more reasonably proportioned than the first. The extra concrete in the larger column is

$$(\pi/4)(21^2 - 17^2) \div 144 = 0.83 \text{ ft}^3 \text{ per ft of column}$$

This probably offsets the cost of the additional steel in the smaller one. Architectural considerations are likely to govern the choice of sizes to be used.

Example 6-5. Design a short spirally reinforced column, having a square cross section, to support a centrally applied load of 450 kips. Assume $f'_c = 3{,}750$ psi, $f_s = 16{,}000$ psi, No. 9 bars, and a standard spiral.

Of course, many different answers are possible. Using the right-hand diagram in Fig. 13*B* of the Appendix, with an assumed p_g of 0.03 (a practicable value) and $P = 450$ kips, the required diameter is approximately 21 in. and A_g = about 350 in.^{2}. Then

$$A_s = 350 \times 0.03 = 10.5 \text{ in.}^2$$

From Table 11 of the Appendix, with a core diameter of 18 in., 12 No. 9 bars can be used but only 11 are necessary ($A_s = 11$ in.^{2}). Therefore, these 11 will be adopted. Then, from Table 9 of the Appendix, with a core diameter of 18 in. and a 3,750-lb concrete, the spiral can be No. 4 bars at 2½-in. spacing. Therefore, the exterior dimensions of the column will be 21 in. square—the circumscribing square for the 21-in.-diameter circle.

6-5. Composite columns. The term "composite column" is used to denote a structural-steel column—or sometimes a cast-iron one—which is thoroughly encased in concrete that often has longitudinal reinforcement and which must have adequate spirals or hoops. Such members are often encountered in the construction of large buildings. The cross section of the concrete is generally large, and it can be relied upon to assist the steel in resisting the applied load.

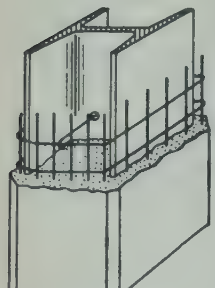


FIG. 6-6.

This classification should not include the ordinary H column which is encased in the minimum amount of concrete that can be used for the purpose of fire protection as pictured in Fig. 6-6. Such steel members are generally heavy; the strength of the concrete is insignificant in comparison with that of the steel; and the encasement is poorly restrained. Therefore, composite columns will be considered to be those in which the strength of the concrete is really substantial. The Code limits the area of the steel section of composite columns to 20 per cent of the total area of the column. Those like Fig. 6-6 will be called combination columns, and they will be discussed in the next article.

Figure 6-7 shows a composite column that was investigated in connection with the construction of the George Washington Bridge at New York City. The longitudinal rods were very small compared with the

structural-steel member and the concrete. However, the concrete is held very well by the bands, by the outside angles of the steel member, and by the internal diaphragms. It is easy to see that the total strength of such a column is made up of three parts, *viz.*, the strength of the structural steel, that of the longitudinal rods, and that of the concrete. Therefore, the ultimate strength of the column might be assumed to be

$$P' = f'_cA_c + f'_sA_s + f'_sA_r \tag{6-7}$$

where A_c is the net area of the concrete in square inches, and A_s and A_r equal the cross-sectional areas in square inches of the longitudinal rods and the structural steel, respectively. The permissible thickness of shell outside the ties or spirals should be limited as for other columns. Preferably there should be 3 in. of concrete between the hoops or spirals and the steel core—of course, circular spirals are far better than hoops. A cover of 2 in. may be accepted for this clear space in the case of steel H columns.

Strictly speaking, the Code does not permit the use of hoops, but they were relied upon in the case of Fig. 6-7. The edge angles and rivet heads helped to bond the concrete to the steelwork.

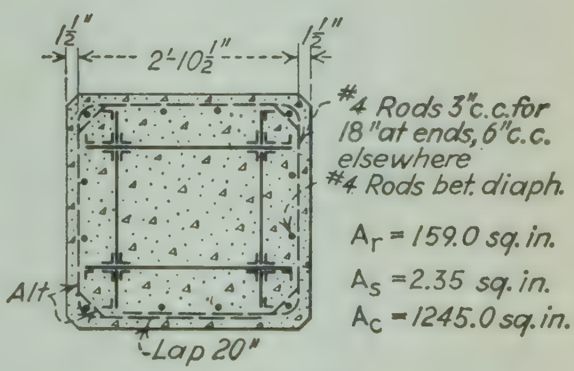


FIG. 6-7.

The spacing of reinforcing bars, their splices, the cover over the spirals, and the percentage of longitudinal steel are to conform to the requirements for spirally reinforced columns. If the metal core is hollow, it is to be filled with concrete.

On the basis of the elastic theory, a steel composite column may be designed by using Eq. (6-2), but p_g must be

$$p_g = \frac{A_s + A_r}{A_g}$$

However, the steel portion is relatively large, and it is not safe to let it be overloaded too severely. If $f'_c = 3{,}000$ psi, $n = 10$, and $f'_s = 36{,}000$ psi, the stress in the concrete when the steel is at its yield point will be $f'_s \div n = 36{,}000/10 = 3{,}600$ psi. It will be seen that the concrete will probably fail before the steel does, unless the former is relieved by plastic flow. Even with a working stress of $f_s = 16{,}000$ psi in the steel the stress in the concrete would appear to be

$$f_c = \frac{f_s}{n} = \frac{16{,}000}{10} = 1{,}600 \text{ psi}$$

This is too great a working stress for a concrete of the stated strength.

The Code has established a formula for the safe load on such a column. It is

$$P = 0.225A_cf'_c + f_sA_s + f_rA_r \tag{6-8}$$

The value of f_r, the safe working stress in the structural steel, is the same as f_s in the rods, *viz.*, 16,000 psi for intermediate grade. Of course, the magnitude of f_r is less in the case of a cast-iron section (10,000 psi).

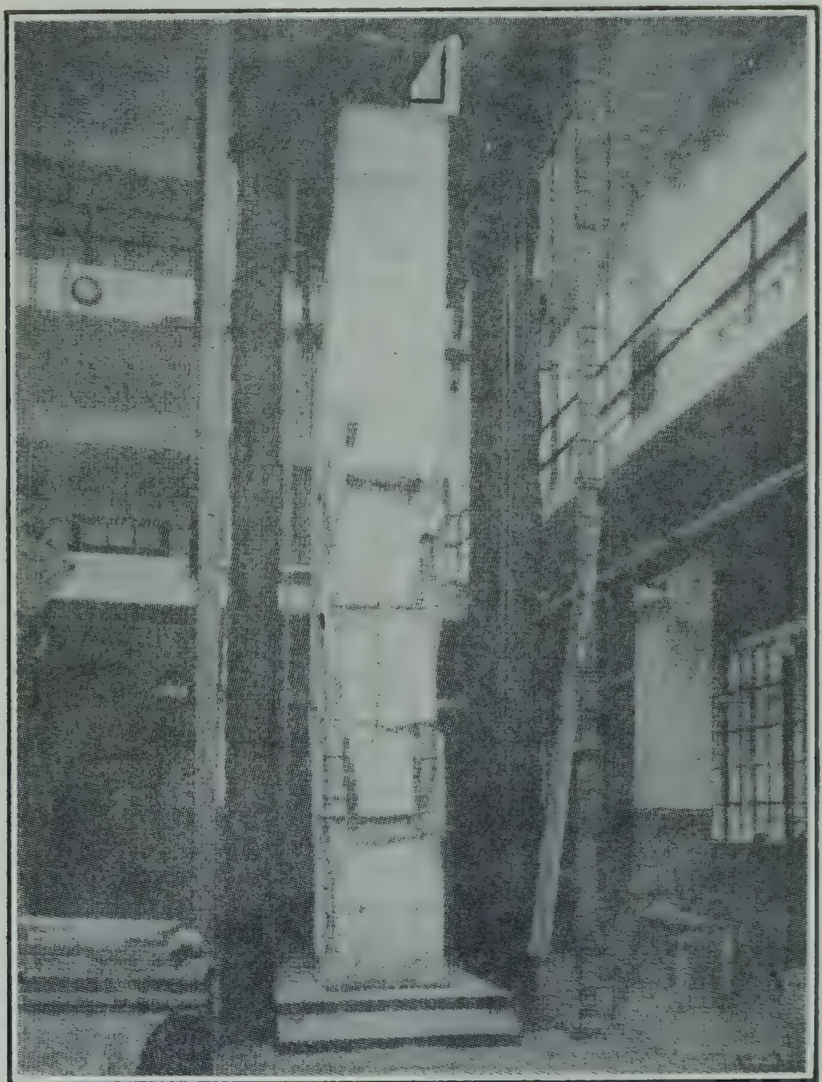


FIG. 6-8. Concrete-encased model of tower columns of the George Washington Bridge at the Bureau of Standards, Washington, D.C.

Equation (6-8) can be used as a limiting formula. It allots the same unit stress to the concrete as Eq. (6-5) does in a spirally reinforced column. However, Eq. (6-8) is empirical, and it does not enable one to find the unit stresses in a column that is not loaded to its capacity, but the important question is whether or not the column is safe.

Figure 6-8 is a picture of one of the columns[1] that is shown in Fig. 6-7 after it had been tested to failure by the Bureau of Standards at Wash-

[1] Stang and Whittmore, *Natl. Bur. Standards Research Paper* R P 831, September, 1935.

ington, D.C. This gives an idea of the size of the member. Figure 6-9 is a view of one end of another specimen after it had been removed from the testing machine. The steel section is clearly visible.

Using Eq. (6-8) with $f'_c = 3{,}000$ psi, $f_s = f_r = 16{,}000$ psi, and the other data as given in Fig. 6-7,

$$P = 0.225 \times 1{,}245 \times 3{,}000 + 16{,}000 \times 2.35 + 16{,}000 \times 159$$
$$P = 3{,}422{,}000 \text{ lb}$$

Actually, the first vertical cracks in the concrete seem to have appeared when the measured values were as follows:

$$P = 3{,}500{,}000 \text{ lb}$$
$$f_c = 1{,}000 \text{ psi} \qquad \text{(compared with } 0.225 \times 3{,}000 = 675 \text{ psi)}$$
$$f_s = 14{,}000 \text{ psi} \qquad \text{(compared with } 16{,}000 \text{ psi)}$$

This, in general, checks Eq. (6-8) very well as far as the total load is concerned.

A review of the results seems to indicate that the stresses in the materials actually were part way between the values that would be computed from Eqs. (6-2) and (6-8), but the lapse of a longer time for the column under load might modify these results.

The ultimate strength of the column was found to be 8,314,000 lb, with a corresponding value of $f'_s = 50{,}400$ psi. When this unit stress is multiplied by A_r, it gives

$$A_r f_r = 159 \times 50{,}400 = 8{,}030{,}000 \text{ lb}$$

This, together with the appearance of the specimen in Fig. 6-9, shows that portions of the concrete failed almost completely, as should be expected.

On the other hand, the concrete encasement increased the strength of the column over that of the structural-steel member alone. Tests of similar bare steel members showed that they failed at an ultimate load of 5,853,000 lb. The increase of strength contributed by the concrete was therefore

$$8{,}314{,}000 - 5{,}853{,}000 = 2{,}461{,}000 \text{ lb, or 42 per cent}$$

Of course, the adjoining structural-steel portions of composite columns should be milled to bear. They are to be spliced at least enough for erection purposes and positively to align one core above another. The cores alone shall be able to resist any erection or permanent loads placed upon them prior to their encasement. The bases must be able to transfer the column loads to the footings safely. The base of the metal core may be designed to transfer the total load to the footing, or it may provide for the core alone if there is sufficient area outside it to provide compressive area and bond for the development of the strength of the reinforced-concrete section.

Loads are to be transferred to metal cores by means of bearing plates, brackets, or framed connections at the top and intermediate levels. If other loads are supported by the concrete alone, the concrete section must be adequate to support these loads as a spirally reinforced column, Eq. (6-5). In this case, A_g is the area of concrete outside the core, but

the allowable load on it when acting in this manner shall be limited to $0.35f'_cA_g$. Of course, adequate concrete and continuity of reinforcement shall be provided at the junctions of beams and girders with a column.

The student should examine Figs. 6-8 and 6-9 carefully because they are extreme illustrations of what takes place in columns that are loaded to failure. The general buckling of the member, the bending and bursting of the ties, the buckling of the longitudinal rods, the spalling and crushing of the concrete— all these are revealed in the pictures.

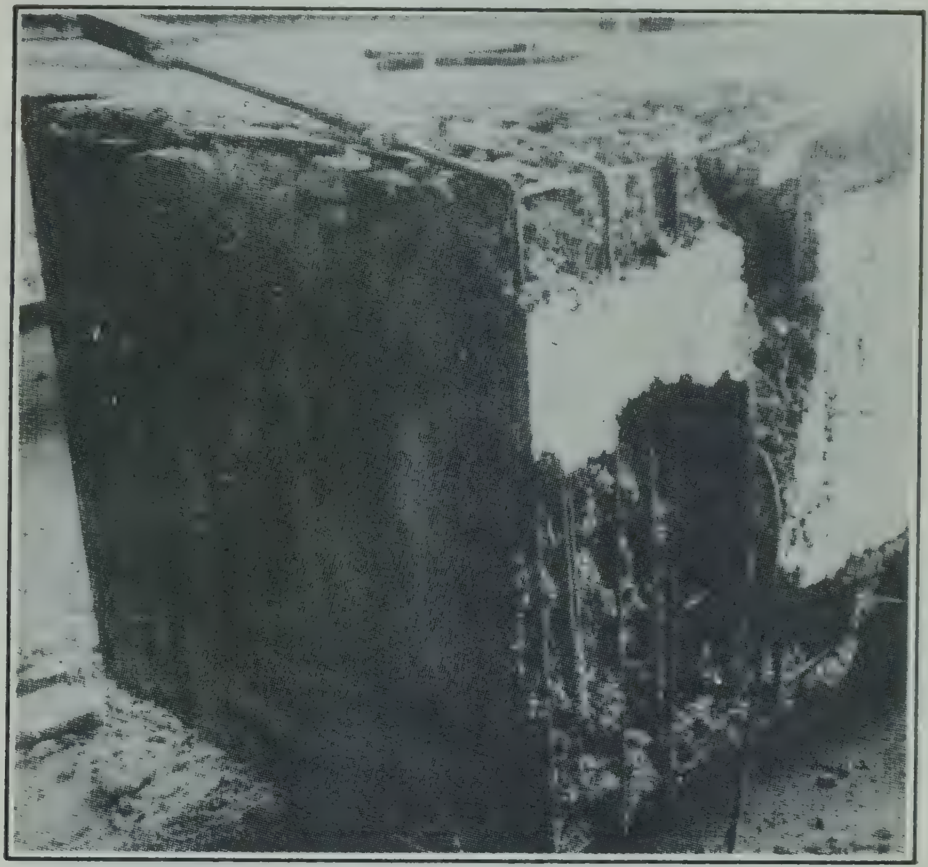


FIG. 6-9. Composite column tested by the Bureau of Standards, Washington, D.C.

6-6. Combination columns. The Code states that the allowable load on a heavy structural-steel column like that of Fig. 6-6 should be computed as follows:

$$P = A_rf'_r\left(1 + \frac{A_c}{100A_r}\right) \tag{6-9}$$

where A_r = cross-sectional area of the steel column, f'_r = allowable stress for the unencased steel column, and $A_c = A_g - A_r$. The symbols are changed slightly to suit those used here.

There may be considerable question as to when the encasement can be assumed to assist in carrying loads and when it is merely fire protection. To be in the former classification, the Code sets a minimum cover of 2½ in. over the steel with adequate wire mesh—No. 10 gage wires and

4 by 8 spacing—about 1 in. from the face of the concrete and having the splices wired and lapped at least 40 times the diameter of the wires. Also, f'_c must be at least 2,000 psi. However, a designer must judge each case to determine the advisability of relying upon the encasement.

The steel column alone must be able to support all loads applied to it prior to the encasement. Floor loads are to be provided for by framing directly into the steel column or by resting upon special brackets connected to it.

Example 6-6. Find the safe load on a column like that of Fig. 6-6 if the steel section is a 14*WF* 202-lb H section and the concrete is 22 in. square with 1-in. chamfers. Assume that the allowable unit stress in the bare steel section is 13.8 ksi.

$$A_r = 59.39 \text{ in.}^2$$
$$A_c = 22^2 - 4 \times \tfrac{1}{2} - 59.39 = 423 \text{ in.}^2$$
$$P = 59.39 \times 13.8 \left(1 + \frac{423}{100 \times 59.39}\right) = 878 \text{ kips}$$

6-7. Pipe columns filled with concrete. Pipes may sometimes be used as columns; if filled with concrete they may be even better for that purpose. However, except as end-bearing piles, their use is rather restricted. In effect, the empirical formula given by the Code for the calculation of the allowable load on such columns is

$$P = 0.225 f'_c A_c + \left(18{,}000 - 70\,\frac{h}{R}\right)\frac{f'_s A_r}{45{,}000} \tag{6-10}$$

where A_r = area of pipe steel, h = unsupported height of column, R = radius of gyration of steel pipe, and f'_s = tensile yield point of pipe material. f'_c should equal at least 2,500 psi. The coefficient of A_r is an expression for the allowable unit stress in the steel pipe. If f'_s is unknown, assume it to be 22,500 psi.

Example 6-7. Assume a 12-in. ID steel pipe, $\frac{3}{8}$ in. thick, with a height of 18 ft. Let f'_c and f'_s = 3,000 and 33,000 psi, respectively. A_r = 14.38 in.², R = 4.38 in.; A_c = 113.1 in.².

$$P = 0.225 \times 3{,}000 \times 113.1 + \left(18{,}000 - \frac{70 \times 18 \times 12}{4.38}\right)\frac{33{,}000 \times 14.38}{45{,}000}$$
$$= 230{,}000 \text{ lb}$$

6-8. Long columns. When the unsupported length of a column exceeds ten times its least lateral dimension, it is arbitrarily classed as a "long" column. Members of such slender proportions are not used frequently in ordinary reinforced-concrete construction. Of course it is obvious that their slenderness increases the possibility that they may buckle under load in the general manner that is illustrated in Fig. 6-8. Therefore, special provision must be made to increase their strength

above that which is required for a short column that carries the same axial load.

Lateral buckling causes a bending moment in the column. This moment acts simultaneously with the direct load, thus tending to cause excessive compressive stresses in one side of the member and tensile stresses in the other. Inasmuch as the column is not composed of one homogeneous material which can resist large tensile stresses as well as compressive ones, an exact theoretical determination of the stress condition in the member due to the tendency to buckle is practically impossible. Therefore, one had better rely upon empirical data which are based upon tests and study by experts.

The recommendations of the Code will be adopted. It gives the following formula for the safe load on long reinforced-concrete and composite columns:

$$P' = P\left(1.3 - 0.03\,\frac{h}{d}\right) \tag{6-11}$$

where P' is the maximum permissible axial load on the long column, P is the maximum axial load for the same column if it is "short," h is the unsupported length, and d equals the least lateral dimension of the member. Equation (6-11) applies whether the column is designed in accordance with the formulas of this chapter for direct load only or in accordance with those of the following one which include bending with the axial load. Especially in the latter case, h/d must not exceed 20.

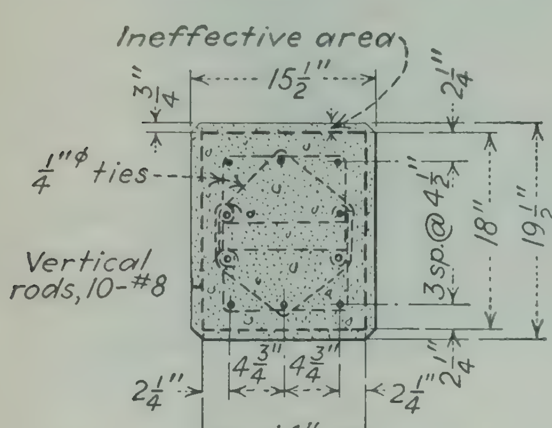


FIG. 6-10.

Example 6-8. Find the safe load for the rectangular tied column which is shown in Fig. 6-10. Use the Code [Eq. (6-4)], and assume a length of 11 ft; then use Eq. (6-11) with an assumed length of 20 ft. Use $f'_c = 3{,}000$ psi and $f_s = 16{,}000$ psi.

The cover over the rods is 3 in., but allow here only 1½ in. beyond the outside of the ties in the computations for the effective area. Therefore, since the main bars are No. 8, ½ + ¼ + 1½ = 2¼ in. beyond the centers of the longitudinal rods is the limit for this area. Then

$$A_g = 14 \times 18 = 252 \text{ in.}^2$$

$$p_g = \frac{A_s}{A_g} = \frac{10 \times 0.79}{252} = 0.0313$$

For the short column, $h/d = 11 \times 12/15.5 = 8.5$. It seems to be satisfactory to use the gross width rather than the net width for d when the difference is not too great.

$$P = A_g(0.18f'_c + 0.8f_sp_g)$$
$$P = 252(0.18 \times 3{,}000 + 0.8 \times 16{,}000 \times 0.0313) = 237{,}000 \text{ lb}$$

For the long column,

$$P' = P\left(1.3 - 0.03\frac{h}{d}\right)$$

$$P' = 237{,}000\left(1.3 - 0.03 \times \frac{20 \times 12}{15.5}\right) = 198{,}000 \text{ lb}$$

The length of this column therefore reduces the safe load to $^{198}\!/_{237} \times 100 = 83$ per cent of its magnitude for a similar short column.

Example 6-9. Design a circular spirally reinforced column to support an axial load of 500 kips at the top if the clear height is 32 ft, $f'_c = 3{,}000$ psi, and $f_s = 16{,}000$ psi.

At first glance it is difficult to tell whether this will be a long column or a short one. To design it quickly, proceed as follows:

1. Assume that it is a short column; then assume p_g. In this case, let $p_g = 0.03$.
2. Make a guess regarding the weight of the column. Here, a 32-ft column of concrete 1 ft square would weigh $32 \times 0.15 = 4.8$ ksf. Then assume the column to have a cross section of 1 ft² for each 100 kips of load. Therefore, the column will perhaps weigh $5 \times 4.8 = 25$ kips. Adding this, call $P = 500 + 25 = 525$ kips.
3. From the middle diagram of Fig. 13*B* of the Appendix, with $P = 525$ kips and $p_g = 0.03$, find $D = 24$ in.
4. The factor $1.3 - 0.03\ h/d$ in Eq. (6-11) then equals

$$1.3 - 0.03 \times {}^{32}\!/_{2} = 0.82$$

5. Now divide the 525-kip load by the factor 0.82.

$$\frac{525}{0.82} = 640 \text{ kips}$$

6. With this increased load of 640 kips for P, Fig. 13*B* of the Appendix gives a required diameter about 25.5 in. Call it 26 in. Using it, Eq. (6-11) now gives

$$P' = 640\left(1.3 - 0.03 \times \frac{32 \times 12}{26}\right) = 507 \text{ kips}$$

This will be called satisfactory. $A_g = 530$ in.².

7. The longitudinal steel required is $530 \times 0.03 = 15.9$ in.². Try 16 No. 9 rods with $A_s = 16$ in.². Table 11 of the Appendix shows that these can be used with a core of $26 - 3 = 23$ in. From Table 9 of the Appendix, No. 3 bars at $1\frac{3}{4}$ in. c.c. will be satisfactory for the spiral. This 26-in. column will therefore be used.

Notice that, in the preceding computations, the effect of the weight of the column is small. It therefore matters little whether the extra load is for the pressure at mid-height or at the base. The latter is more conservative and desirable. Some allowance should be made for the column itself in any case.

6-9. Estimating column loads. It is properly the custom to design the roof members of a building first, then the floor below, etc., proceeding down to the foundation. The loads on the columns are determined in the same order, and these members are designed accordingly, including the live load and the weight of the columns themselves. A tentative size of column may be found by dividing the estimated load by $\frac{1}{3}f'_c$. It is also obvious that a particular column will usually be somewhat larger

and heavier than the one above it, and the upper one is known when one has to estimate the size of the lower column.

If the Code allows a reduction in the computed live loads for a series of floors in a multistory building, the dead loads and live loads should be summed separately so that advantage may be taken of the allowed percentage reduction for the latter. Figure 6-11 has been prepared so that the reader may get a little practice in calculating column loads.

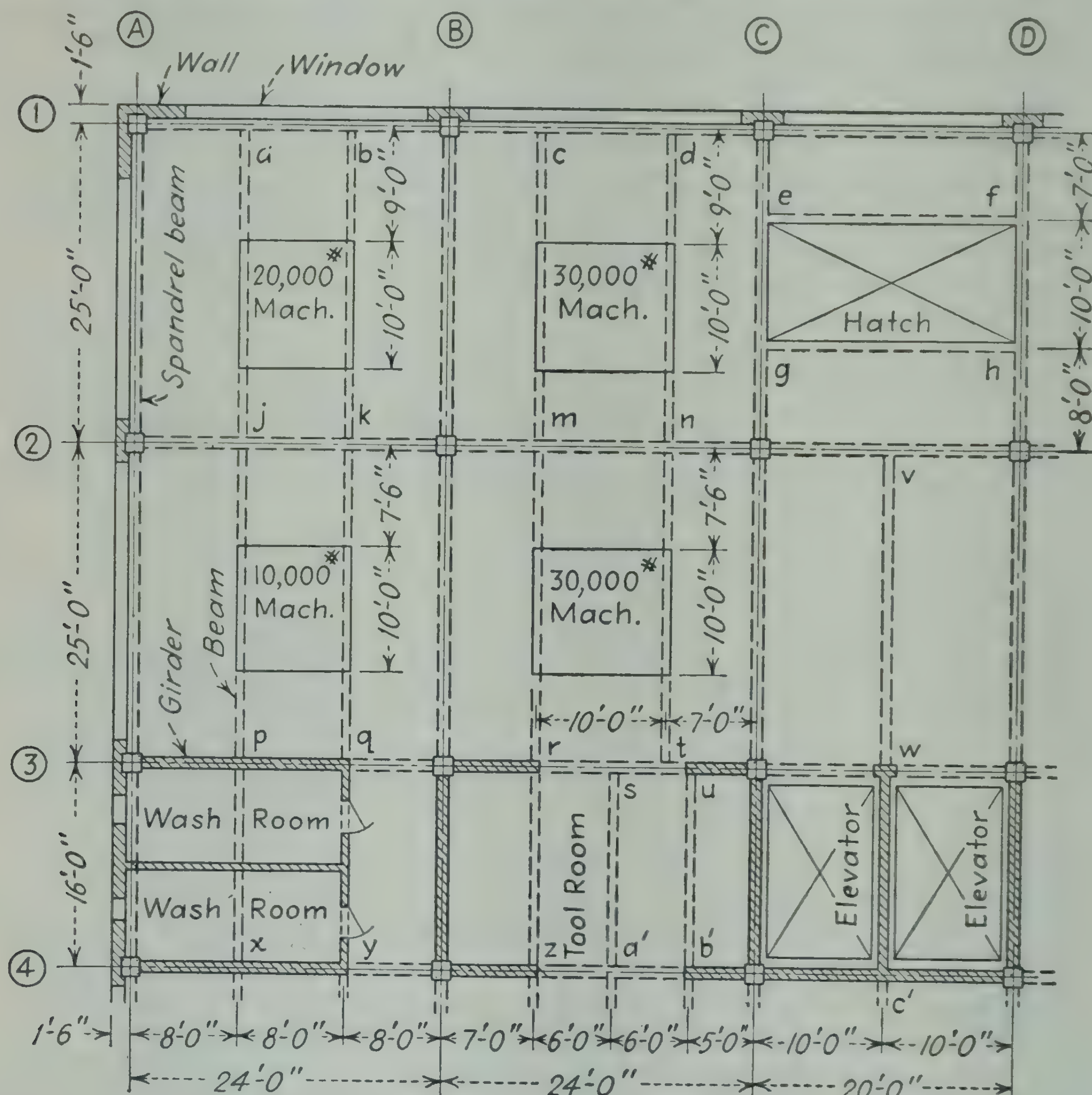


FIG. 6-11. Partial plan of a heavy factory floor.

Assume that Fig. 6-11 pictures a portion of the second floor of a large multistory factory building. The floor framing has been designed, and the column loads from the floor above have been computed. It is now necessary to calculate the loads in the columns under this floor in order to design them.

Using the numbering system shown in Fig. 6-11, the following outlines indicate the general procedure to be followed in computing the column loads:

1. Col. A1:
   Load from Col. A1 above =
   Reaction of girder A1-B1 at A1 =
   Reaction of beam A1-A2 at A1 =
   Estimated weight of Col. A1 itself = ____________
   Total =
2. Col. B1:
   Load from Col. B1 above =
   Reaction of girder A1-B1 at B1 =
   Reaction of girder B1-C1 at B1 =
   Reaction of beam B1-B2 at B1 =
   Estimated weight of Col. B1 itself = ____________
   Total =
3. Col. B2:
   Load from Col. B2 above =
   Reaction of girder A2-B2 at B2 =
   Reaction of girder B2-C2 at B2 =
   Reaction of beam B1-B2 at B2 =
   Reaction of beam B2-B3 at B2 =
   Estimated weight of Col. B2 itself = ____________
   Total =

Practice Problems

6-1. Compute the total safe load for a short tied column 16 in. square with eight No. 7 rods. The ties are No. 3 at 12 in. c.c. and have a cover of 2 in. Assume 3,500-lb concrete and structural-grade steel. Use Eq. (6-4). *Ans.* $P = 203{,}000$ lb.

6-2. Design a square short tied column to support a total load $P = 700{,}000$ lb. Let $f_c' = 3{,}000$ psi, $f_s = 16{,}000$ psi, and $p_g = 0.02$. Design the ties.

Ans. $D = 31$ in., with 12 No. 11 rods, and No. 4 ties 20 in. c.c.

6-3. Design a square short tied column to support a total load P of 600,000 lb. Let $f_c' = 3{,}750$ psi, $f_s = 20{,}000$ psi, and $p_g = 0.025$. Draw a sketch of the column. Design the ties.

6-4. A rectangular column is 18 by 20 in. The rods are eight No. 10. The ties are No. 3 at 16 in. c.c. The cover over the ties is $2\frac{1}{4}$ in. If $f_c' = 3{,}000$ psi and $f_s = 16{,}000$ psi, what is the total safe load P?

6-5. Determine the safe load on a short circular spirally reinforced column having an outside diameter of 24 in., No. 3 spiral with $1\frac{3}{4}$-in. spacing, 2-in. cover, and 12 No. 8 bars. Assume $f_c' = 3{,}000$ psi and $f_s = 16{,}000$ psi. *Ans.* 274,000 lb.

6-6. Find the safe load on the column of Prob. 6-5 if $f_c' = 3{,}500$ psi, $f_s = 20{,}000$ psi, the reinforcement is 12 No. 9 rods, and the clear height is 20 ft.

6-7. Compute the safe load on a circular spirally reinforced column 20 in. in outside diameter and having a clear height of 25 ft. The spirals are No. 3 at $1\frac{3}{4}$ in. c.c. with 2 in. of cover. Assume $f_c' = 3{,}000$ psi, and $f_s = 16{,}000$ psi.

6-8. By the Code [Eq. (6-5)], design a circular spirally reinforced column to support a centrally applied load of 380,000 lb, using $f_c' = 2{,}500$ psi, $f_s' = 40{,}000$ psi, and $f_s = 18{,}000$ psi. Use $p_g =$ about 3 per cent. Design the spiral reinforcement for this column. Compare with Fig. 13*B* in the Appendix.

6-9. Calculate the safe load for a tied column 25 ft high and 20 by 24 in. in cross section. Let $f_c' = 3{,}000$ psi, and $f_s = 18{,}000$ psi. There are 16 No. 7 longitudinal rods. The ties have a cover of 2 in. of concrete.

6-10. Determine the safe load for a spirally reinforced column 20 ft high and 18 in. in diameter if f'_c = 3,500 psi, f_s = 18,000 psi, and the reinforcement = 10 No. 7 rods. Use Eqs. (6-5) and (6-11), and assume that the spiral has $1\frac{1}{2}$ in. of cover.

6-11. Design a square tied column to support a load of 400,000 lb if its height is 26 ft and f'_c and f_s = 3,000 and 16,000 psi, respectively. Use the Code [Eq. (6-4)], also Eq. (6-11) if necessary. Use a cover of 2 in. over the ties. Choose sizes and spacings of main rods and ties.

Discussion. From Fig. 13*B* in the Appendix, with f'_c = 3,000, compute a trial A_g and width, first choosing a tentative value for p_g, say 0.02. Test to see if Eq. (6-11) affects the case. Add more area if it seems to be needed. Analyze the trial section; then modify the design until it is satisfactory. Remember to have $\frac{1}{2}$ in. more cover than the minimum $1\frac{1}{2}$ in.

Ans. One satisfactory column is 24 in. square over all with 12 No. 9 rods and No. 3 ties 16 in. c.c. similar to Fig. 6-1(*d*).

6-12. A spirally reinforced column is made 30 in. square for architectural reasons. The diameter of the core is 26 in.; the longitudinal reinforcement consists of 12 No. 10 rods; the spiral is made of a No. 4 rod with a pitch of $2\frac{1}{2}$ in.; and the allowable f'_c and f_s are 3,750 and 16,000 psi, respectively. The clear height is 18 ft. The column is supposed to carry a load of 850,000 lb. Is it safe according to Eqs. (6-5) and (6-11)? If not, what can be done to make it safe?

Ans. Not safe because allowable P = 800,000 lb. Add 4 more No. 10 rods or make the section larger.

Data for Probs. 6-13 *to* 6-24, *Inclusive.* Assume the framing shown in Fig. 6-11. Also assume the following (neglect windows in washroom):

Walls = 100 psf
Partitions = 60 psf
Live load on floor exclusive of machines = 200 psf
Live load in washroom = 75 psf
Live load in toolroom = 150 psf
Story height = 15 ft
Walls below window sills = 3 ft
Slab thickness = 6 in. (no hung ceiling)
Stems of beams:

Span, ft	Total depth below slab, in.	Width, in.
16	14	10
20	18	14
25	22	16

Stems of girders:

Span, ft	Total depth below slab, in.	Width, in.
20	22	16
24	26	18

For all columns assume f'_c = 3,500 psi. Use the Code formulas. Compute the

beam and girder reactions. P_1 represents the load from the column above the one being designed. Various numbers of stories are assumed.

6-13. Design column A1 as a square tied column. The load from the column above is 260 kips.

6-14. Design column A2 as a square tied column. $P_1 = 380$ kips.

6-15. Design column B1 as a square tied column. $P_1 = 420$ kips.

6-16. Design column B2 as a square spirally reinforced column. $P_1 = 550$ kips.

6-17. Design column A3 as a tied column that is to have a width of 20 in. in one direction. $P_1 = 220$ kips.

6-18. Design column B3 as a square spirally reinforced column. $P_1 = 300$ kips.

6-19. Design a rectangular tied column for column C3. Assume one side is to be 24 in. wide. $P_1 = 500$ kips.

6-20. Design a circular spirally reinforced column for column C2. $P_1 = 600$ kips.

6-21. A designer proposes to use for column C1 (as a tied column) a section 20 in. square with 8 No. 9 rods. $P_1 = 240$ kips. Is the column satisfactory?

6-22. A designer proposes to use a 24-in. square spirally reinforced column for column C2. It is to have 16 No. 8 rods. $P_1 = 450$ kips. Is this column satisfactory?

6-23. A tied column 18 by 22 in. with 10 No. 11 rods is proposed for column C3. $P_1 = 350$ kips. Is the column satisfactory?

6-24. A designer has selected a tied column 16 in. square with 8 No. 8 rods for column A2. $P_1 = 240$ kips. Is the design satisfactory?

7

COMBINED BENDING AND COMPRESSION

7-1. Introduction. In ordinary construction, there are many cases in which members are subjected to a combination of bending moments and direct axial loads. Generally, in reinforced-concrete work, the direct load in such combinations is a compressive force. Lateral earth pressures which act upon subway and foundation walls, columns to which beams are connected eccentrically, frame action between beams and columns whereby the deflections of the former compel the latter to bend also because of the rigidity of the connections, wind loads which force the columns of a building to bend sideways—all these are ordinary causes of combined compressive and flexural stresses in the members that are affected by them.

The designer must not disregard these combined stresses. He must visualize what forces will exist and what deformations will occur in any proposed structure. He must find a way of designing the members to withstand the combined forces, doing so with reasonable accuracy and without undue labor.

Problems involving compression and bending generally come into one of two classes: the first includes those members which have compression upon all their cross section; the second covers those which have compression upon part of the section and tension upon the remainder. The former case, being more simple, will be considered first. The analysis of the latter situation generally involves considerable complicated calculation. Therefore, because of limited space, this volume will deal only with a few methods of analysis. Its purpose is to state fundamental principles and to illustrate the action of members that are subjected to combined bending and compression.

Unless the cover over the bars is excessive, it is customary to use the over-all dimensions of the member when computing its resistance to this combined action.

7-2. Construction procedure affecting columns. In attacking the problem of combined bending and compression, it is advisable first to analyze the probable construction conditions in so far as they affect the

design. Because of frame action, the maximum bending moment in a column will occur at the top of the floor or at the bottoms of the beams that frame into the column. In the case of lateral pressure against a column, there may be bending at the ends as well as near the middle of the member.

However, as pictured in Fig. 7-1, in ordinary building construction it is customary to place the concrete in the column forms after the lower floor has been constructed. Then the concrete of the column is carried up to, or nearly to, the bottoms of the beams of the next higher floor. The concrete of the beams and the floor is poured next; then that of the column in the next story follows. In any case, each stopping point

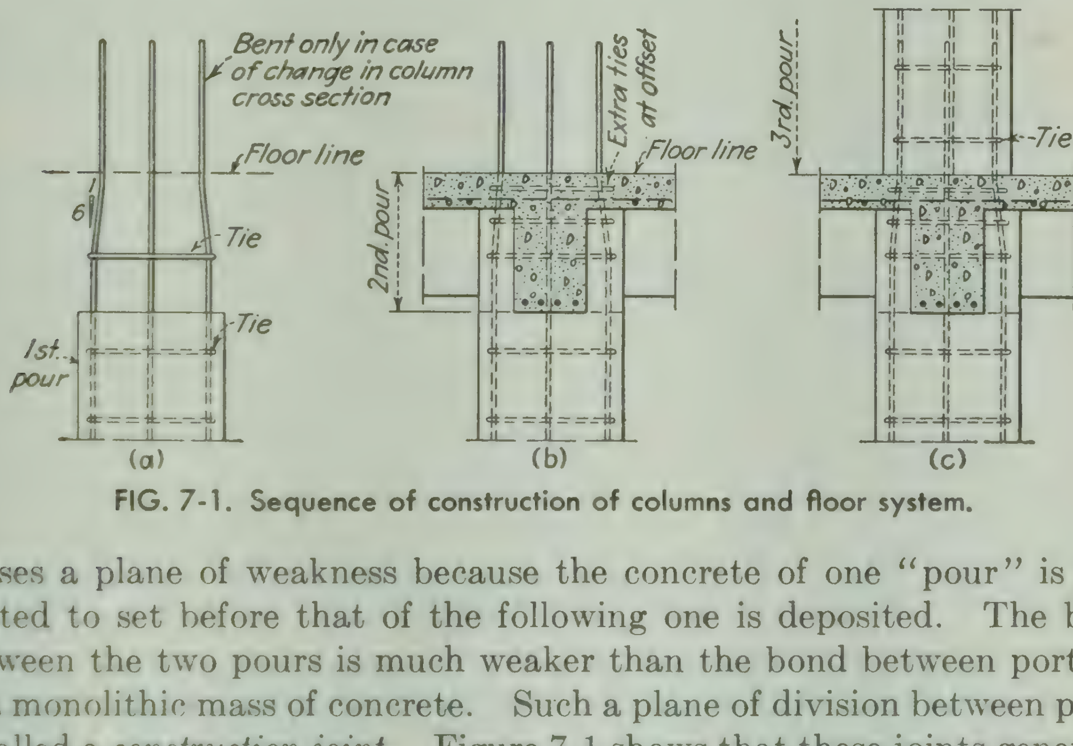


FIG. 7-1. Sequence of construction of columns and floor system.

causes a plane of weakness because the concrete of one "pour" is permitted to set before that of the following one is deposited. The bond between the two pours is much weaker than the bond between portions of a monolithic mass of concrete. Such a plane of division between pours is called a *construction joint.* Figure 7-1 shows that these joints generally occur at the points of maximum bending in the columns.

A construction joint may be roughened or keyed to lock the two sections together to resist shear. The full compressive strength can be relied upon, but the concrete at this point is much weaker than usual in its resistance to tension. Therefore, it seems reasonable to assume that the concrete of ordinary columns cannot withstand tensile stresses at the critical points.

7-3. Combined compression and bending without resulting tension upon the section. Let *ABCD* (Fig. 7-2) represent a short piece of a square column with a symmetrical cross section. Let P equal the direct load and M equal the bending moment, whatever its cause may be. Then *CEFD* represents the pressure diagram for the direct load if it is uniformly distributed, A_t being the transformed area in terms of the concrete. $A_t = A_g + (n - 1)A_s$. Assuming linear distribution

of the stresses, the figure $CDHG$ pictures the diagram for the internal stresses which resist the bending moment M, where I_t is the moment of inertia of the transformed section. $I_t = I_c + (n - 1)I_s$. Then $CKLD$ is the diagram for the combined stresses. Of course there is no true neutral axis or point of zero stress within the section of the column.

It must be realized that a tensile stress like HD in Fig. 7-2 annuls an equal compressive stress, but the effect is unchanged as far as bending is concerned. Furthermore, the rods are assumed to be conveniently symmetrical in this case, and plastic flow is neglected so that the diagrams may be assumed to represent the stresses in the concrete and in the steel in accordance with the elastic properties of the materials. Therefore, from Fig. 7-2,

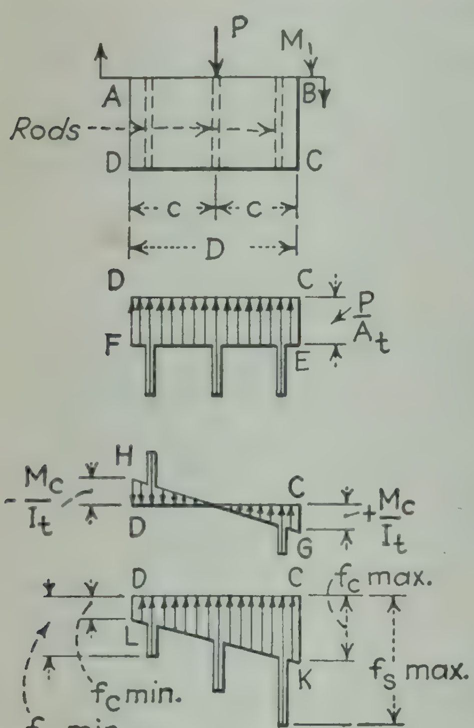


FIG. 7-2.

$$CK = \max f_c = \frac{P}{A_t} + \frac{Mc}{I_t} \quad (7\text{-}1)$$

where $c = D/2$—from the center of gravity axis.

This may be expressed in another form. Let R equal the radius of gyration of the transformed section. Then $R^2 = I_t/A_t$. Furthermore, let $M = Pe$, where e is the eccentricity of the direct load with respect to the axis of the column. In the cases where P acts at the center of the column but an external bending moment is applied, the latter can always be treated in the analysis as though $M = Pe$, so that $e = M/P$. Substituting $M = Pe$ and $I_t = R^2A_t$ in Eq. (7-1) yields the following:

$$\max f_c = \frac{P}{A_t} + \frac{Pec}{R^2A_t} = \frac{P}{A_t}\left(1 + \frac{ec}{R^2}\right)$$

However, $A_t = A_g[1 + (n - 1)p_g]$. Therefore,

$$\max f_c = \frac{P[1 + (ec/R^2)]}{A_g[1 + (n - 1)p_g]} \quad (7\text{-}2)$$

Correspondingly,

$$\min f_c = \frac{P[1 - (ec/R^2)]}{A_g[1 + (n - 1)p_g]} \quad (7\text{-}3)$$

Equations (7-2) and (7-3) are based upon the elastic theory. They can be used for preliminary design purposes. The simultaneous stresses

in the steel are theoretically n times the stress in the concrete at the location of the steel. They can be found by proportion from the trapezoidal diagram $DCKL$ of Fig. 7-2.

On the other hand, the Code limits the allowable compressive stress in such members to

$$f_c = f_a \left[\frac{1 + (ec/R^2)}{1 + C(ec/R^2)} \right] \tag{7-4}$$

where $C = f_a/0.45f_c'$. The term f_a is the average permissible unit stress on an equivalent axially loaded column:

For spiral columns:

$$f_a = \frac{0.225f_c' + f_s p_g}{1 + (n - 1)p_g} \tag{7-5}$$

For tied columns:

$$f_a = \frac{0.18f_c' + 0.8f_s p_g}{1 + (n - 1)p_g} \tag{7-5a}$$

The symbols have the same meanings as the similar ones in Eqs. (6-4) and (6-5).

The Code expresses the preceding formulas in a slightly different form by using a constant to represent $D^2/2R^2$. Using B to designate this constant, although the Code uses a different symbol,

$$\frac{ec}{R^2} = \frac{e(D/2)}{D^2/2B} = \frac{Be}{D}$$

This substitution can be made in Eqs. (7-2), (7-3), and (7-4) if desired. However, the basic term is preferred here in order to show its meaning clearly.

If there is a bending moment acting about both axes of symmetry, 1-1 and 2-2 of the column, the term ec/R^2 should be replaced by

$$\frac{e_1c_1}{R_1^2} + \frac{e_2c_2}{R_2^2}$$

When a tied column is subjected to axial and bending stresses, the Code permits the allowable steel ratio p_g to be increased from the limit of 0.04 to 0.08 if the reinforcement lapped in any 3-ft length of column is less than that represented by $p_g = 0.04$. However, such a large amount of steel is not desirable for ordinary construction. Of course, the section designed in this way must not be less than that required by Eq. (6-4) for axial loads alone.

Example 7-1. If the spirally reinforced concrete column shown in Fig. 7-3 carries a centrally applied load of 150,000 lb and a bending moment of 360,000 in.-lb, acting about the axis X-X, find the maximum and minimum values of f_c and f_s, using $n = 12$.

Use Eqs. (7-2) and (7-3). Also, in order to avoid undue labor in the calculations, it is satisfactory to replace the longitudinal rods with an annular ring having the same area as the entire group of rods and having a mean diameter equal to that of the circle in which the rods are set. Call this mean diameter D_2. This substitution of a ring of steel for the individual rods will cause no appreciable error unless there are only a very few rods in the column. Then

$$A_t = A_g[1 + (n-1)p_g] = \frac{\pi \times D^2}{4} + (n-1)A_s = \frac{\pi \times 21^2}{4} + 11 \times 7.2$$
$$= 346 + 79 = 425 \text{ in.}^2$$

$$I_t = \frac{\pi D^4}{64} + \frac{(n-1)A_s D_2^2}{8} = \frac{\pi \times 21^4}{64} + \frac{11 \times 7.2 \times 16^2}{8} = 9{,}540 + 2{,}530$$
$$= 12{,}070 \text{ in.}^4$$

since I for an annular ring is

$$\frac{\pi}{64}(D_a^4 - D_b^4) = \frac{\pi(D_a^2 - D_b^2)}{4}\,\frac{(D_a^2 + D_b^2)}{16} = \frac{\text{area} \times D_2^2}{8} \qquad \text{(approx)}[1]$$

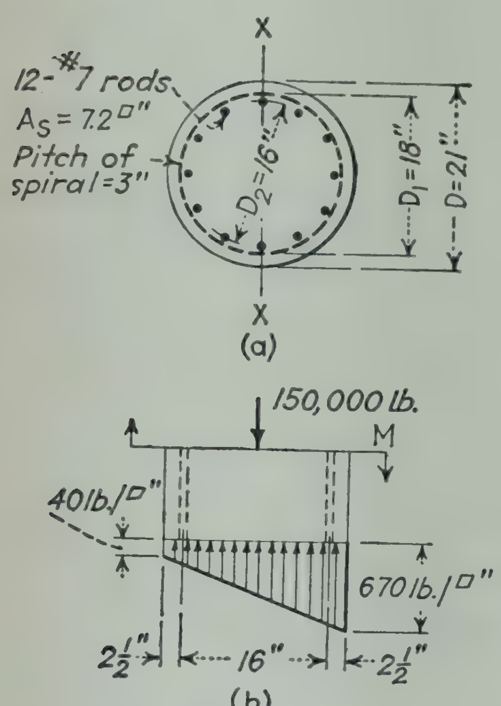


FIG. 7-3.

Therefore, $I_s = [(n-1)/8]A_s D_2^2$.

$$R^2 = \frac{I_t}{A_t} = \frac{12{,}070}{425} = 28.4$$

$$p_g = \frac{A_s}{A_g} = \frac{7.2}{346} = 0.0208$$

$$e = \frac{M}{P} = \frac{360{,}000}{150{,}000} = 2.4 \text{ in.}$$

Since $c = 10.5$ in.,

$$\max f_c = \frac{P[1 + (ec/R^2)]}{A_g[1 + (n-1)p_g]}$$
$$= \frac{150{,}000(1 + 2.4 \times 10.5/28.4)}{425} = 670 \text{ psi}$$

Then, from Eq. (7-3),

$$\min f_c = \frac{150{,}000(1 - 2.4 \times 10.5/28.4)}{425} = 40 \text{ psi}$$

The steel stress at the locations of the extreme rods is found from Fig. 7-3(*b*) as follows:

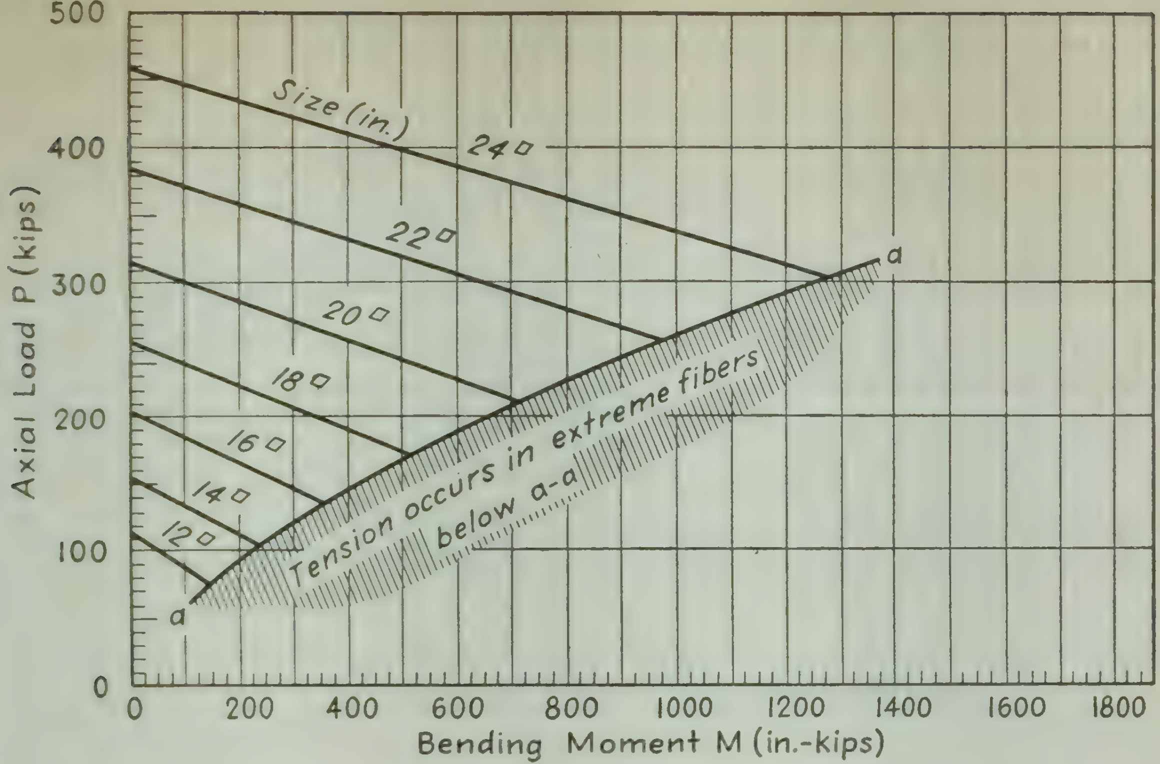
$$\max f_s = nf_c = 12\left(40 + \frac{630 \times 18.5}{21}\right) = 7{,}140 \text{ psi}$$

$$\min f_s = nf_c = 12\left(40 + \frac{630 \times 2.5}{21}\right) = 1{,}380 \text{ psi}$$

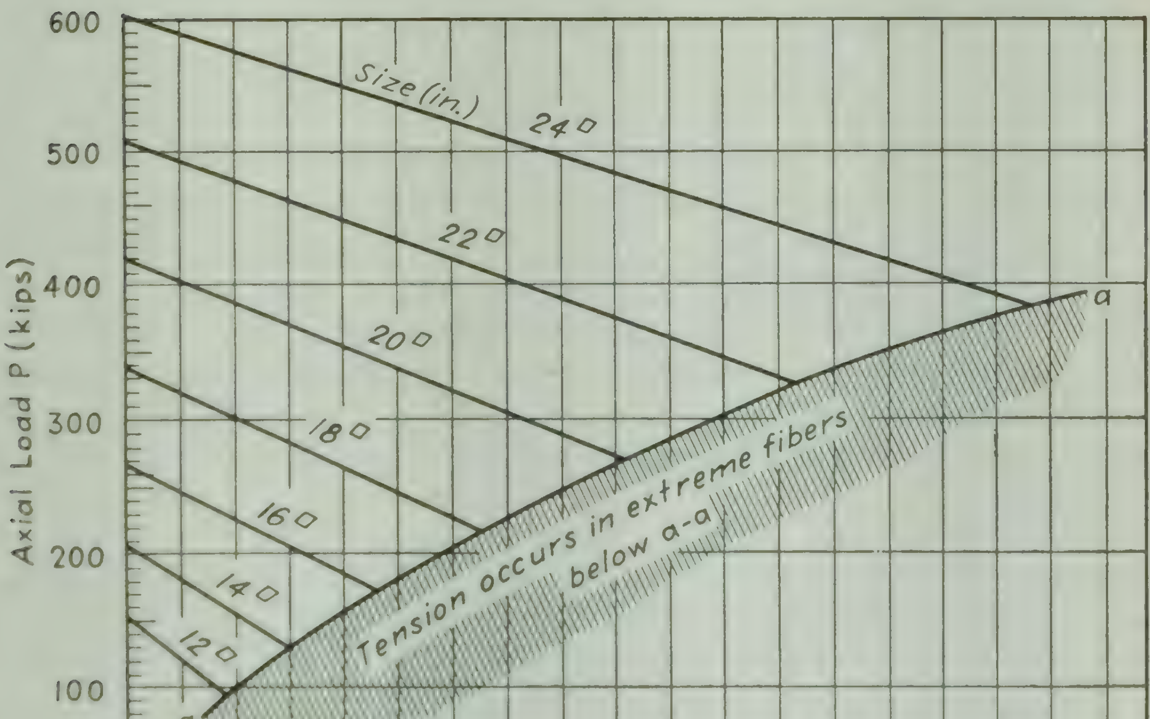
A checkup with Eqs. (7-4) and (7-5), using $f'_c = 2{,}500$ psi and $f_s = 16{,}000$ psi, gives

[1] Similarly, for a hollow square substituted for the rods in a square tied column with equal reinforcement near all faces,

$$I_s = \frac{(n-1)}{6} A_s D_2^2$$



(a) Tied Columns: $f_s = 16{,}000$, $f_c' = 3{,}000$, $P_g = 0.02$



(b) Tied Columns: $f_s' = 16{,}000$, $f_c' = 3{,}000$, $P_g = 0.04$

FIG. 7-4. Diagrams for estimating trial sizes of square tied columns with bending and compression.

$$f_a = \frac{0.225 \times 2{,}500 + 16{,}000 \times 0.0208}{1 + 11 \times 0.0208} = 730 \text{ psi}$$

$$C = \frac{730}{0.45 \times 2{,}500} = 0.650$$

$$f_c = 730 \left(\frac{1 + 2.4 \times 10.5/28.4}{1 + 0.650 \times 2.4 \times 10.5/28.4} \right) = 870 \text{ psi}$$

Since the permissible value of f_c exceeds 670 psi, the latter will be called satisfactory.

In situations involving direct compression combined with bending, it is difficult to obtain a fairly accurate preliminary design for a member to meet a specific set of conditions. Nevertheless, a good first guess is very desirable if fruitless analyses are to be avoided. Of course, one could "guess and test," but this is tedious. Figures 7-4 and 7-5 have been prepared to show approximately what direct loads P and simultaneous bending moments M can be withstood safely by certain square tied columns having equal reinforcement near all four sides, and by circular spirally reinforced columns, using 3,000-lb concrete. The diagrams are based upon Eqs. (7-2) to (7-5*a*), inclusive, with no tension in the columns. They are helpful in choosing trial sizes.

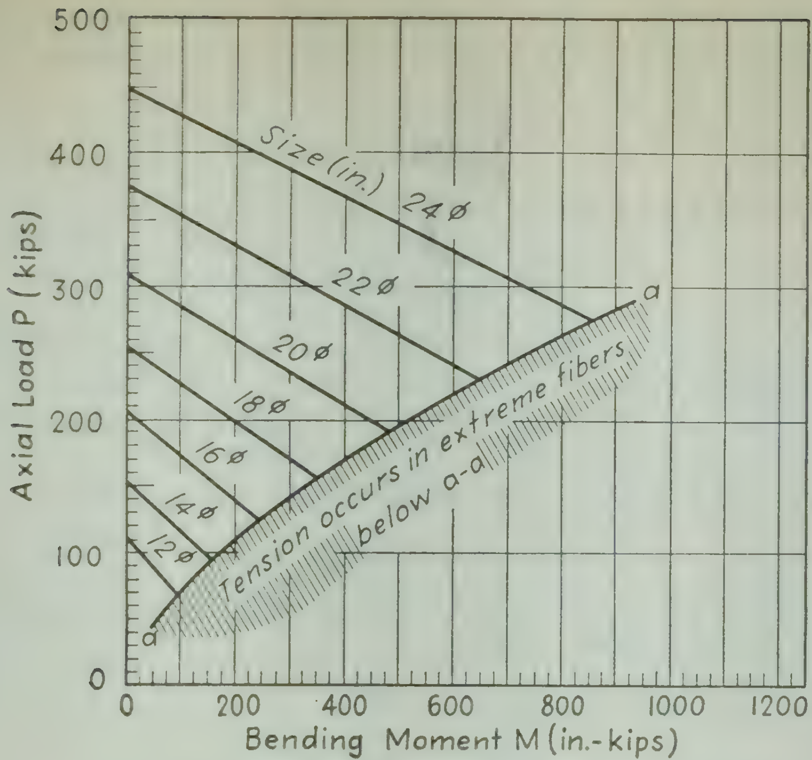
In making estimates of sizes when concrete of other than 3,000-lb strength is to be used, increase the loads and moments about 15 per cent for 2,500-lb concrete or decrease them approximately 15 or 20 per cent for 3,750-lb concrete. With the adjusted value of P as an ordinate and with the adjusted M as an abscissa, find their intersection point on the diagram that most nearly meets the contemplated type and percentage of reinforcement. The estimated size can be interpolated from the diagram, or a somewhat larger size may be assumed. If the intersection of the P and M values falls beyond the ends of the inclined lines in Figs. 7-4 and 7-5, it indicates that the column will probably have tension on part of its cross section, and the curves and assumed formulas are not applicable. The use of these diagrams will be illustrated by the following example.

Example 7-2. A square tied column is to support a load $P = 250$ kips and a bending moment $M = 450$ in.-k. Its clear height is to be 12 ft 6 in. The concrete strength is to be 2,500 psi. Reinforcement is to be equal along all four sides. Design the column.

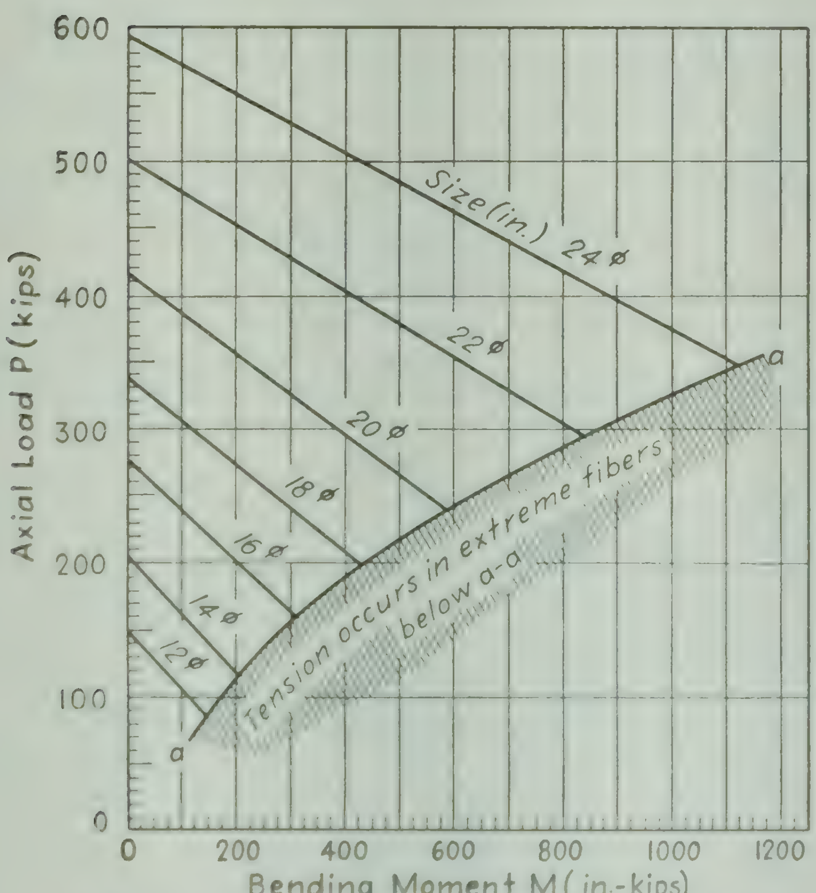
Assume that $p_g = 0.02$ will be satisfactory. The adjusted values for P and M for use in Fig. 7-4(*a*) are

$$P = 250 \times 1.15 = 288 \text{ kips}$$
$$M = 450 \times 1.15 = 520 \text{ in.-k}$$

From the diagram, the intersection of these values lies about one-half the distance between the lines for columns 20 and 22 in. square. A 21-in. square will be assumed. Since the height divided by the depth of this column is $12.5 \times {}^{12}\!/_{21} = 7.1$, which is less than the value of 10 allowed for short columns, no reduction need be made for



(a) Spiral Columns: $f_s = 16{,}000$, $f_c = 3{,}000$, $P_g = 0.02$



(b) Spiral Columns: $f_s = 16{,}000$, $f_c' = 3{,}000$, $P_g = 0.04$

FIG. 7-5. Diagrams for estimating trial sizes of round spirally reinforced columns with bending and compression.

slenderness. If the reverse is true, compute $(1.3 - 0.03h/d)$ from Eq. (6-11), then divide the adjusted load P (but not M) by this factor to find the trial value. For example, if $h/d = 12$ for this case,

$$1.3 - 0.03 \times 12 = 0.94$$

and the trial magnitude of P for use in Fig. 7-4(a) would be

$$P = \frac{288}{0.94} = 306 \text{ kips}$$

With $P = 306$ kips and $M = 520$ in.-k, Fig. 7-4(a) would indicate that the trial section should be 22 in. square.

The 21-in. column previously assumed will be analyzed for purposes of illustration, using $n = 12$ and $p_g = 0.02$ approx.

As a first step, select the reinforcement to be used. If $p_g = 0.02$,

$$A_s = 21^2 \times 0.02 = 8.8 \text{ in.}^2$$

Use 12 No. 8 bars, giving $A_s = 9.48$ in.^{2}. These can be used satisfactorily in a 21-in. column. Allow 2 in. for cover. Then

$$p_g = \frac{9.48}{21 \times 21} = 0.0215$$

$$A_t = 21 \times 21\,[1 + (12 - 1)0.0215] = 546 \text{ in.}^2$$

$$I_t = I_c + I_s = \frac{D^4}{12} + \frac{(n-1)}{6} A_s D_2^2$$

$$I_t = \frac{21^4}{12} + \frac{(12-1)}{6} \times 9.48 \times (21 - 6)^2 = 20{,}110 \text{ in.}^4$$

where D_2 is the depth D minus the following:

$$2 \times 2 \text{ in. of cover} + 2 \times \tfrac{1}{2} \text{ in. ties (assumed)} + 2 \times \text{radius of 1-in. bars}$$

$$R^2 = \frac{I_t}{A_t} = \frac{20{,}110}{546} = 36.8 \text{ in.}^2$$

$$e = \frac{M}{P} = \frac{450}{250} = 1.8 \text{ in.}$$

using the real values, not the adjusted magnitudes of M and P that were employed in using Fig. 7-4(a).

$$\frac{ec}{R^2} = \frac{1.8 \times 10.5}{36.8} = 0.51$$

From Eq. (7-2),

$$f_c = \frac{250{,}000}{546}(1 + 0.51) = 692 \text{ psi}$$

Equation (7-5a) shows that

$$f_a = \frac{0.18 \times 2{,}500 + 0.8 \times 16{,}000 \times 0.0215}{1 + (12 - 1)0.0215} = 586 \text{ psi}$$

$$C = \frac{f_a}{0.45f'_c} = \frac{586}{0.45 \times 2{,}500} = 0.52$$

Equation (7-4) gives the maximum allowable f_c as

$$f_c = 586\left(\frac{1 + 0.51}{1 + 0.52 \times 0.51}\right) = 586\left(\frac{1.51}{1.265}\right) = 700 \text{ psi}$$

which is slightly above the 692 psi found from Eq. (7-2). This is satisfactory.

Example 7-3. Determine the approximate size of a short circular spirally reinforced column to withstand a direct load of $P = 320$ kips, a bending moment M_1 about one axis $= 200$ in.-k, and a bending moment M_2 about an axis perpendicular thereto $= 300$ in.-k. Assume that there is to be no tension on the section, that $f_c' = 3{,}750$ psi, and that $p_g = 0.04$.

Since the section is symmetrical about any axis, the resultant of the two bending moments may be computed and used directly. Therefore,

$$M = \sqrt{M_1^2 + M_2^2} = \sqrt{200^2 + 300^2} = 360 \text{ in.-k}$$

The adjusted trial values for P and M for use with 3,750-lb concrete are

$$P = 0.8 \times 320 = 256 \text{ kips}$$
$$M = 0.8 \times 360 = 288 \text{ in.-k}$$

Using these in Fig. 7-5(b) for $p_g = 0.04$, find a trial diameter of approximately 19 in.

This column can now be checked in a manner similar to that used in Example 7-1. If the analysis shows that the column is too small or too large, a correction can be made easily since the correction generally does not have to be very great. On the other hand, as stated previously, it may be found that it is desirable to reduce p_g below 0.04 in order to have less congestion of reinforcing bars.

Example 7-4. Determine the approximate size of a square tied column with a 22-ft clear height if it supports an axial load of 300 kips that has an eccentricity of 1 in. about one rectangular axis and an eccentricity of 2 in. about the other rectangular axis. The reinforcement is equally distributed on all four sides. Assume $f_c' = 3{,}750$ psi, $n = 8$, and $p = 0.03$ approx. Use $1\frac{1}{2}$ in. of cover.

The first trial values of P and M, in order to use Figs. 7-4(a) and (b) for 3,750-lb concrete, are

$$P = 0.8 \times 300 = 240 \text{ kips}$$
$$M_1 = 0.8 \times 300 \times 1 = 240 \text{ in.-k}$$
$$M_2 = 0.8 \times 300 \times 2 = 480 \text{ in.-k}$$

Since the effects of these moments will both add to the compressive stress at one corner of the column, add these moments directly and call

$$M = 240 + 480 = 720 \text{ in.-k}$$

Then, with this adjusted M and with $P = 240$ kips, Fig. 7-4(a) indicates that a 22-in. column may be satisfactory as a short column with 2 per cent of steel. Figure 7-4(b) indicates that, with 4 per cent of reinforcement, the column might be 20 in. square. For $p_g = 0.03$, interpolate the size as 21 in. However, Eq. (6-11) indicates that a column 21 in. square and 22 ft long is $(1.3 - 0.03 \times 22 \times 12/21) = 0.92$ times as strong as if it were a short column. Therefore, revise the trial load $P = 240$ kips to $P = 240/0.92 = 260$ kips. However, in this case, this does not make enough change to affect the choice of the first trial size, but the real load of 300 kips will be correspondingly increased. Thus, for a trial analysis, call

$$P' = \frac{300}{0.92} = 326 \text{ kips}$$

Therefore, test a 21-in. square column for $P' = 326$ kips, $M_1 = 300$ in.-k, and $M_2 = 600$ in.-k. For $p_g = 0.03$, $A_s = 0.03 \times 21^2 = 13.2$ in.^{2}. Use 12 No. 9 bars. Then $A_s = 12$ in.^{2} and the real $p_g = 12/21 \times 21 = 0.0272$ and

$$(n-1)p_g = 7 \times 0.0272 = 0.19$$
$$A_t = 21^2(1 + 0.19) = 524 \text{ in.}^2$$
$$I_t = I_c + \frac{1}{6}(n-1)A_sD_2^2 = \frac{21^4}{12} + \frac{1}{6} \times 7 \times 12 \times 16^2 = 19{,}800 \text{ in.}^4$$

where $D_2 = 21 - 2 \times 2.5 = 16$ in., approx.

$$R_1^2 = R_2^2 = \frac{19{,}800}{524} = 37.8 \text{ in.}^2$$

Using the actual eccentricities,

$$\frac{e_1c_1}{R_1^2} = \frac{1 \times 10.5}{37.8} = 0.278 \quad \text{and} \quad \frac{e_2c_2}{R_2^2} = \frac{2 \times 10.5}{37.8} = 0.556$$

$$f_a = \frac{0.18 \times 3{,}750 + 0.8 \times 16{,}000 \times 0.0272}{1 + 7 \times 0.0272} = 860 \text{ psi}$$

$$C = \frac{860}{0.45 \times 3{,}750} = 0.51$$

f_c allowed from Eq. (7-4), using ec/R^2 for both axes, is

$$f_c = 860\left[\frac{1 + 0.278 + 0.556}{1 + 0.51(0.278 + 0.556)}\right] = 1{,}110 \text{ psi}$$

Equation (7-2) gives f_c for a short column. For present purposes, since the direct load is to be increased for the allowance caused by slenderness but the bending moments are not increased correspondingly, separate Eq. (7-2) into two parts as follows:

$$\max f_c = \frac{P'}{A_t} + \frac{P}{A_t}\left(\frac{e_1c_1}{R_1^2} + \frac{e_2c_2}{R_2^2}\right)$$

Then

$$\max f_c = \frac{326{,}000}{524} + \frac{300{,}000}{524}(0.278 + 0.556) = 1{,}100 \text{ psi}$$

which is almost exactly the maximum allowed. Although this is an approximation and may not perfectly allow for slenderness, the column will be accepted.

7-4. Combined compression and bending with resulting tension upon the section. The analysis of this type of problem will be based upon the following fundamental principles and assumptions:

1. Regardless of the existence of the direct compressive load, the intensities of the stresses upon a cross section of the member are assumed to vary as the ordinates to a straight line.

2. The summation of all the axial loads and the internal stresses upon the section must be zero in order to have equilibrium. Hence, $\Sigma V = 0$, or $\Sigma H = 0$.

3. The summation of the bending moments caused by the axial loads and by the internal stresses must be zero about any axis in order to have equilibrium. Thus $\Sigma M = 0$.

Figure 7-6 illustrates the ordinary stress distribution in a beam by the triangles ACO and BDO. The neutral axis for a simple beam is usually

above the middle of the beam's depth. However, when a longitudinal load P is applied, causing a compressive stress EA, the rectangle $EABG$ is assumed to be superimposed upon the previous figure so that the new diagram is made up of the triangles ECF and GDF. It is thus apparent that the neutral axis is shifted downward in the figure, and kd increases. Therefore, a column that carries bending and a beam that is subjected to a compressive force are the same thing in reality as far as analysis is concerned.

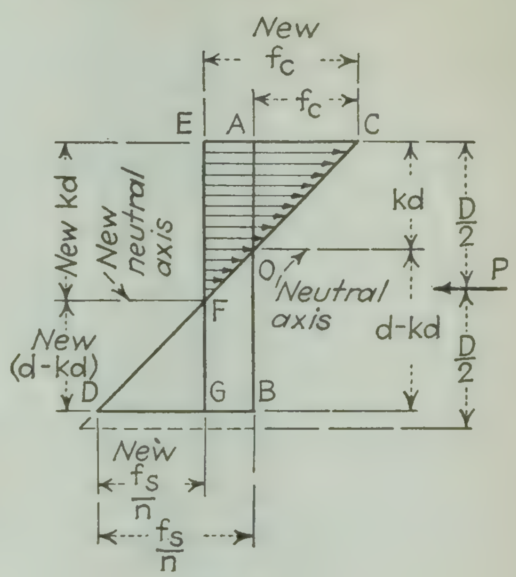


FIG. 7-6.

It is not always easy to determine by inspection whether or not there is tension upon any section of a column that is subjected to bending. If the factor ec/R^2 of Eq. (7-2) exceeds unity, then CK of Fig. 7-2 must be greater than twice the average stress CE. Also, min f_c from Eq. (7-3) will have a minus sign. In order to expedite the work of making a preliminary test, it is usually sufficient to neglect the transformed section of the steel in computing ec/R^2 unless the percentage of steel is large. For a rectangular column, the approximate value of

$$\frac{ec}{R^2} = \frac{eDA}{2I} = \frac{eD \times b \times D \times 12}{2 \times bD^3} = \frac{6e}{D} \tag{7-6}$$

For a circular column,

$$\frac{ec}{R^2} = \frac{eDA}{2I} = \frac{eD \times \pi D^2 \times 64}{2 \times 4 \times \pi D^4} = \frac{8e}{D} \tag{7-7}$$

Therefore, using these approximate formulas, it is possible to determine whether or not the member in question is likely to have tension upon part of its cross section.

Let $ABCD$ of Fig. 7-7(a) represent the cross section of a rectangular tied column that supports a centrally applied longitudinal compressive load P and a bending moment M that causes compression on edge BC and a tendency for tension at AD. The neutral axis is EF and the area under compression is $EBCF$. The five rods near BC are also compressed. They are represented by A'_s. The five bars near AD are in tension and are represented as A_s. The two bars near EF are useful in resisting compression, but their lever arm from EF is so short that they are not of much help in resisting bending.

Figure 7-7(*b*) shows a partial side elevation of this column with the internal resisting forces pictured as acting upon AB. Since $\Sigma V = 0$ for

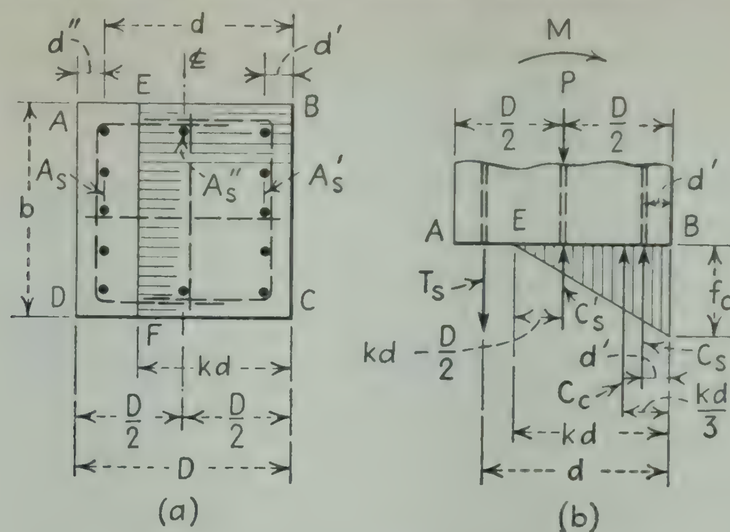


FIG. 7-7. A column with tension on part of the section.

equilibrium, the elastic theory will give the following expression if the external force P is equated to the sum of the internal resisting forces:

$$P = C_c + C_s + C_s' - T_s \tag{7-8}$$

$$C_c = \frac{f_c}{2} bkd \tag{7-9}$$

$$C_s = (n - 1)A_s'f_1 = (n - 1)A_s'\left[f_c \frac{(kd - d')}{kd}\right] \tag{7-10}$$

because

$$f_1 \text{ at the steel}: f_c \text{ at } B::kd - d':kd$$

Similarly,

$$C_s' = (n - 1)A_s''f_2 = (n - 1)A_s''\left[f_c \frac{(kd - D/2)}{kd}\right] \tag{7-11}$$

$$T_s = A_sf_s = nA_sf_3 = nA_s\left[f_c\left(\frac{d - kd}{kd}\right)\right] \tag{7-12}$$

because

$$f_3 \text{ at the steel}: f_c \text{ at } B::d - kd:kd$$

Therefore, an equation for P can be set up in terms of the compression in the concrete at B and various dimensions.

Furthermore, M can be expressed as P times some eccentricity e. Then, equating Pe to the sum of each of the internal resisting forces times its respective lever arm from the center of the column, one can obtain an expression for $\Sigma M = 0$ about the center line. Therefore,

$$Pe = C_c\left(\frac{D}{2} - \frac{kd}{3}\right) + C_s\left(\frac{D}{2} - d'\right) + C_s'\left(\frac{D}{2} - \frac{D}{2}\right) + T_s\left(d - \frac{D}{2}\right) \tag{7-13}$$

By substituting the values of C_c, C_s, C'_s, and T_s from Eqs. (7-9) to (7-12), inclusive, an expression is obtained for Pe in terms of f_c and various dimensions.

Now, if Eq. (7-13) is divided by Eq. (7-8), an equation is found for e in terms of the unknown dimension kd as follows:

$$e = \frac{\frac{b}{2}(kd)^2\left(\frac{D}{2} - \frac{kd}{3}\right) + (n-1)A'_s(kd - d')\left(\frac{D}{2} - d'\right) + nA_s(d - dk)[d - (D/2)]}{(b/2)(kd)^2 + (n-1)A'_s(kd - d') + (n-1)A''_s\{[kd - (D/2)]/kd\} - nA_s(d - kd)} \tag{7-14}$$

This can be solved by trial for kd, or by means of any approved method for the solution of cubic equations. The value of kd thus found can be substituted in Eqs. (7-9), (7-10), (7-11), and (7-12). Then these can be substituted in Eq. (7-8), from which f_c can be computed to see if the column is safe for P and M.

Obviously, the preceding calculations involve a great deal of labor. If there are more bars along faces AB and DC of Fig. 7-7(a) than the central ones shown, there will be still more terms in the equations. However, these are seldom of great importance.

7-5. Approximate method for the design of rectangular columns with tension on part of the section. The previously explained method of analysis of members having combined direct loads and bending moments requires considerable labor in its application. As a practical matter, the actual conditions do not justify such a degree of accuracy in many cases because there are so many unknown factors or empirical assumptions in the initial determination of the loads and their distribution that the subsequent calculations need not be more exact than the fundamental data upon which they are based. It is therefore desirable to use a reasonably accurate, approximate method of analysis which is easy to apply. Such a method will now be explained. It was originally developed and used by Frederick C. Lowy[1] and Erick M. Black.[2] It has been tested in many cases in the office of The Port of New York Authority.

Consider the distribution of the stresses in a column for an eccentrically applied load. Figure 7-8(a) shows a column with the load P which has an eccentricity of sufficient magnitude to make the stress upon the section vary from zero at A to a maximum at B. If there were no steel, e would equal $D/6$. In other words,

$$e = r_k = \frac{D}{6} = \frac{bD^2}{6} \div bD = \frac{\text{section modulus}}{\text{area}}$$

[1] Formerly designer, The Port of New York Authority, New York.

[2] Formerly assistant designing engineer, City Railway, Newark, N.J.

where r_k will be called the *kern radius*. Similarly, for the reinforced-concrete member, let

$$r_k = \frac{\text{section modulus of the entire transformed section}}{\text{area of the entire transformed section}}$$

Let I'_c equal the moment of inertia of the complete section including $(n - 1)$ times the area of the steel. The whole section must be used in this calculation. Therefore,

$$r_k = \frac{I'_c}{0.5D[A_g + (n - 1)A_{sg}]} \tag{7-15}$$

where A_{sg} = the total area of steel in the entire cross section. Thus, the value of r_k may be considered as the limit of the central portion, or kern

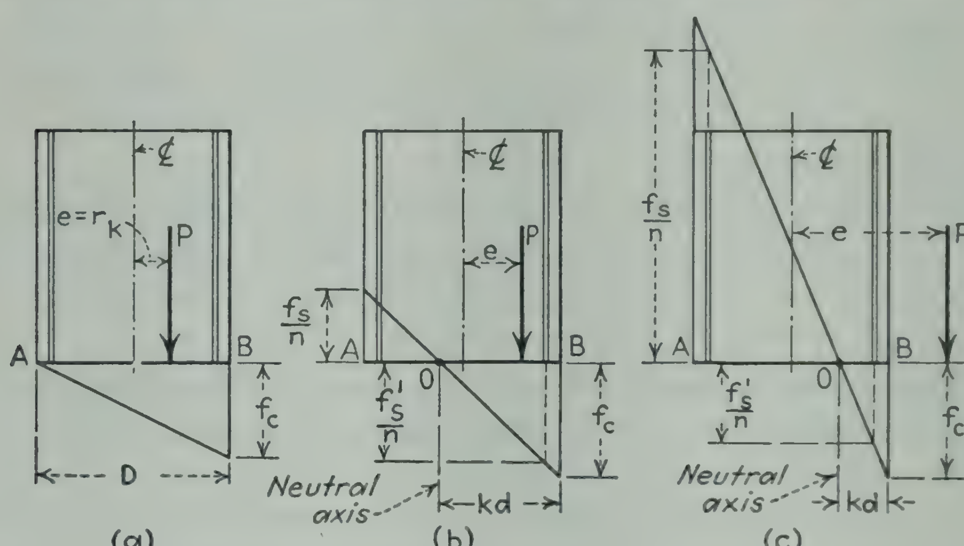


FIG. 7-8.

(core), of the member. If the load acts within this kern, there will be no tension upon the section; if it is applied at the limit of r_k, the stress at A will be zero, but at B, f_c = twice the average stress; if e exceeds r_k, the neutral axis O will move from A toward B, as shown in Figs. 7-8(b) and (c), causing tension near A and making f_c greater than twice the average stress in direct compression. However, when the magnitude of e becomes large compared with the depth of the section D, the member becomes primarily a beam, the direct stress being insignificant. Usually, if $e = 2D$, the direct load may be neglected; and for such a case, kd will approximate $D/3$.

Considering the fact that r_k is about equal to $D/6$, it is seen that the kern radius comes approximately at the neutral axis of the section when the latter is acting as an ordinary beam. Therefore, it will be assumed that the stress diagram for any position of P, where e exceeds r_k, is made up of one part which is the triangle of Fig. 7-9(b), where P is assumed to be at r_k, plus a second diagram as shown in Fig. 7-9(c), the latter being

caused by the load P acting, with a lever arm of $e - r_k$, upon the section as a pure beam. The resultant stress diagram is shown in Fig. 7-9(d), being indicated in the picture as bounded by a broken line merely for the reasons which will be explained later.

In the practical design of columns, it will be found that the stress in the concrete f_c is the critical one. Since f'_s, the compressive stress in the longitudinal reinforcement, is only n times the stress in the concrete at the same point, it is usually low except for the indefinite increase which is caused by plastic flow. Also, f_s is usually unimportant unless the bending moment is very severe Therefore, it is satisfactory to solve for f_c alone in most cases, it being equal to $f_1 + f_2$ (Fig. 7-9).

In any such problem, the procedure is simply as follows:

(1) Assume a section to be analyzed

(2) Find $r_k = \dfrac{I'_c}{0.5D[A_g + (n-1)A_{sg}]}$ (7-16)

(3) Find $f_1 = \dfrac{2P}{A_g + (n-1)A_{sg}}$ (7-17)

(4) Find $f_2 = \dfrac{P(e - r_k)kd}{I_c}$ (7-18)

(5) $f_c = f_1 + f_2$ (7-19)

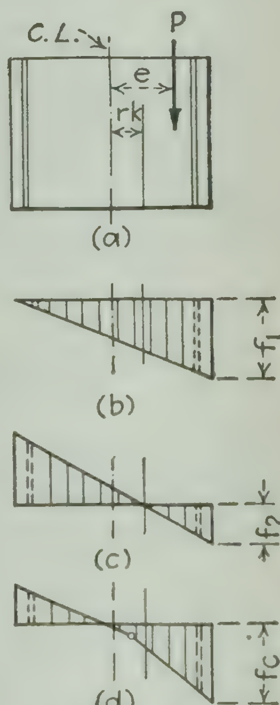


FIG. 7-9.

It must be noted that I_c in step (4) is the moment of inertia of the transformed section of the member as an ordinary beam, allowing no tension upon the concrete.

This method still appears to require a great deal of work. However, in any structure that has a large number of columns, it will be found advisable to simplify the work by using a moderate variety of sections. Therefore, the tentative members can be chosen; their areas, section moduli, and kern radii can be computed; and then the members can be used for any combination of P and M or P and Pe for which they are safe. Having the properties of the sections, the labor of finding f_1 and f_2 is very small.

If it is necessary to find the stresses in the steel, it is not advisable to use the results of this approximate method because the errors which are introduced by the approximations often multiply considerably in any attempt to use them in Eqs. (7-10) and (7-12). That is why Fig. 7-9(d) is shown with a broken line so as to warn the reader not to attempt such a calculation.

For important structures it is advisable to design the members by

this approximate method and then to check them by more exact means.

As a start in the solution of a design problem, the methods of Art. 7-3 can be used to find a trial member as though the column could resist tension in the concrete or as though it had none. Then a somewhat larger size can be selected and analyzed more carefully.

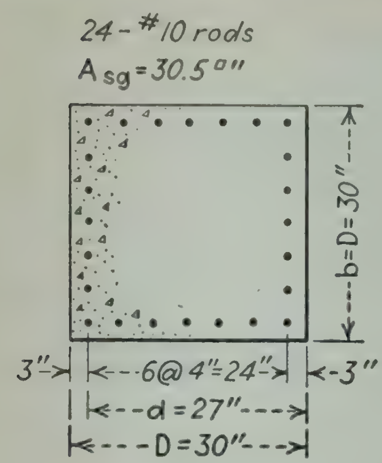


FIG. 7-10.

Example 7-5. Find f_c for the column shown in Fig. 7-10 if $P = 250{,}000$ lb, $e = 11$ in., and $n = 8$.

The first step is the calculation of I'_c:

$$I'_c = \frac{bD^3}{12} + (n-1)I_s$$

$$I'_c = \frac{30 \times 30^3}{12} + (8-1)(14 \times 1.27 \times 12^2 + 4 \times 1.27 \times 8^2 + 4 \times 1.27 \times 4^2) = 88{,}260 \text{ in.}^4$$

Then, from Eq. (7-16),

$$r_k = \frac{I'_c}{0.5D[A_g + (n-1)A_{sg}]} = \frac{88{,}260}{15[900 + 7 \times 30.5]} = 5.28 \text{ in.}$$

The next step is the calculation of I_c. For a square member with the rods symmetrically placed, neglecting the fact that the area of the steel in compression should be multiplied by $(n - 1)$ rather than by n, and taking moments about the neutral axis,

$$\frac{D(kd)^2}{2} = nA_{sg}\left(\frac{D}{2} - kd\right)$$

$$kd = -\frac{nA_{sg}}{D} + \sqrt{\frac{(nA_{sg})^2}{D^2} + nA_{sg}} \qquad (7\text{-}20)$$

$$kd = -\frac{8 \times 30.5}{30} + \sqrt{\frac{(8 \times 30.5)^2}{30^2} + 8 \times 30.5} = 9.5 \text{ in.}$$

$$I_c = \frac{30 \times 9.5^3}{3} + 8(14 \times 1.27 \times 12^2 + 4 \times 1.27 \times 8^2 + 4 \times 1.27 \times 4^2) + 8 \times 30.5(15 - 9.5)^2 = 39{,}700 \text{ in.}^4$$

$$f_1 = \frac{2P}{A_g + (n-1)A_{sg}} = \frac{2 \times 250{,}000}{900 + 7 \times 30.5} = 449 \text{ psi}$$

$$f_2 = \frac{P(e - r_k)kd}{I_c} = \frac{250{,}000(11 - 5.28)9.5}{39{,}700} = 342 \text{ psi}$$

$$f_c = f_1 + f_2 = 449 + 342 = 791 \text{ psi}$$

Example 7-6. Assume the same column as that of Fig. 7-10, and test it for the following combinations of direct load and bending, assuming $n = 8$ and the allowable $f_c = 800$ psi.

(1) $P = 200{,}000$ lb, $M = 2{,}800{,}000$ in.-lb
(2) $P = 300{,}000$ lb, $M = 2{,}000{,}000$ in.-lb
(3) $P = 175{,}000$ lb, $M = 3{,}000{,}000$ in.-lb
(4) $P = 350{,}000$ lb, $M = 2{,}400{,}000$ in.-lb

For all these cases $[A_g + (n-1)A_{sg}] = 1{,}114$ in.², $kd = 9.5$ in., $I_c = 39{,}700$ in.⁴, and $r_k = 5.28$ in. as for Example 7-5.

$$(1)\quad e = \frac{2{,}800{,}000}{200{,}000} = 14 \text{ in.}$$

$$f_1 = \frac{2 \times 200{,}000}{1{,}114} = 359 \text{ psi}$$

$$f_2 = \frac{200{,}000(14 - 5.28)9.5}{39{,}700} = 417 \text{ psi}$$

$$f_c = 359 + 417 = 776 \text{ psi}$$

$$(2)\quad e = \frac{2{,}000}{300} = 6.67 \text{ in.}$$

$$f_1 = \frac{2 \times 300{,}000}{1{,}114} = 539 \text{ psi}$$

$$f_2 = \frac{300{,}000(6.67 - 5.28)9.5}{39{,}700} = 100 \text{ psi}$$

$$f_c = 539 + 100 = 639 \text{ psi} \qquad \text{(too low for economy)}$$

$$(3)\quad e = \frac{3{,}000}{175} = 17.1 \text{ in.}$$

$$f_1 = \frac{2 \times 175{,}000}{1{,}114} = 314 \text{ psi}$$

$$f_2 = \frac{175{,}000(17.1 - 5.28)9.5}{39{,}700} = 494 \text{ psi}$$

$$f_c = 314 + 494 = 808 \text{ psi}$$

$$(4)\quad e = \frac{2{,}400}{350} = 6.86 \text{ in.}$$

$$f_1 = \frac{2 \times 350{,}000}{1{,}114} = 628 \text{ psi}$$

$$f_2 = \frac{350{,}000(6.86 - 5.28)9.5}{39{,}700} = 132 \text{ psi}$$

$$f_c = 628 + 132 = 760 \text{ psi}$$

These calculations illustrate the ease with which any given member can be tested for its stresses when it is subjected to various combinations of loading.

7-6. Analysis by ultimate-load theory.

Figure 7-11 pictures a rectangular member that is subjected to a load P with an eccentricity e that causes compression and bending in it. Assume a symmetrical concrete section with bars at the top and at the bottom but with steel areas A_s' and A_s not necessarily equal.

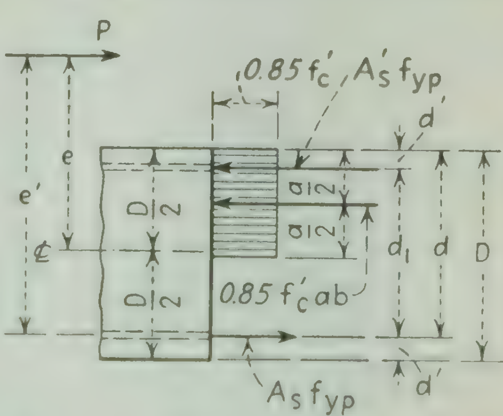


FIG. 7-11. Stress diagram assumed for balanced design.

If the resistance of the member were controlled by compression alone under flexural action, Eq. (2-34) states that

$$M = A_s' f_{yp}\, d_1 + \tfrac{1}{3} b d^2 f_c'$$

Then, according to Whitney's method, if one considers this to be the limit of the bending resistance of the compressive strength of the member, no matter what the cause of the compression may be, the internal resistances of the materials may be pictured as in Fig. 7-11.

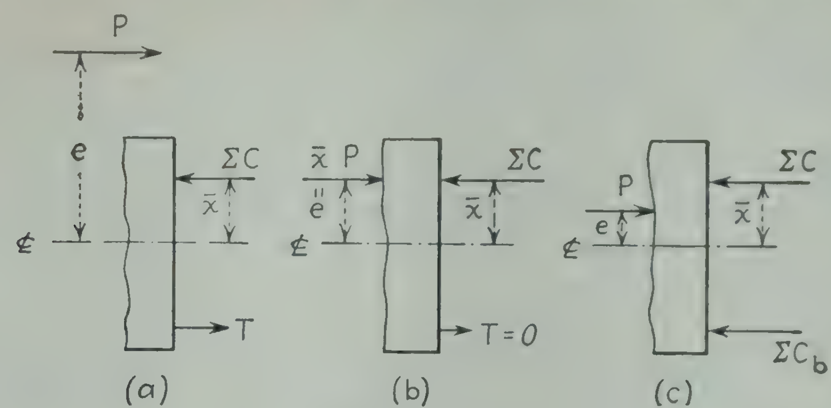


FIG. 7-12.

Taking moments about the tensile reinforcement, Fig. 7-11 gives

$$M = Pe' = P\left[e + \left(d - \frac{D}{2}\right)\right] \tag{7-21}$$

Therefore,

$$P\left(e + d - \frac{D}{2}\right) = A'_s f_{yp} d_1 + \frac{1}{3} bd^2 f'_c \tag{7-22}$$

If the cover of the bars d' in Fig. 7-11 is the same for top and bottom,

$$d = \frac{d_1 + D}{2} \tag{7-23}$$

Substituting Eq. (7-23) in the left side of Eq. (7-22) gives

$$P = \frac{2A'_s f_{yp} d_1}{2e + d_1} + \frac{2bd^2 f'_c}{3(2e + d_1)}$$

Then, dividing the last term by d^2 and multiplying by D,

$$P = \frac{A'_s f_{yp} d_1}{e + (d_1/2)} + \frac{bDf'_c}{(3eD/d^2) + [(6Dd - 3D^2)/2d^2]} \tag{7-24}$$

Equation (7-24) applies when the eccentricities are large. For example, a qualitative picture of the situation may be obtained by referring to Fig. 7-12. Sketch (*a*) shows a case where e exceeds $\bar{x}$, the distance from the center of the member to the resultant of the assumed compressive resistances, ΣC Here T must be a tensile force in order to stop clockwise rotation. In Sketch (*b*), e is assumed to equal $\bar{x}$. Then T will be zero for equilibrium. In (*c*), e is less than $\bar{x}$; hence a compressive

force, pictured by ΣC_b, is needed to stop counterclockwise rotation. Since Eq. (7-24) is based upon compression in the top of Fig. 7-11 and tension in the bottom steel, it cannot apply to the condition of Figs. 7-12 (*b*) and (*c*).

For small eccentricities, Whitney has modified Eq. (7-24) to make it reasonably applicable, and the results are reported to agree reasonably well with tests. His procedure is the following:

1. Modify Eq. (7-24) so that, as e approaches zero, P approaches the proper value for the member as an axially loaded column.

2. If $A_s = A'_s$, as for a truly symmetrical member, the resistance of the steel becomes $2\,A'_s f_{yp}$.

3. The total strength of the concrete, using $0.85f'_c$ as a maximum, and not deducting the area occupied by the bars, becomes $0.85bDf'_c$ when $e = 0$.

4. With $e = 0$, $3eD/d^2$ in Eq. (7-24) $= 0$ and, to obtain the coefficient 0.85 for the last term of Eq. (7-24), the remainder of the denominator must be 1.178 (to give $1/1.178 = 0.85$). Therefore, let

$$P = \frac{2A'_s f_{yp}}{(2e/d_1) + 1} + \frac{bdf'_c}{(3De/d^2) + 1.178} \tag{7-25}$$

express the equation for P for eccentricities less than $\bar{x}$ of Fig. 7-12. This, of course, assumes that the load is controlled by the compressive strength.

In order to have Eq. (7-25) reduce to the standard formula

$$P = 0.225A_c f'_c + 0.4A_s f_{yp} \qquad \text{or} \qquad P = 0.225A_c f'_c + A_s f_s$$

for axially loaded columns with a safety factor of $2\frac{1}{2}$ and $e = 0$, this equation becomes

$$P = \frac{2A'_s f_{yp}}{(2e/d_1) + 1} + \frac{A_c f'_c}{(3De/d^2) + 1.775} \tag{7-26}$$

If Eq. (7-26) is multiplied by 0.8, it reduces to the formula for axially loaded tied columns ($e = 0$).

For a square column with spiral reinforcement like that of Fig. 7-14(*a*), Whitney finds that Eq. (7-26) yields results that agree fairly well with tests if one-half of the total steel A_{st} is assumed to be effective on each side of the section, and if $0.67\,d_1$ is substituted for d_1. Therefore, based upon compressive strength, and counting upon all the concrete,

$$P = \frac{A_{st} f_{yp}}{(3e/d_1) + 1} + \frac{0.85A_c f'_c}{[10.2De/(D + 0.67d_1)^2] + 1.51} \tag{7-27}$$

For a round spirally reinforced column like Fig. 7-14(*b*), substitute $0.8D$ for D in Eq. (7-27). Then

$$P = \frac{A_{st}f_{yp}}{(3e/d_1) + 1} + \frac{0.85A_cf'_c}{[8.16De/(0.8D + 0.67d_1)^2] + 1.51} \tag{7-28}$$

Equations (7-26), (7-27), and (7-28) are sufficient for the present purposes of this text.

When e is very large so that bending is predominant (perhaps when e exceeds $1.5D$), the effect of the direct compression caused by P is small.

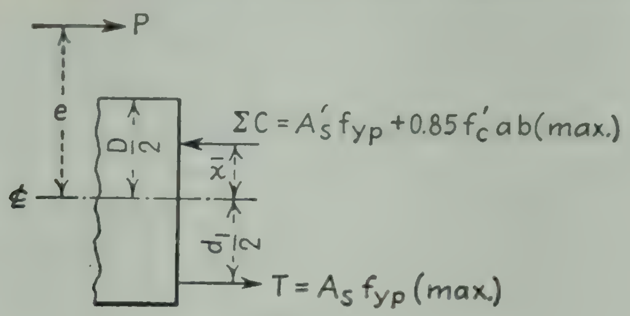


FIG. 7-13.

The member can then be analyzed as a flexural member when the tensile steel limits the strength.

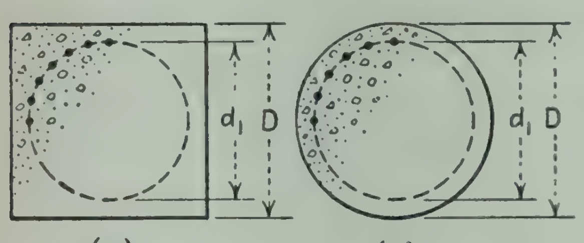


FIG. 7-14.

For moderate eccentricities and members that are considerably underreinforced, it may be satisfactory to approximate the tensile force T as follows, referring to Fig. 7-13:

1. Compute or approximate the center of gravity of the compressive resistances, called $\bar{x}$, as though the member were subjected to flexure alone and fully stressed in compression—a balanced design. Therefore, taking moments about the tensile steel,

$$\bar{x} = \frac{A'_sf_{yp}(d_1/2) + 0.85f'_cab(D/2 - a/2)}{A'_sf_{yp} + 0.85f'_cab} \tag{7-29}$$

In this equation, let $a = 0.537d$.

2. Then $P(e - \bar{x}) = T(d_1/2 + \bar{x})$. (7-30)

3. Solve for T and make $T = A_sf_{yp}$.

4. Check to see that ΣC does not exceed $A'_sf_{yp} + 0.85f'_cab$, where $a = 0.537d$.

Beams analyzed by the ultimate-load theory in Chap. 2 used load factors of approximately 1.5 for dead load and 2.0 for live load. Even with axially loaded columns the Code requires a safety factor of $2\frac{1}{2}$ to 3, and properly so. Now, when bending is combined with axial loads, what should the load factors be? In other words, when does a beam with a longitudinal thrust become a column with bending added to

direct load? If a beam fails, the loss may not be irreparable. If a column fails, the structure will probably be wrecked. Therefore, until this question is studied much further, treat members with a heavy thrust as though they were columns, and use load factors appropriate thereto—$2\frac{1}{2}$ for spirally reinforced columns and 3 for tied columns. In doing so, use the same factors for both dead load and live load. When thrusts are relatively small, consider the member to be a beam. The safety factor may be reduced for special combinations of loads if the specifications permit it.

Example 7-7. Figure 7-15(a) pictures a 1-ft section through part of a rigid-frame bridge. Assume f'_c = 3,500 psi, f_{yp} = 40,000 psi, the compressive loads

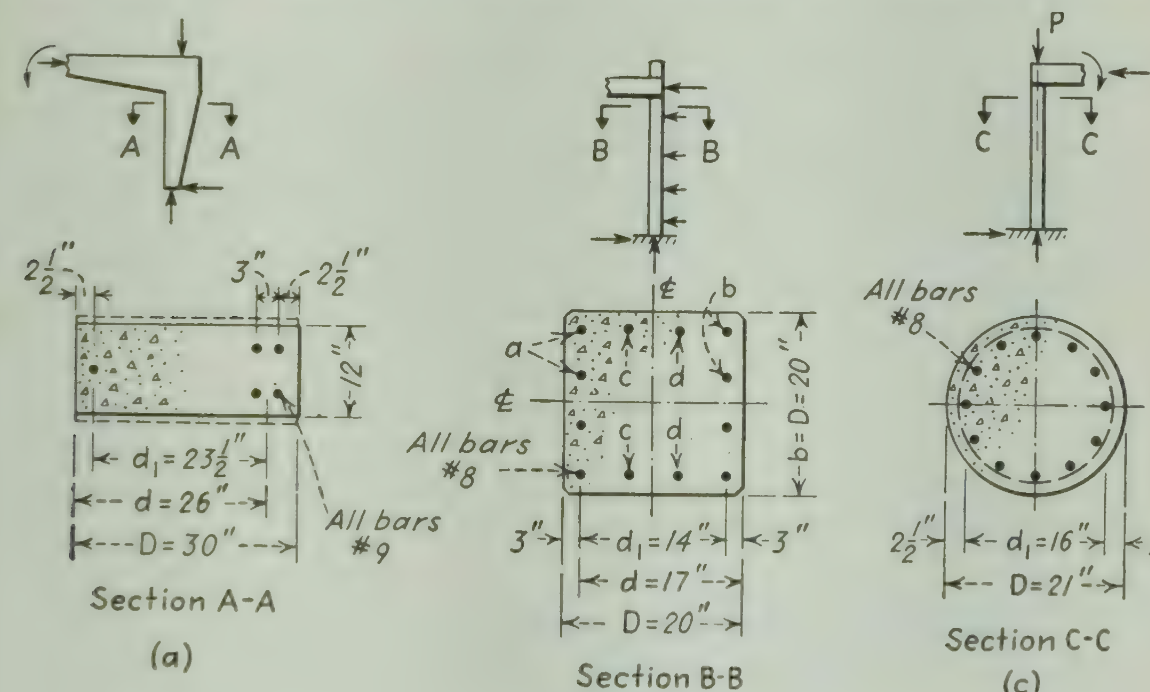


FIG. 7-15.

$P_{D.L.}$ = 9,000 lb and $P_{L.L.}$ = 5,000 lb, the bending moments at section A-A are $M_{D.L.}$ = 50,000 ft-lb and $M_{L.L.}$ = 30,000 ft-lb, and the tension is in the right-hand side. Use a load factor = 3 as for a tied column. Is the design satisfactory?

Loads for design:

$$P = (9{,}000 + 5{,}000)3 = 42{,}000 \text{ lb}$$
$$M = (50{,}000 + 30{,}000)3 = 240{,}000 \text{ ft-lb}$$
$$e = \frac{M}{P} = \frac{240{,}000 \times 12}{42{,}000} = 68 \text{ in.}$$

This shows that the eccentricity is so large compared with the stated limit of $1.5D$ that the member is primarily a beam, and the direct compression will not be important. In this case, if the ultimate direct load of 42,000 is divided by the assumed compression area ab

$$f_c = \frac{42{,}000}{(0.537 \times 26)12} = 250 \text{ psi}$$

This is very small compared with the 3,500 psi that the concrete can stand. Furthermore, the member is obviously underreinforced, as shown by the fact that

$$p = \frac{A_s}{bd} = \frac{4}{12 \times 26} = 0.0128$$

whereas the critical percentage from Eq. (2-27) is far above this. Therefore, the member will not be injured by this small pressure.

Example 7-8. The tied column in Fig. 7-15(*b*) is to support a direct load of $P_{\text{D.L.}} = 120$ kips and $P_{\text{L.L.}} = 90$ kips. Wind loads and other lateral forces cause a bending moment of 75 ft-k near the ends of the column. Assume $f'_c = 3{,}000$ psi, $f_{yp} = 36{,}000$ psi, and the load factor = 3. Is the column safe?

For design:

$$\begin{aligned} P &= 120 \times 3 + 90 \times 3 = 630 \text{ kips} \\ M &= 75 \times 3 = 225 \text{ ft-k} \\ e &= \frac{M}{P} = \frac{225 \times 12}{630} = 4.3 \text{ in.} \end{aligned}$$

This is a case where the steel is spread around the periphery of the column. One approximation that can be made for the steel areas is the assumption that all of bars *a* of Fig. 7-15(*b*) are in tension, and all of *b* are in compression. Then assume that bars *c* are partially effective at the location of bars *a* on the basis of the ratio of their relative distance from the center line. Similarly, part of bars *d* may be considered to act at *b*. Thus,

$$A_s = A'_s = \left(4 \text{ bars} + 2 \times \frac{0.5}{1.5}\right) 0.79 = 3.8 \text{ in.}^2$$

The center of gravity $\bar{x}$ for the assumed compressive forces is found from Eq. (7-24). Thus, separating the terms for convenience,

$$\begin{aligned} A'_s f_{yp} &= 3.8 \times 36{,}000 = 137{,}000 \text{ lb} \\ 0.85 f'_c ab &= 0.85 \times 3{,}000 \times (0.537 \times 17)20 = 465{,}000 \text{ lb} \\ \bar{x} &= \frac{137{,}000 \times 7 + 465{,}000(10 - 4.5)}{137{,}000 + 465{,}000} = 5.8 \text{ in.} \end{aligned}$$

Since $e = 4.3$ in., Eq. (7-26) can be used if the answer is multiplied by 0.8. Therefore,

$$\begin{aligned} \frac{P}{0.8} &= \frac{2A'_s f_{yp}}{(2e/d_1) + 1} + \frac{bDf'_c}{(3De/d^2) + 1.775} \\ \frac{P}{0.8} &= \frac{2 \times 3.8 \times 36{,}000}{(2 \times 4.3/14) + 1} + \frac{20 \times 20 \times 3{,}000}{(3 \times 20 \times 4.3/17^2) + 1.775} = 619{,}000 \text{ lb} \end{aligned}$$

$P = 495{,}000$ lb ultimate or $495{,}000/3 = 165{,}000$ lb as a safe load. This is less than the $120{,}000 + 90{,}000 = 210{,}000$ lb required. Hence the column is not safe.

Example 7-9. The column in Fig. 7-15(*c*) has a direct load $P_{\text{D.L.}} = 100{,}000$ lb, and $P_{\text{L.L.}} = 50{,}000$ lb acting with an eccentricity of 10 in. The safety factor (load factor) of $2\frac{1}{2}$ is to be used for this spirally reinforced column. Assume $f'_c = 4{,}000$ psi and $f_{yp} = 40{,}000$ psi. Is the column safe?

$$\text{Ultimate } P = 150{,}000 \times 2.5 = 375{,}000 \text{ lb}$$

From Eq. (7-28) and the data in Sketch (*c*), find

$$P = \frac{A_{st}f_{yp}}{(3e/d_1) + 1} = \frac{9.48 \times 40{,}000}{(3 \times 10/16) + 1} = 132{,}000 \text{ lb for steel}$$

$$+ \frac{0.85A_cf'_c}{[8.16De/(0.8D + 0.67d_1)^2] + 1.51}$$

$$= \frac{0.85 \times 345 \times 4{,}000}{8.16 \times 21 \times 10/(0.8 \times 21 + 0.67 \times 16)^2 + 1.51} = 311{,}000 \text{ lb for concrete}$$

or a total of 443,000 lb. This exceeds the 375,000 lb required, and is satisfactory.

Practice Problems

7-1. Assume a short circular column like that of Fig. 7-3(*a*). Let $D = 22$ in., $D_1 = 19$ in., $D_2 = 17.5$ in., the rods = 12 equally spaced No. 8 ($A_s = 9.48$ in.^{2}), and

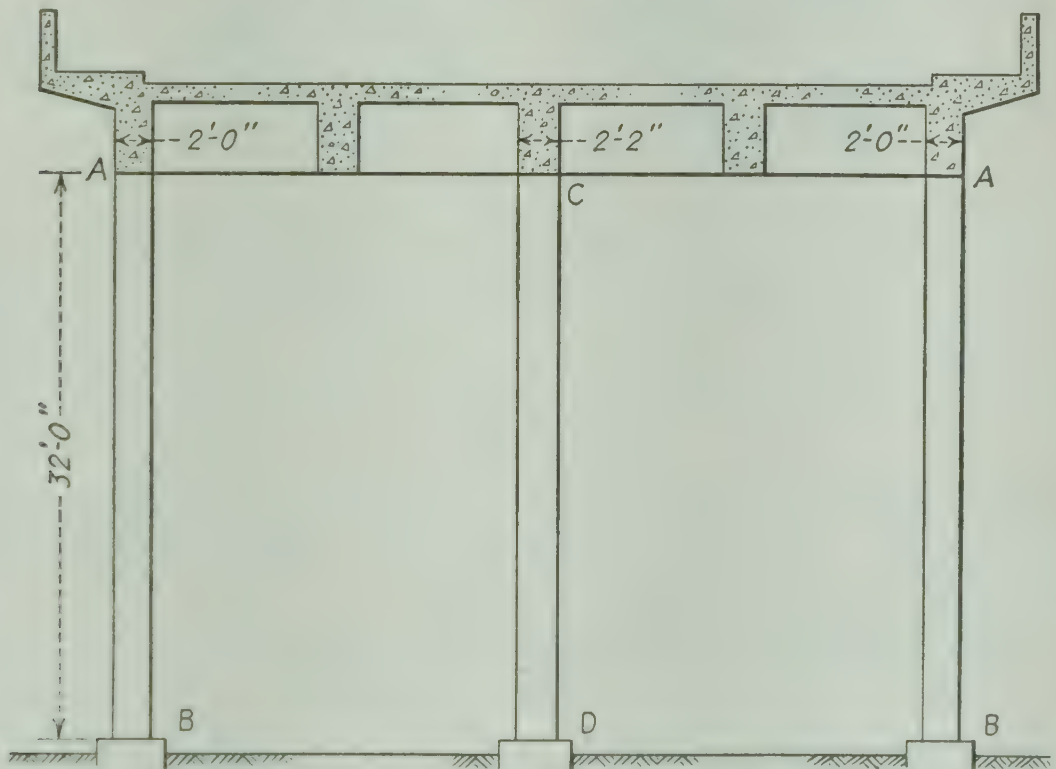


FIG. 7-16. Cross section of a viaduct.

$n = 10$. Find the stresses in the steel and the concrete for a centrally applied load of 200,000 lb and a bending moment of 250,000 in.-lb about the axis *X-X*.

Discussion. Follow the same procedure as in Example 7-1.

7-2. Recompute Prob. 7-1 with all data remaining the same except for the use of 12 No. 6 rods.

7-3. Compute the maximum compressive stress f_c in a square tied column for the following data: $f'_c = 4{,}000$ psi, $n = 8$, $f_s = 18{,}000$ psi, rods = 16 No. 10, $D = 24$ in., core width = 20 in., $P = 400$ kips, $M = 30$ ft-k. Is the column safe?

Ans. $f_c = 680$ psi, 1,060 psi allowed. Safe.

7-4. Compute the maximum compressive stress f_c in a circular spirally reinforced column for the following data: $f'_c = 3{,}000$ psi, $n = 10$, $f_s = 16{,}000$ psi, rods = 15 No. 9, $D = 24$ in., core diameter = 21 in., $P = 350$ kips, $M = 50$ ft-k. Is the column safe?

7-5. A tied column is to be 20 ft high, 24 in. wide, and 20 in. thick with a cover of 2 in. It is to support a load of 250 kips and a bending moment of 50 ft-k acting in a

plane parallel to the 24-in. width. Determine the reinforcement to be used provided $f'_c = 4{,}000$ psi and $f_s = 18{,}000$ psi.

Discussion. First increase the 250 kips to allow for h/d. There is to be no tension on the section.

7-6. Design the rectangular tied columns AB to support the viaduct shown in Fig. 7-16. The reactions at A are 155 kips. Include the estimated weights of the columns themselves. The transverse bending moments at the tops and bottoms of the columns are assumed to be 65 ft-k and the longitudinal bending moments at these points are 50 ft-k. Assume $f'_c = 3{,}500$ psi, $f_s = 16{,}000$ psi, cover over the ties = 2 in., and no tension is to be on the section. Assume that the width is to be the same as that of the longitudinal girders.

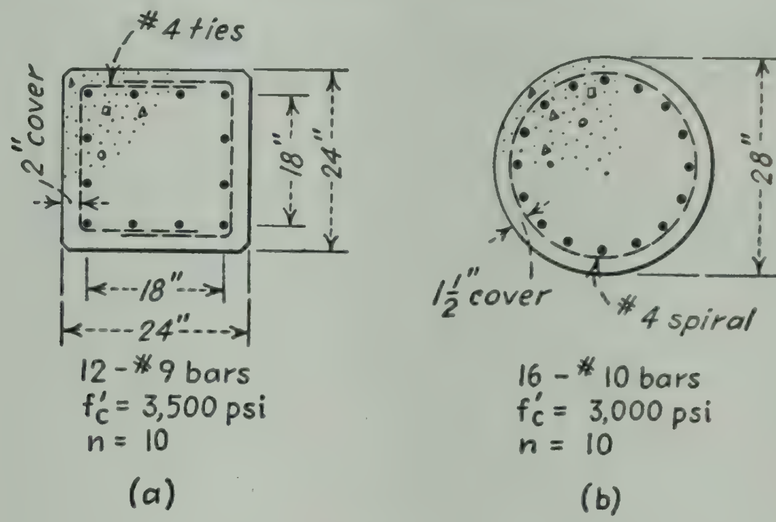


FIG. 7-17.

Discussion. Find a trial size by extrapolation from Fig. 7-4 as though no tension exists on the section. Then increase the size somewhat. Check by the approximate method of Art. 7-5. Find two values of f_2, one for each axis, then add these to f_1. Assume that the maximum allowable $f_c = 900$ psi.

7-7. Design the central column CD for the viaduct of Fig. 7-16 for the same conditions as given in Prob. 7-6 except that the load is to be 180 kips and the width is to be 26 in.

7-8. By the methods of Art. 7-5, compute the maximum stress f_c in the column of Fig. 7-17(*a*) for the following combinations of loading; assuming 900 psi to be the allowable maximum:

(*a*) $P = 300$ kips, $M = 120$ ft-k
(*b*) $P = 400$ kips, $M = 200$ ft-k
(*c*) $P = 320$ kips, $M = 280$ ft-k
(*d*) $P = 250$ kips, $M = 175$ ft-k

7-9. By the methods of Art. 7-5, compute the maximum stress f_c in the column of Fig. 7-17(*b*) for the following combinations of loading, assuming 1,000 psi to be the allowable maximum:

(*a*) $P = 500$ kips, $M = 200$ ft-k
(*b*) $P = 450$ kips, $M = 250$ ft-k
(*c*) $P = 320$ kips, $M = 150$ ft-k
(*d*) $P = 350$ kips, $M = 175$ ft-k

7-10. The section of Fig. 7-18(a) has a direct load of 200 kips and a bending moment of 60 ft-k. The safety factor is to be 3.0 for a tied column. f'_c = 3,500 psi, f_{yp} = 40,000 psi. Is the column safe?

7-11. Figure 7-18(b) shows a column that has a direct load of 230 kips and a bending moment of 250 ft-k. Assume f'_c = 4,000 psi, f_{yp} = 40,000 psi, and the load factor = 3.0. Is the column safe?

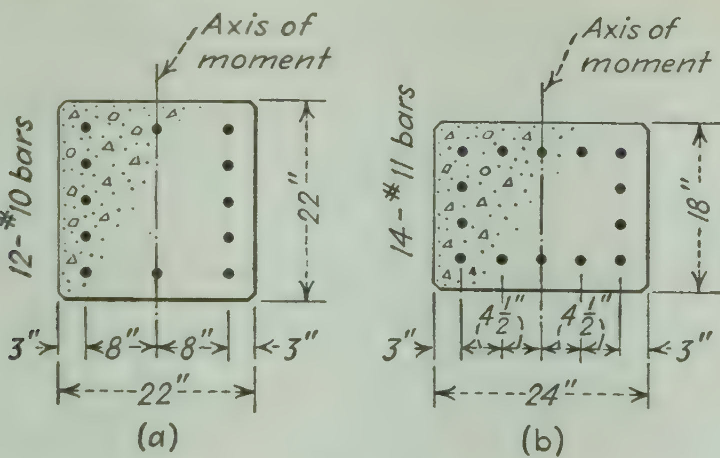


FIG. 7-18.

7-12. Design a short circular spirally reinforced column to support a load P = 200 kips and a moment M = 100 ft-k if f'_c = 3,000 psi, f_{yp} = 36,000 psi, and the safety factor = 2.5.

Discussion. From Fig. 13B of the Appendix, with p_g = about 4 per cent, select a column that is able to support a load that is considerably larger than the 200 kips required. Test it; then choose a new size if necessary, basing the selection upon the results of the first trial.

8

RETAINING WALLS

8-1. Introduction. Retaining walls are used to provide lateral support for a mass of earth or other material the top of which is at a higher elevation than the earth or rock in front of the wall, as shown in Fig. 8-1.

Gravity retaining walls such as that in Sketch (*a*) depend mostly upon their own weight for stability. They are usually low in height. They are expensive because of their inefficient use of materials; sometimes they can be cheapened by using cyclopean concrete—concrete in which fairly large rocks are buried.

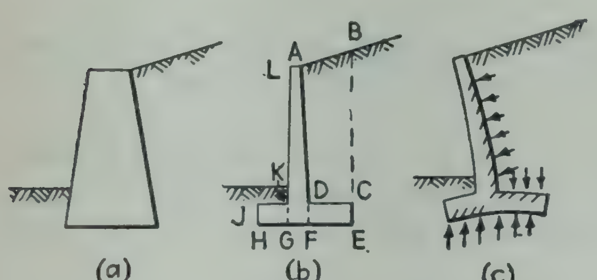


FIG. 8-1.

In contrast to them, Fig. 8-1(*b*) pictures an ordinary "cantilever" retaining wall. Part of its stability is obtained from the weight of the earth mass *ABCD*, but the wall's resistance to collapse depends upon the strength of its individual parts as cantilever beams. This action is pictured in Fig. 8-1(*c*).

Figure 8-2 shows some of this type of work as it looks in a large construction job. It is part of the Exit Plaza of the South Tube of the Lincoln Tunnel at New York City. In the background is part of a wall for which the concrete of the lower half has been placed and from which the forms have been stripped. Beside it is another portion for which the reinforcement has been placed and the forms are being built. The next part is a combined wall and pump room with the inside forms and part of the reinforcement clearly visible. The nearest portion shows the forms in place, braced, and ready for the pouring of the concrete.

The design of retaining walls requires a combination of theory and practical engineering sense. The designer must think of them in terms of the procedures that are involved in their construction. Therefore, in order to show these things clearly and to illustrate all the principles that may be involved in such construction, the problems that are used in this chapter are practical cases which are taken directly from such

work as that in Fig. 8-2. The solutions of more simple problems will be relatively easy.

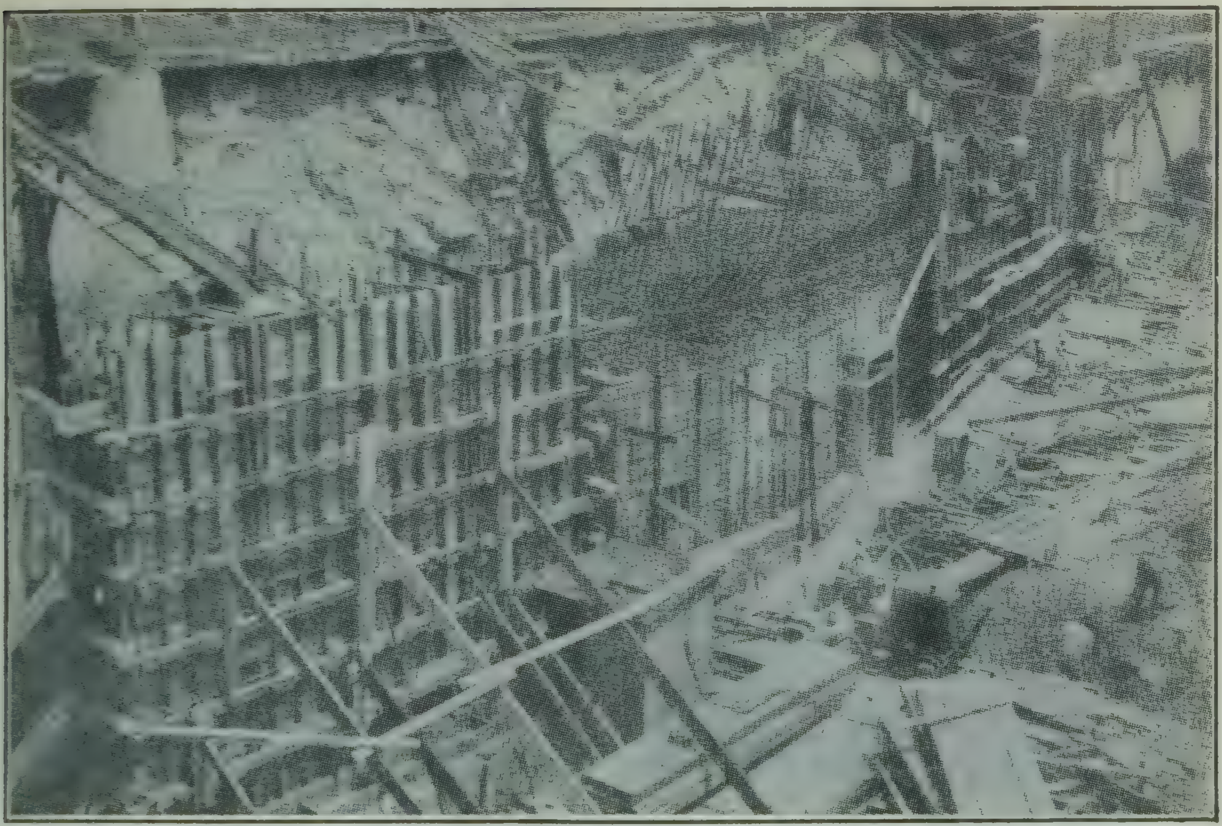


FIG. 8-2. Construction of retaining walls, Exit Plaza of the Lincoln Tunnel, New York City.

8-2. Definitions of parts. The various portions of a typical reinforced-concrete retaining wall are defined as follows, using Fig. 8-3(a) for reference:

1. Stem—the portion $ADKL$.
2. Footing, or base—the part $JCEH$.
3. Toe—the projecting part of the footing on the side toward which the wall tends to tip $JKGH$.
4. Heel—the projecting portion of the footing on the side from which the wall tends to tip $CDFE$.
5. Back—the surface AD.
6. Front—the surface LK.
7. Foundation—the material under the footing, below HE.

8-3. Types of reinforced-concrete retaining walls. *T-shaped Wall.* A "T-shaped" retaining wall is shown in Fig. 8-3(a). This is the most simple and common type of cantilever wall. The base should be from 0.4 to 0.6 times the total height AH, but it will vary somewhat with the position of the stem along the base and with the strength of the foundation. The length of the toe JK should be about one-fourth to one-half of the base, the stem being located nearer the rear when it is desired to obtain foundation pressures that are as small and as nearly uniformly distributed as it is possible to have them—as for foundations

upon clay. Possible troubles from sliding because of the decrease in the dead load will be discussed later.

L-shaped Wall. An "L-shaped" wall such as that in Fig. 8-3(*b*) is used when the wall is along a property line or in other situations where a toe cannot be provided. Its disadvantages are excessive pressure at the front edge B and difficulty in resisting the bending moment at the junction between the stem and the heel. The base should be about 0.5 to 0.55 times the height AB.

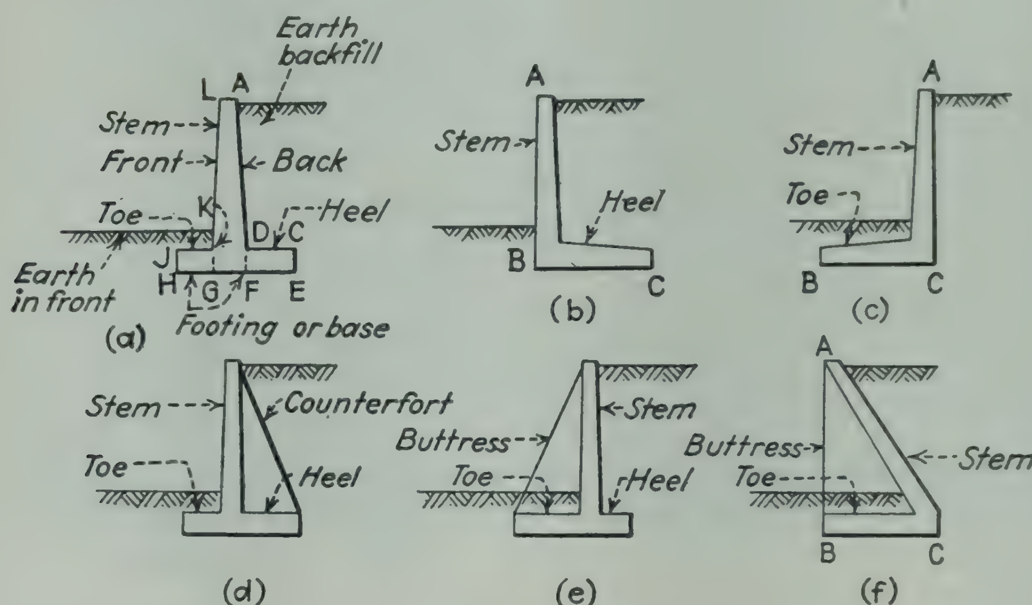


FIG. 8-3.

Reversed L-shaped Wall. A "reversed L-shaped" wall is illustrated in Fig. 8-3(*c*). It is usually difficult to make such a wall stable and to keep it from sliding if the height is great, because of the fact that the dead load is relatively small. However, such walls are useful in cases where it is expensive or impossible to provide a heel. The base BC should be about 0.5 to 0.6 times the height AC.

Counterforted Wall. A "counterforted" wall such as that of Fig. 8-3(*d*) is a modification of the T- or L-shaped ones. It has intermittent vertical ribs called counterforts. This is advantageous for very high walls because the counterforts can be heavily reinforced so as to act as ties to connect the stem and the heel, really transforming the last two parts into continuous slabs which are supported by the counterforts. Although the stem and the heel can be relatively thin, the extra formwork and details may offset the economy in materials. The base should be about the same width as for a T- or an L-shaped wall.

Buttressed Wall. A "buttressed" wall such as that in Fig. 8-3(*e*) is like a reversed counterfort type with ribs or walls that serve the same general functions as the corresponding parts in a counterforted wall

except that they are compression members instead of ties. The toe and the stem are continuous slabs. Such a wall may be built with an inclined stem as shown in Fig. 8-3(f). The base BC should ordinarily be about 0.5 to 0.6 times the height AB, depending upon the size of the heel.

8-4. Stability and safety factor. The stability of a retaining wall is its ability to hold its position and to perform its function safely. The safety factor is a measure of the magnitudes of the forces that are required to cause failure of the structure, compared with the forces that are really acting upon it. Thus, if the safety factor is 1, the wall will be upon the point of failure. If, for any given design, it is 2, then the overturning moment or the horizontal forces may be doubled before the wall will fail. The magnitude of the safety factor to be used in a design will depend upon the engineer's judgment, the specifications, or the building code that is to be followed. In general, it may vary from 1.5 to 2.

A retaining wall may fail in one of four ways: by the collapse of its component parts, by overturning about the front of its toe, by excessive pressure upon its foundation, or by sliding upon its foundation. In a well-balanced design, the wall should be equally safe in all respects.

The bending of the individual parts and the pressures that act upon them are illustrated in Fig. 8-1(c). Each part must act as a cantilevered beam.

In order to illustrate overturning, let W of Fig. 8-4 represent the resultant of all the vertical forces, including the weight of the wall and the vertical component of the lateral earth pressure (if any) and of the earth on top of the base; let H = the resultant of all the horizontal forces. Then Hn represents a moment that tends to overturn the wall as an entity, rotating it about the point A, but Wm is the "righting," or "stabilizing," moment which resists this overturning. If Hn exceeds Wm, the wall will tip over because the resultant of H and W will pass outside the base (beyond A).

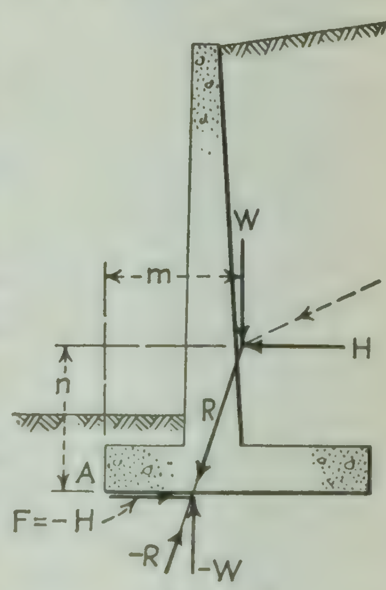


FIG. 8-4.

Failure due to excessive pressure on the foundation will result in the tipping or overturning of the structure. When the wall is founded upon rock, it may actually rotate about the corner A, because the rock is very strong, but when it is on earth, the latter will settle, and the wall will tilt about a point to the right of A if the concentration of pressure becomes too great. The effective value of m will be decreased. If the overturning moment exceeds the reduced righting moment, the wall will overturn.

Sliding on the foundation may be demonstrated by considering Fig.

8-4. The resultant of W and H is represented by R. This, in turn, must be resisted by an equal and opposite reaction which may be called $-R$ and which will have components equal to $-W$ and $-H$. The latter force is caused by the friction of the base upon the foundation so that $F = -H$. Then, if f = the coefficient of sliding friction, $F = Wf$. When F can actually counteract H, the wall is said to be "stable against sliding." Of course, when the wall is built upon an irregular rock surface, there is no difficulty about sliding.

A retaining wall needs weight in order to resist overturning and sliding. Therefore, it is not usually advisable to use high-strength concrete and excessively thin sections for ordinary walls because they will be too light in weight. Web reinforcement in retaining walls is very troublesome. It should be avoided by keeping the shearing stresses low.

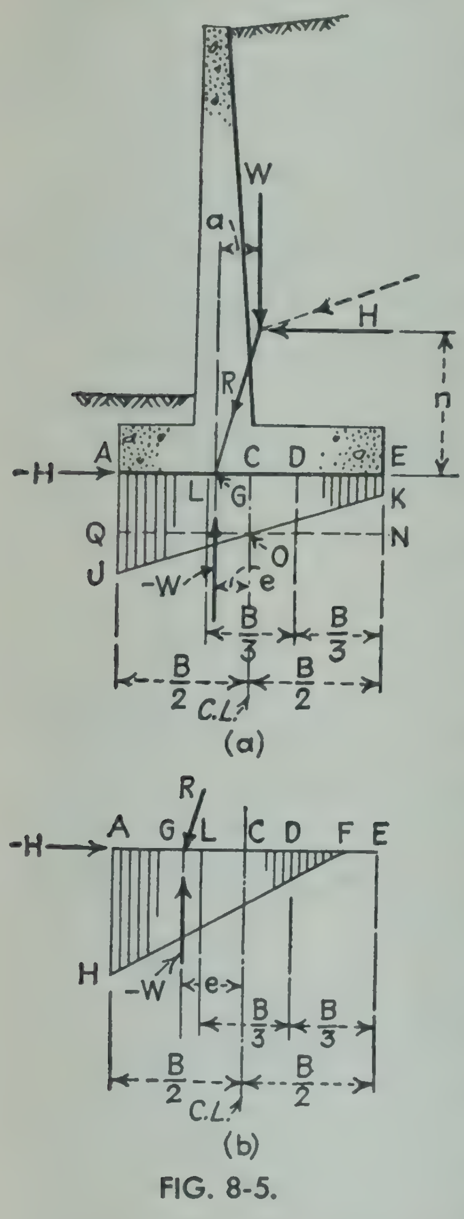


FIG. 8-5.

It is often difficult to secure the desired safety factor against sliding in the case of earth-borne walls. The ground in front of the structure may have considerable abutting power or passive resistance to being shoved away, but the wall should stand without depending upon this force. Absence of the earth in front of the wall when the backfill is placed behind the structure, thoughtless excavation of the earth along the toe by someone in the future, possible scouring or washing away of this material —all these are reasons for this statement. Sometimes this passive resistance is relied upon, but this should be done with caution.

8-5. Foundations. Retaining walls that are founded upon earth present more of a problem than do those which are supported upon rock, because the high pressures that can be applied to the rock are not permissible upon the earth. Those pressures are the result of the combined action of the direct vertical load W and the overturning moment Hn.

Two different sets of pressure distributions are shown in Fig. 8-5. These differences are caused by the relative effects of the direct load and the overturning moment, but it is convenient to look upon them in terms of the position of the point at which the resultant of W and H intersects

the bottom of the footing. If G is this point, its location with respect to W can be found by taking moments about G itself. Therefore,

$$a = \frac{Hn}{W} \tag{8-1}$$

This enables one to compute e, the eccentricity of the resultant with respect to C, the center of the base.

When G is at the right of L, it is inside the middle third of the footing. Therefore, the pressure diagram $AEKJ$ of Sketch (a) is made up of a uniformly distributed pressure equal to EN which equals W/B for a 1-ft strip of wall, and a uniformly varying pressure which is caused by the moment

$$M = We$$

$$QJ \text{ or } KN = \frac{Mc}{I} = We \times \frac{6}{1 \times B^2} \tag{8-2}$$

Therefore,

$$p = \frac{W}{B} \pm 6\frac{We}{B^2} = \frac{W}{B}\left(1 \pm \frac{6e}{B}\right) \tag{8-3}$$

where p is the intensity of the greatest or the least pressures, and B is the width of the footing. The positive sign is to be used to find the maximum pressure AJ; the negative sign will give the minimum pressure EK. For earth-borne walls, the pressure diagram should be as nearly rectangular as it is practicable to make it.

When the resultant falls outside the middle third (left of L), the pressure diagram in Fig. 8-5(b) results. Since there can be no tension upon the base near E, Eq. (8-3) cannot be used. The pressure diagram is assumed to be the triangle AFH. Its total area must equal the direct load W, and its center of gravity must lie vertically below G. Therefore, for a strip 1 ft wide, $AH \times AF/2 = W$. Therefore, the maximum pressure AH is

$$AH = p = \frac{2W}{AF} = \frac{2W}{3AG} \tag{8-4}$$

The exact distribution of the resisting pressure of the foundation may not vary as a straight line, but it is sufficient to assume that it does so.

The safe bearing value of any given soil and its probable deformation are so problematical and so dependent upon the qualities of the material itself that it is unwise to set any exact magnitudes for them, but the data of Table 8-1 may be used as a general guide. However, the conditions at the site must be examined and the bearing value of the soil should be tested before one designs an important retaining wall.

TABLE 8-1. General Data Regarding Foundation Materials

Material	Safe bearing capacity, ksf	Angle of repose, deg	Maximum coefficient of friction of concrete on foundation
Sound rock	80		
Poor rock	30		
Gravel and coarse sand	10–12	37	0.6–0.7
Sand (dry)	6– 8	33	0.4–0.6
Fine sand (wet but confined)	4	25	0.3–0.4
Clay and sand mixed	4– 5	36	0.4–0.5
Hard clay	5– 6	36	0.4–0.6
Soft clay	2	26	0.3

The frictional resistance of the foundation when a wall tends to slide upon it is also uncertain. For some soils, the resistance of the material

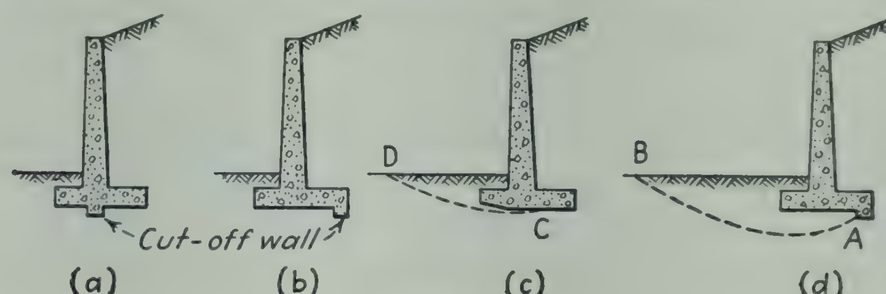


FIG. 8-6.

against sliding upon itself is greater than that for the concrete sliding upon it. In order to take advantage of this greater frictional resistance, the footings of walls are often made with projections on the bottom, called "cutoff walls," as shown in Figs. 8-6(*a*) and (*b*). Sometimes the bottom of the toe is sloped upward, as in Fig. 8-6(*c*). The first method is likely to cause disturbance of the earth as the result of digging the trench in it. Probably the real value of these measures comes from the increase of the shearing and abutting values of the confined earth when it is subjected to the large pressures that exist under the forward portion of the footing. It seems to be best to put a cutoff wall at the rear of the heel as shown in Fig. 8-6(*b*) because this is the region where the soil pressures under the footing are a minimum. Furthermore, it is probable that the line of slippage may be somewhat as shown by the dotted line AB in Fig. 8-6(*d*). This requires the movement of a much heavier mass of earth with more resistance from friction and cohesion than would occur if the movement took place along CD of Sketch (*c*).

It is very important to found all walls upon undisturbed material.

The consolidation that has been produced by nature has probably given the soil the best treatment that it can have for use as an ordinary foundation. In no case should a wall be placed upon newly deposited fill when avoidance of settlement is important. Furthermore, the excavation for the footing should be sufficient to remove organic matter and to get below the frost line—about 4 to 5 ft in cold climates.

Another foundation problem which must not be overlooked in long walls is that of founding them partly upon rock and partly upon earth. If such conditions cannot be avoided, the structure should be designed so that the last portion that rests upon rock has a seat to receive the footing of the adjacent earth-borne structure, the latter being a rather long section which can act like a vertical beam as shown in Fig. 8-7. The compaction of the earth will then merely open the joint at A slightly,

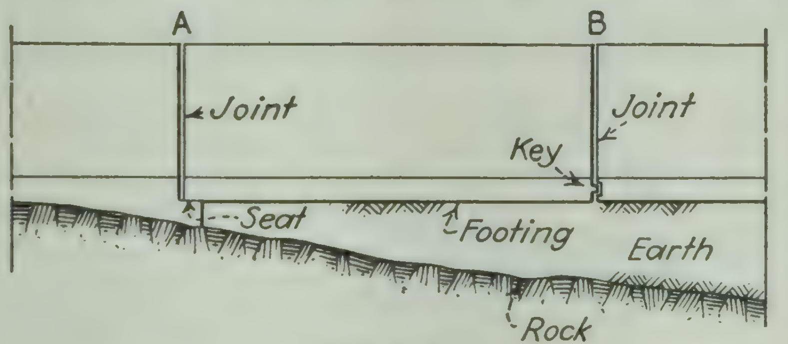


FIG. 8-7.

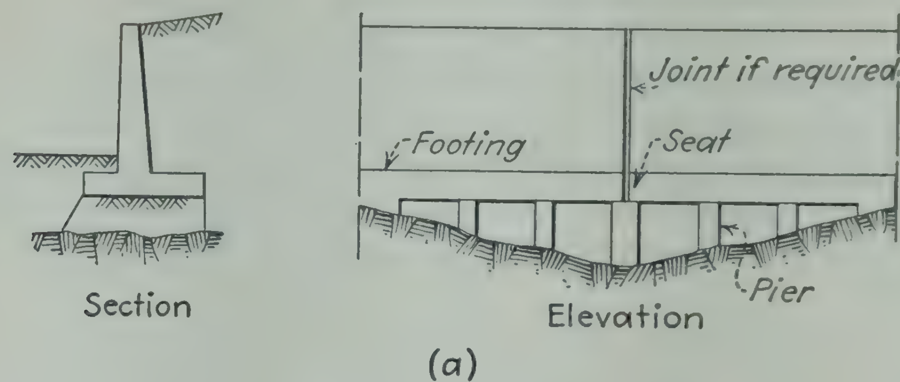
and it will close the one at B somewhat, but the motions will not be very apparent. On the other hand, if the rock has only local gullies of shallow depth, concrete piers without reinforcement may be built to carry the loads to the surface of the rock as shown in Fig. 8-8(a).

If a retaining wall rests upon a shelf at one end as shown in Fig. 8-7, or upon piers as in Fig. 8-8, one should remember that the presence of a hard support may cause considerable local torsion in the footing because of the tendency to concentrate the resistance at these points instead of at the adjacent but more yieldable soil.

If a retaining wall supports vertical loads upon its top, its heel, or its toe, these loads are to be included along with the weight of the wall, earth, etc., in finding the total load W and the location of its resultant. Similarly, if horizontal forces are applied to the top or anywhere else, they are to be included in the calculations for the magnitude and position of H. These resultants are then used as for any other wall.

Figure 8-8(b) illustrates the construction used at one wall of an industrial plant that is located on a hillside. The back of the wall laps under the siding. The heavy crane columns are supported partly on the top of the stem and partly on the local pilaster. Wind and cranes can cause lateral forces at the top of the wall. The toe should be wide enough to

avoid excessive edge pressures. The local column load can be assumed to be spread over a length of wall equal to from one to two times the height of the stem. The horizontal projection or wing at the top of the wall is to spread the horizontal load from the column over a considerable length of the wall. Incidentally, if local projections of the toe are made at the columns, as shown by the dotted lines in Fig. 8-8(*b*), these must



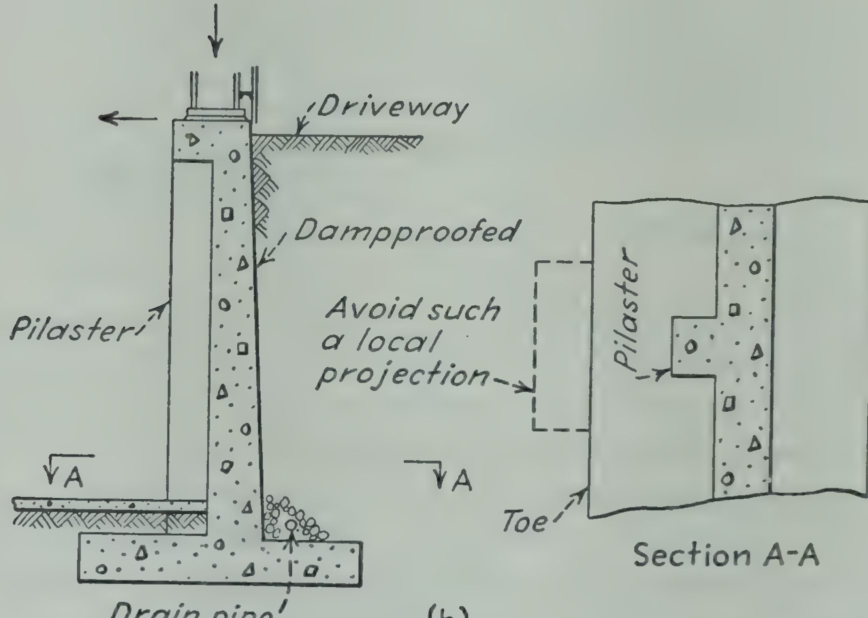


FIG. 8-8.

be adequate to resist heavy localized pressures because, when the wall tends to tip over, the pressures will be concentrated upon these projections which act somewhat as fulcrums.

8-6. Lateral earth pressure. When an excavation is made in earth or when earth is piled up, the soil tends to slump and move sideways as shown in Fig. 8-9. Force is required to prevent this motion. The force exerted by the earth against any opposing structure is called the *active* earth pressure in order to differentiate it from the *passive* pressure—the resistance of the earth to being shoved aside by an outside force. The greatest angle at which the earth slope will remain in equilibrium is

called the *angle of repose*, and it is usually denoted by ϕ. It ordinarily varies from 30 to 40° from the horizontal.

The magnitude of the active earth pressure—called "earth pressure" hereafter—is rather indeterminate. The soil behind any wall may vary greatly in its characteristics from place to place; when the backfill is deposited behind the wall in Fig. 8-10, it may exert a certain lateral pressure upon the structure; but when the wall deforms slightly, as shown by the dotted lines, it may tend to relieve itself of some of the pressure because of the cohesion and friction of the earth upon itself; if traffic or some other force causes vibrations that break down this internal frictional resistance, it may cause an increase of pressure upon the wall;

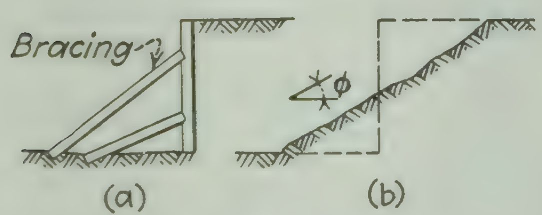


FIG. 8-9.

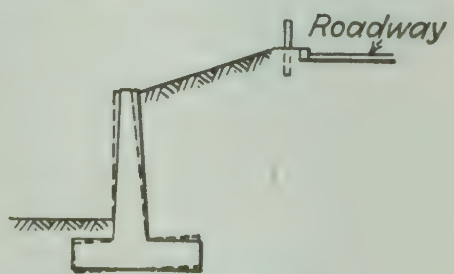


FIG. 8-10.

when the earth dries out, it may shrink and settle as it dries; and when the soil is saturated by a rain, it may expand again.

It is not possible to discuss in this text the various theories of earth pressures that have been developed by others. However, it is necessary to adopt one theory of earth pressures for use as a basis of design. Coulomb's theory[1] will be used in this text. This theory is based upon the assumption that the wedge of earth that lies above the plane of rupture—a plane at or above the one that is established by the angle of repose—will tend to slide downward and will shove the wall before it or will tip it over. A general case is pictured in Fig. 8-11(a), where ϕ is the angle of repose of the soil and P is the total earth pressure per linear foot of wall, P being parallel to the plane of rupture. The meaning of each of the other symbols is obvious.

Coulomb's general formula for the total thrust of the earth per foot of the length of the wall is

$$P = \frac{1}{2} wh^2 \frac{\sin^2 (\theta - \phi)}{\sin^2 \theta \sin (\theta + z) \left[1 + \sqrt{\dfrac{\sin (z + \phi) \sin (\phi - \delta)}{\sin (\theta + z) \sin (\theta - \delta)}} \right]^2} \qquad (8\text{-}5)$$

where the meanings of all the terms are indicated in Fig. 8-11(a). When applied to a reinforced-concrete cantilever wall, the conditions become

[1] For brief explanation of both Coulomb's and Rankine's theories, see Milo S. Ketchum, "Structural Engineers' Handbook."

as shown in Fig. 8-11(b). The vertical line BE—labeled h_1—through the end of the heel can be taken as the effective back of the wall unit, of which $ABCD$ is earth. Then $z = \phi$ and $\theta = 90°$. Therefore, Eq. (8-5) becomes

$$P = \frac{1}{2} wh_1^2 \frac{\cos \phi}{(1 + \sqrt{2 \sin^2 \phi - 2 \sin \phi \cos \phi \tan \delta})^2} \tag{8-6}$$

The point of application of this force is $h_1/3$ above the bottom of the footing, assuming a triangular pressure diagram. Roughly, this is about

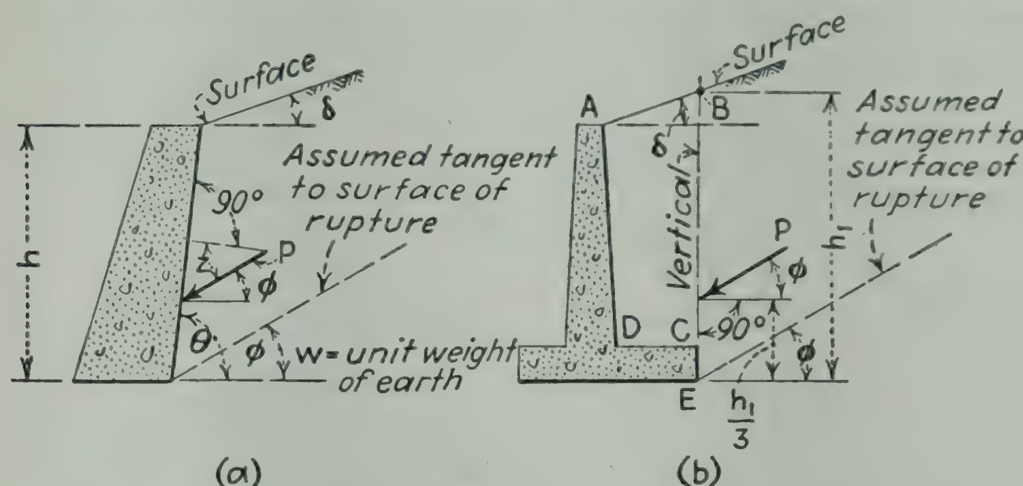


FIG. 8-11.

equivalent to the fluid pressure caused by a liquid weighing 30 to 35 pcf.

If the angle of repose $\phi = 30°$, then Eq. (8-6) becomes

$$P = \frac{1}{2} wh_1^2 \frac{0.87}{(1 + \sqrt{0.5 - 0.87 \tan \delta})^2} \tag{8-7}$$

It has been stated that the force P is assumed to be parallel to the plane of rupture. This is a debatable question when the embankment or fill is subjected to vibrations due to trains, trucks, etc., which tend to break down the plane of rupture. In these cases, it is safer to assume that the earth pressure acts horizontally because its vertical component otherwise exerts a stabilizing effect theoretically.

8-7. Surcharge. "Surcharge" generally denotes a temporary or live load which is applied on top of the earth behind a retaining wall, tending to increase the earth pressure. These loads may be caused by trains, vehicles, or even piles of materials.

Surcharge diagrams are shown in Fig. 8-12. Sketch (a) pictures a highway upon an embankment; W' is the wheel load of a truck. Obviously, the earth spreads the load W' over increasingly large areas of soil as the depth is increased. The pressure diagram may be assumed to be a cone. The horizontal components of the pressure lines upon a vertical

plane like AB of Fig. 8-12(a) must vary with the pressures and their directions. Experiments that have been made by M. G. Spangler[1] indicate that the diagram of horizontal pressures on AB which are due to the

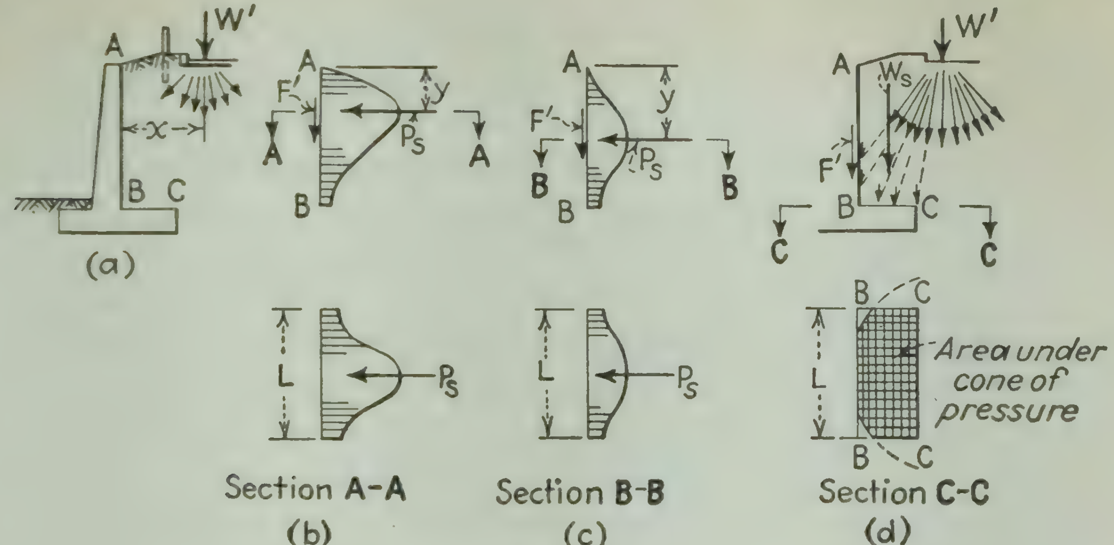


FIG. 8-12.

surcharge is somewhat as shown in Fig. 8-12(b)—a protuberance. They also indicate that the distance x has a great effect upon this diagram. When x is small, the magnitude of the resultant horizontal surcharge P_s

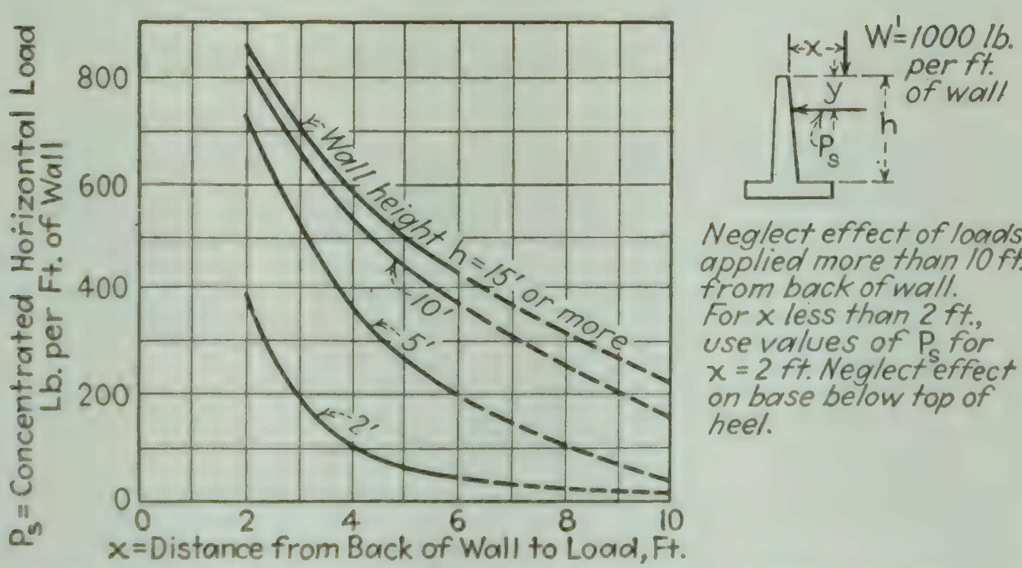


FIG. 8-13. Lateral pressure due to concentrated surcharge loads.

is large, and its distance from the top of the wall is small. As x increases, the maximum intensity of pressure decreases as in Fig. 8-12(c); the forces are spread over a greater area; and y increases.

[1] Associate structural engineer, Iowa Engineering Experiment Station, Iowa State College, Ames, Iowa. The data have been published in *Paper* J-1, Vol. 1, p. 200, of *Proceedings* of the International Conference on Soil Mechanics and Foundation Engineering; also in Iowa Engineering Experiment Station, Iowa State College, *Bull.* 140.

At the same time, the pressures from the surcharge have vertical components as shown in Fig. 8-12(d). Part of these cause downward stabilizing forces upon the heel BC; others may cause a downward frictional force F'. However, the latter does not seem to be reliable because of the effect of vibrations. Therefore, it will be allocated to those things which may increase the safety of the structure but are not included in the design.

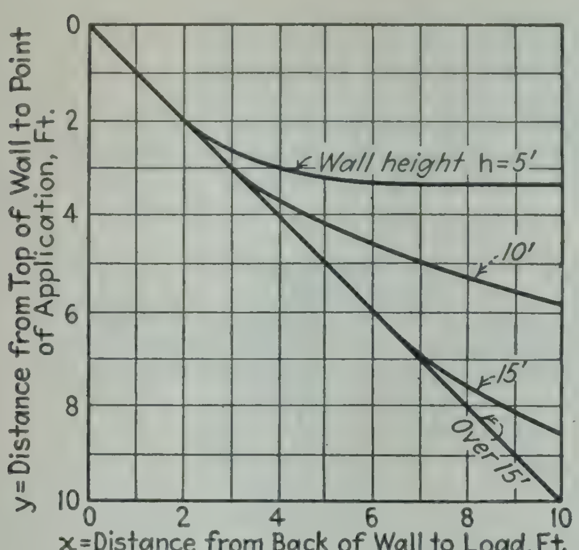


FIG. 8-14. Location of point of application of resultant lateral pressure due to concentrated surcharge loads.

With Spangler's report as a starting point, curves have been drawn as shown in Figs. 8-13, 8-14, and 8-15. The first two enable one to determine and to locate a single horizontal force which should be included in the equations for moment and for sliding. On the other hand, Fig. 8-15 is prepared in order to give an approximate uniform load for the downward pressure on the heel as shown in Fig. 8-12(d). In preparing these surcharge diagrams,[1] W' was assumed to be 18,000 lb. The resultant of the horizontal components of the surcharge forces was

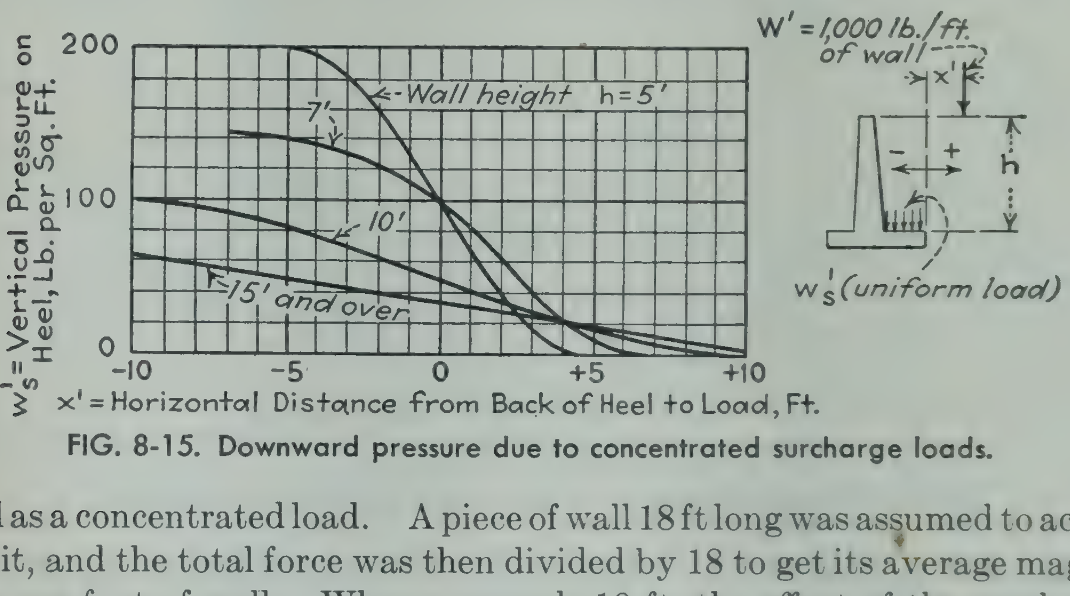


FIG. 8-15. Downward pressure due to concentrated surcharge loads.

used as a concentrated load. A piece of wall 18 ft long was assumed to act as a unit, and the total force was then divided by 18 to get its average magnitude per foot of wall. When x exceeds 10 ft, the effect of the surcharge is neglected; when h is greater than 15 ft, the effect of the surcharge is assumed to be constant in magnitude and position because the effect of the bottom of the diagram of Fig. 8-12(b) is negligible. However if the

[1] The curves have been prepared by Dr. A. H. Baker, formerly designer, The Port of New York Authority.

load W' differs from that assumed here—1,000 lb per ft over 18 ft—the values in Fig. 8-13 may be modified in proportion but Fig. 8-14 should not be greatly affected. It is unnecessary to include impact in these loads. Of course, the real downward pressure at any point on the heel (Fig. 8-15) varies with the location of the load, the height of the wall, and the relative position of the point on the heel. However, great refinement is not justified. Therefore, a conical distribution at a maximum of 45° is assumed, and the curves have been made accordingly. This downward pressure becomes relatively so small when a wall is more than 15 ft high that one may neglect it, but it is included in problems here for purposes of illustration.

If the surcharge is a uniform load—say 300 psf—it may be converted into an equivalent depth of earth. Using 100 pcf as the weight of earth, this surcharge is similar to an extra fill 3 ft deep. The wall can be designed as though the top of the earth really came to a point 3 ft above its actual surface—as far as the earth pressure alone is concerned.

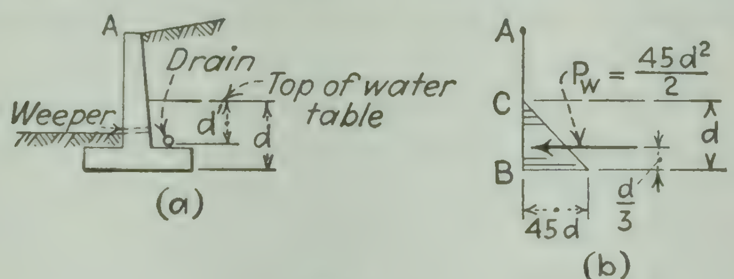


FIG. 8-16.

8-8. Water pressure. Although a wall may have longitudinal drains behind it or small holes called "weepers" through the stem, as pictured in Fig. 8-16(a), the ground water behind the wall may become impounded if the outlets clog up or become filled with ice. If the wall is built upon solid rock, the water may be almost entirely held back. If the foundation is composed of earth, water may be able to escape under the footing, but it is likely to require some head to force it through the soil.

The presence of water in the soil increases the lateral pressure on the wall. If the weight of saturated earth is assumed to be 120 pcf, and if the lateral pressure that it exerts is 60 per cent[1] of this weight, then this pressure equals 72 psf per ft of depth. However, using Eq. (8-7) for dry earth, assuming $\delta = 0$, and using $w = 100$ pcf, $P = 0.15wh_1^2$. If $h_1 = 1$ ft, $P = 0.15w = 0.15 \times 100 = 15$ lb. This shows that the pressure for the dry earth is 30 pcf per ft of depth, since it varies uniformly from zero to a maximum. (P = area of pressure triangle = $p \times 1/2$.) The saturated earth exerts a pressure that is 42 psf larger than that of the dry earth, but this excess will be called 45 psf per ft of depth. How-

[1] Karl Terzaghi, Pressure of Saturated Sand, *Eng. News-Record*, Feb. 22, 1934.

ever, the top of the "water table" must be ascertained before one can determine the total pressure caused by the water.

Theoretically, the existence of water pressure on the back of the wall would be accompanied by hydrostatic uplift under the footing and downward pressure on top of it, all but the last of these pressures tending to tip over the wall. However, these forces are rather indeterminate, and the inclusion of them causes needless complication of the calculations. On this account, the horizontal hydrostatic pressure will be used alone; the maximum distance from the invert of the drain to the top of the water table—d' in Fig. 8-16(a)—will be assumed to have an arbitrary magnitude of 8 ft. (For small walls, the water will be assumed never to be less than 5 ft below the top of the earth even when it reduces d'.) The diagram of horizontal hydrostatic pressures will therefore be taken as shown in Fig. 8-16(b), the resultant being called P_w. If the back of the wall is sloped, the water pressure also has a vertical component, but it is usually negligible because of the steepness of the back.

8-9. Local thrust at the top of a wall due to temperature. Some retaining walls are likely to have a localized thrust (P_T of Fig. 8-17) which will act near their tops. One cause of such a horizontal thrust is the action of frost. If the soil were saturated and then it froze solid, no ordinary wall could withstand the resultant pressure. With properly drained soil, the pressure from freezing must be greatly reduced from that which would be caused by solid ice. However, it seems that the expansion of backfill upon freezing and then its contraction and settlement upon thawing may account for the failure of some retaining walls after years of service, particularly small ones.

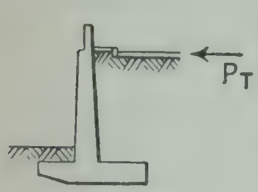


FIG. 8-17.

In order to make some arbitrary allowance for this pressure from ice, 700 plf of wall, applied at the top of the ground and parallel to the surface of the earth, will be assumed. This figure has no experimental verification. It is the product of experience and judgment, and is being used as indicated because of its relatively large effect upon small walls for which the thrust of the earth alone is small. If the ground is nearly level and the conditions facilitate the formation and collection of ice, this pressure will be included in the calculations; if the earth slopes and is thoroughly drained, it may be omitted. It should also be omitted from all calculations for sliding because the ground in front of the wall will be frozen also and it will resist the shear caused by ice pressure behind the structure.

8-10. Preliminary steps in the design of a cantilever retaining wall. In order to illustrate the theory of the design of a reinforced-concrete retaining wall, a practical example will be worked out in detail,

using a problem that will illustrate most of the forces to be encountered in such work. For this, assume that the wall is to support a roadway with a 4-ft sidewalk; it is to have a concrete parapet to protect traffic; the top of the pavement is to be 20 ft above the adjacent ground. Assume also that the following data are specified:

w = weight of earth = 100 pcf
Safe bearing value of earth = 7,000 psf; its ultimate value = 14,000 psf
Angle of repose of earth = $\phi = 30°$
Maximum coefficient of sliding friction of concrete on earth = 0.45
Maximum coefficient of shearing friction of earth on earth = 0.55

$f'_c = 2{,}500$ psi and $n = \dfrac{30{,}000}{2{,}500} = 12$

$f_c = 750$ psi
$f_s = 18{,}000$ psi elastic-limit stress = 36,000 psi
$v_L = 0.03f'_c = 75$ psi
$u = 0.07f'_c = 175$ psi

The maximum stresses in shear and bond shall not exceed two times those given above when testing for the safety factor.

Safety factor = 2, because the wall supports an important highway, and this value is very conservative. Its use will emphasize the relative seriousness of failure due to sliding, also the difficulty of securing the necessary safety factor against such action.

Equation (8-6) will be used to calculate the earth pressure.

The surcharge force and its position will be determined from Figs. 8-13 and 8-14; its downward pressure on the heel, from Fig. 8-15.

Water pressure will be 45 psf per ft of depth, using a height of 8 ft above the weepers.

Ice pressure will be 700 plf at the top of the roadway.

The design is to be based upon the straight-line theory.

In any problem like this, there generally are certain fundamentals and details that are desired by the engineer who is in charge. These automatically influence the design. In this case, some such points will be explained for the purpose of illustration, because these practical things are important and because they should be decided upon before the calculations are made. They are shown in Fig. 8-18.

1. The wall is to be the T type with a large heel and short toe—the latter being about $0.25B$—so as to utilize the dead load of the earth.

2. The bottom of the toe is to be sloped up 2 in. per ft to help resist sliding.

3. The front face is to be battered ¼ in. per ft for appearance.

4. The parapet is used as a guardrail. It is to be 12 in. thick at the top, and it is to be 3 ft 6 in. high.

5. The top of the main wall is to be at least 20 in. thick. This provides a shelf which may improve the appearance of the wall by reducing its apparent height. It also enables men to work inside the reinforcement and the forms during the placing of the concrete. (It is very difficult to avoid honeycombing when concrete is poured into forms for high

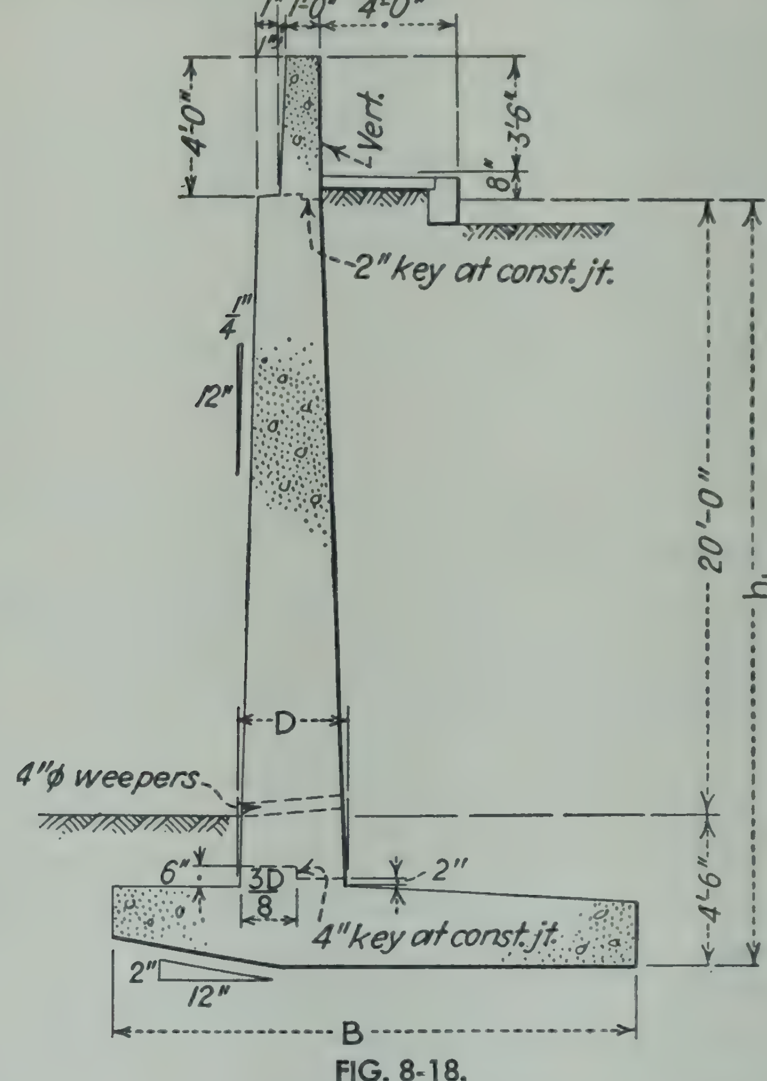


FIG. 8-18.

thin walls, but it can be avoided if portions of the upper part of the forms are left off until the concrete is placed in the lower part of the structure.)

6. The footing is to be poured in sections about 30 to 40 ft long with a construction joint for the stem as shown in Figs. 8-18 and 8-19, the latter being a picture of a footing for a heelless wall. The projections above the toe and heel are important as stops against which the forms for the stem can be held so as to keep them from being displaced or becoming wavy during the pouring of the concrete. The 4-in. key is made so as to resist the shear. The width of the forward raised portion of it should be from one-fourth to one-half of the thickness of the stem,

because the concrete that will be in the tensile side of the stem is not so effective in resisting shear as that which is in compression.

7. The stem is to be poured in one operation so as to avoid the marks

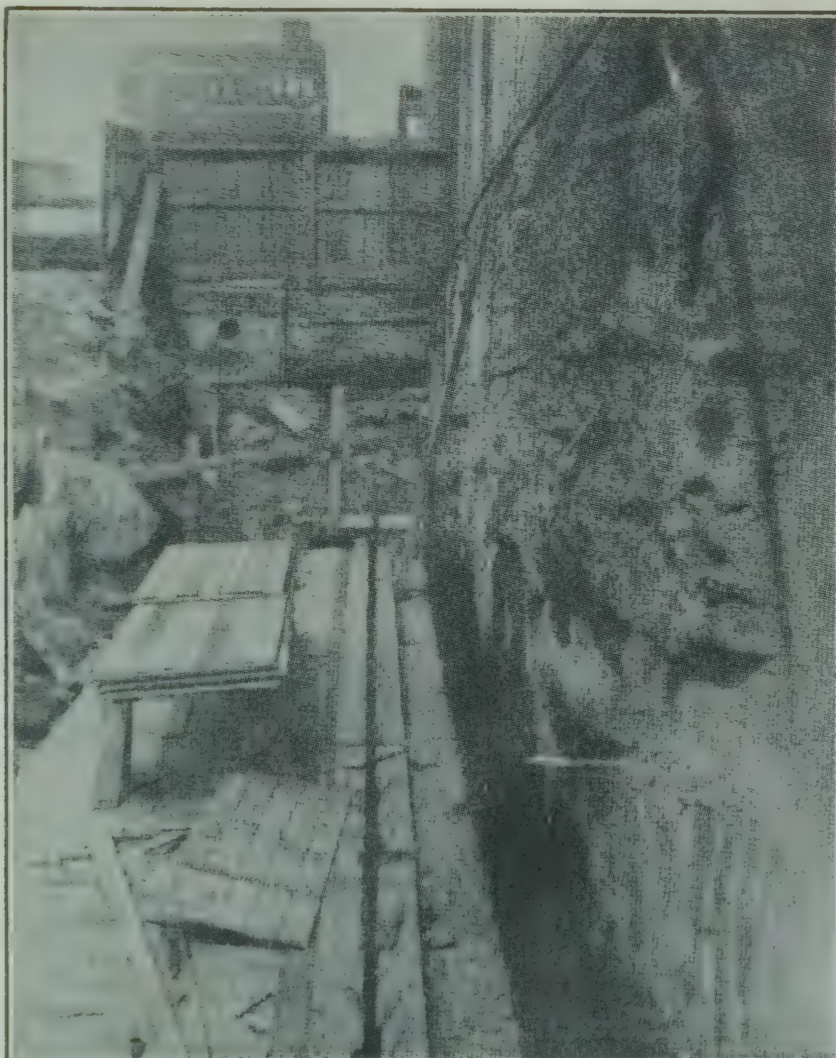


FIG. 8-19. Footing of reversed L-shaped retaining wall, New York approach to Lincoln Tunnel.

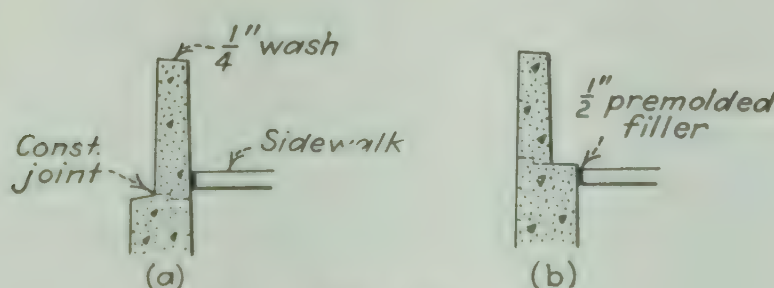


FIG. 8-20.

and the change of appearance that may occur at horizontal construction joints. (Such deep lifts are difficult to make, and they are likely to have detrimental effects upon the quality of the concrete.)

8. The parapet is to be placed flush with the back of the wall with a 7-in. step at the outside as shown in Fig. 8-20(a). The step is placed

below the sidewalk so that the top construction joint will not be visible. This arrangement with the offset on the outside face of the wall avoids the likelihood of settlement of the sidewalk away from the ledge, which might occur if it were constructed as in Fig. 8-20(*b*). On the other hand, if the sidewalk in the latter case is supported upon a shelf on the stem, settlement of the fill may tilt the slab or even crack it.

9. The bottom of the footing is to be 4 ft 6 in. below the finished grade in front of the wall so as to be below the frost line.

10. Weepers 4 in. in diameter and 10 ft c.c. are to be used at the level of the surface of the ground in front of the wall.

8-11. Design of the stem. The effective height of the stem will be assumed to be 22 ft. The surcharge for trucks will be assumed to be

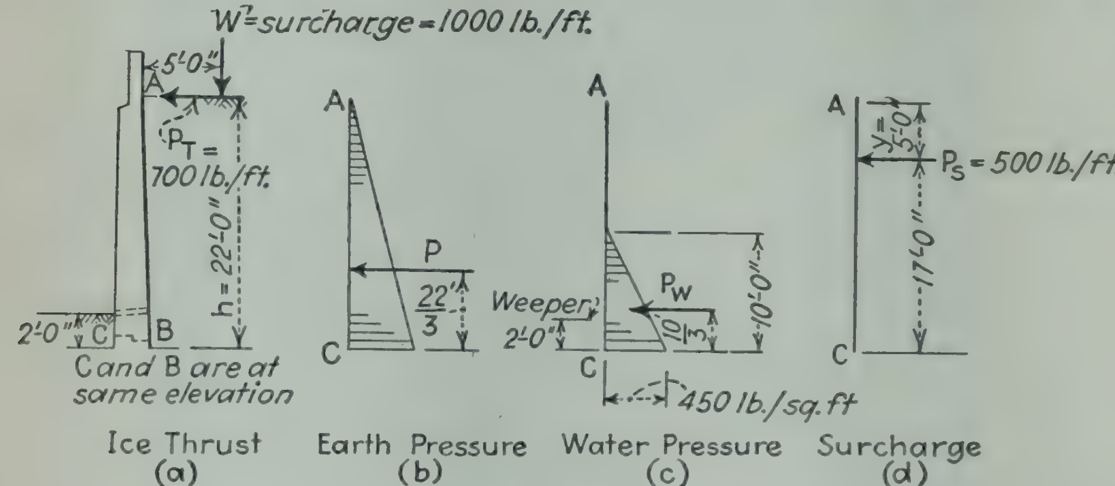


FIG. 8-21.

about 1 ft from the curb. The loading diagrams are then as pictured in Fig. 8-21.[1]

From Eq. (8-7), with $\delta = 0$ and $w = 100$,

$$P = \frac{1}{2} wh_1^2 \frac{0.87}{(1 + \sqrt{0.5 - 0.87 \tan \delta})^2} = 15h_1^2 \tag{8-8}$$

$$P = 15 \times 22^2 = 7{,}260 \text{ lb}$$

where h is used instead of h_1. The force P is assumed to be horizontal because of the vibrations caused by traffic. When there are no such vibrations, the horizontal force may be assumed to equal $P \cos \phi$ (see Prob. 8-1).

Writing the equation for moments about C, the top of the toe in Fig. 8-21, since the stem is a vertical cantilever beam,

$$M = P_T h + P\frac{h}{3} + P_w \frac{(8+2)}{3} + P_s(h - y)$$

$$M = 700 \times 22 + 7{,}260 \times \frac{22}{3} + 10 \times \frac{450}{2} \times \frac{10}{3} + 500 \times 17$$

$$M = 84{,}600 \text{ ft-lb}$$

[1] Note that the answers in such problems as these are not carried out to more than three significant figures in most cases.

From Table 5 in the Appendix, with f_s, f'_c, and f_c equal to 18,000, 2,500, and 750 psi, respectively, K for a balanced design is 111, and $j = 0.889$. Then, using Eq. (2-8),

$$bd^2 = \frac{M}{K} \qquad \text{or} \qquad d = \sqrt{\frac{84{,}600 \times 12}{12 \times 111}} = 27.6 \text{ in.}$$

Assume $d = 28$ in. and the cover of the rods = 3 in. (to provide thorough protection against the moisture in the adjacent earth). $D = 31$ in., giving a batter of ¼ in. in 12 in. for the back below the sidewalk.

$$A_s = \frac{M}{f_s jd} = \frac{84{,}600 \times 12}{18{,}000 \times 0.889 \times 28} = 2.27 \text{ in.}^2 \text{ per ft (approx)}$$

If No. 8 rods are used at 4 in. c.c., $A_s = 2.37$ in.²; if 4½ in. c.c., $A_s = 2.11$ in.². Inasmuch as the direct compressive load due to the weight of the stem will decrease the tension in the bars, the latter will be assumed.

The intensities of the shear and the bond stresses at the base of the stem are

$$v_L = \frac{H}{bjd} = \frac{P_T + P + P_w + P_s}{bjd} = \frac{700 + 7{,}260 + 2{,}250 + 500}{12 \times 0.889 \times 28} = 36 \text{ psi}$$

$$u = \frac{H}{(\Sigma o)jd} = \frac{10{,}710}{3.14 \times 12/4.5 \times 0.889 \times 28} = 51 \text{ psi}$$

It is unnecessary to extend all these heavy rods for the full height of the stem. The required areas of steel at intermediate heights can be found with sufficient accuracy by assuming $j = 0.88$ and designing the wall for bending alone, neglecting any compressive steel. At mid-height,

$$M = P_T \frac{h}{2} + P \frac{h}{2 \times 3} + P_s \left(\frac{h}{2} - y\right)$$

$$M = 700 \times 11 + (15 \times 11^2) \times 11\tfrac{1}{3} + 500 \times 6 = 17{,}400 \text{ ft-lb}$$

$$A_s = \frac{M}{f_s jd} = \frac{17{,}400 \times 12}{18{,}000 \times 0.88 \times 22.5} = 0.59 \text{ in.}^2$$

From such calculations, the curve of Fig. 8-22 can be drawn, showing the permissible cutoff of the rods. It is customary to extend them beyond the theoretical points a distance that need not exceed what is required to develop them through bond. In this case, with $u = 0.07f'_c$,

$$L_s = \frac{A_s f_s}{\Sigma o \times u} = \frac{0.79 \times 18{,}000}{3.14 \times 175} = 26 \text{ in.}$$

The rods should not be much farther apart at the top than the thickness of the wall at this point.

The stem has a safety factor of 2 because, if M is doubled, the stresses in the steel and the concrete will not exceed the elastic limit of the rods or the ultimate strength of the concrete.

The need for good judgment and common sense in engineering has been emphasized repeatedly. One glaring case[1] of the lack of thinking and of a sense of proper proportion occurred in the building of an L-shaped retaining wall 20 ft high. The main bars in the stem were to be 1¼-in. round at 12 in. c.c. The figure 1 happened to be on a dimension line.

[1] *Eng. News-Record*, Feb. 8, 1951, p. 51.

The bars were ordered as ¼-in. round. They were detailed, shipped, erected, and approved by an inspector. As the backfill was placed, the wall progressively failed and had to be replaced. It seems strange that no one would ask "What are ¼-in. bars doing in a 20-ft retaining wall?"

8-12. Stability and foundation pressure. Before the base of the wall is designed in detail, it is advisable to assume its size, to test the retaining wall for stability, and to see that the foundation pressure is satisfactory. To do so for this case, assume that the thickness of the footing is about equal to that of the base of the stem, that $B = 0.6h_1$, and that the toe is about $0.25B$.

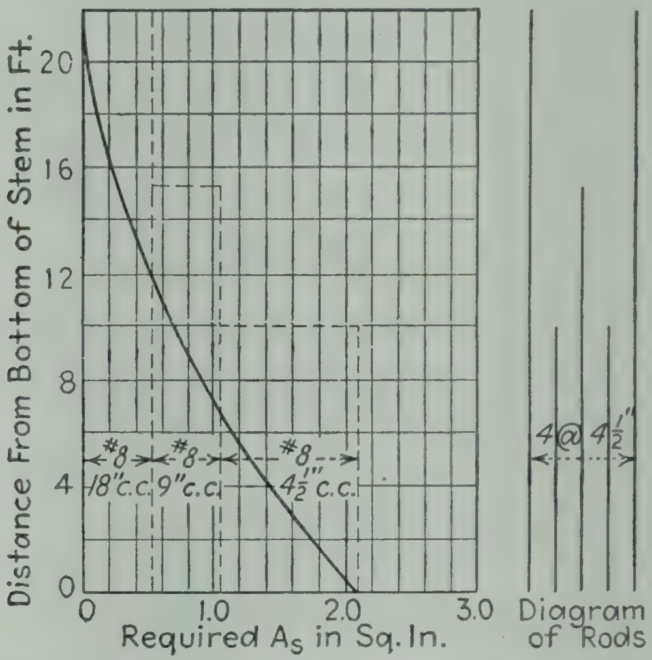


FIG. 8-22. Diagram for determining the required lengths of reinforcement in stem.

Thus, let $D = 30$ in.

$B = 0.6(22 + 2.5) = 14.7$ ft. Assume $B = 14$ ft 9 in. and the toe $= 3$ ft 6 in.

Figure 8-23(*a*) diagrammatically pictures the forces that are to be considered.[1] The stabilizing effect of the surcharge W_s is found from Fig. 8-15 for

$$-8.67 - 0.46 + 5 = -4.13$$

It is 45 psf. Also, from Eq. (8-8),

$$P = 15 \times 24.5^2 = 9{,}000 \text{ lb}$$

$$P_w = \frac{12.5^2}{2} \times 45 = 3{,}500 \text{ lb}$$

The weight of the earth on the toe will be neglected.

The magnitudes and lines of action of the resultant of the vertical forces W and the resultant of the horizontal forces H are found as follows, taking moments about E because of convenience in finding the proper lever arms:

[1] Notice that P is assumed to be horizontal so that $P_v = 0$. This is conservative, but there is much difference of opinion among engineers as to whether or not P may be assumed to slope at the angle ϕ under such conditions. If P is sloped, P_v is its vertical component.

Vertical forces		*Weight*	*Lever arm*	*Moment*
Stem W_c	=	7,640	$\times$ 9.93 =	75,900
Footing $W_F = 2.5 \times 14.75 \times 150$	=	5,530	$\times$ 7.38 =	40,800
Earth $W_e = 8.67 \times 22 \times 100$	=	19,100	$\times$ 4.34 =	82,900
Earth $W'_e = 0.46 \times 22 \times 0.5 \times 100$	=	500	$\times$ 8.82 =	4,400
Surcharge $W_s = 45 \times 8.67$	=	390	$\times$ 4.34 =	1,700
	W =	33,160 lb	ΣM =	205,700 ft-lb

$$x = \frac{\Sigma M}{W} = \frac{205{,}700}{33{,}160} = 6.2 \text{ ft}$$

Horizontal forces		*Force*	*Lever arm*		*Moment*
Temperature P_T	=	700	$\times$ 24.5	=	17,200
Surcharge P_s	=	500	$\times$ 19.5	=	9,800
Earth P	=	9,000	$\times$ 8.17	=	73,500
Water P_w	=	3,500	$\times$ 4.17	=	14,600
	H =	13,700 lb	ΣM	=	115,100 ft-lb

$$n = \frac{\Sigma M}{H} = \frac{115{,}100}{13{,}700} = 8.4 \text{ ft}$$

These two forces are shown in Fig. 8-23(b).

The next step is the determination of the foundation pressures. The eccentricity

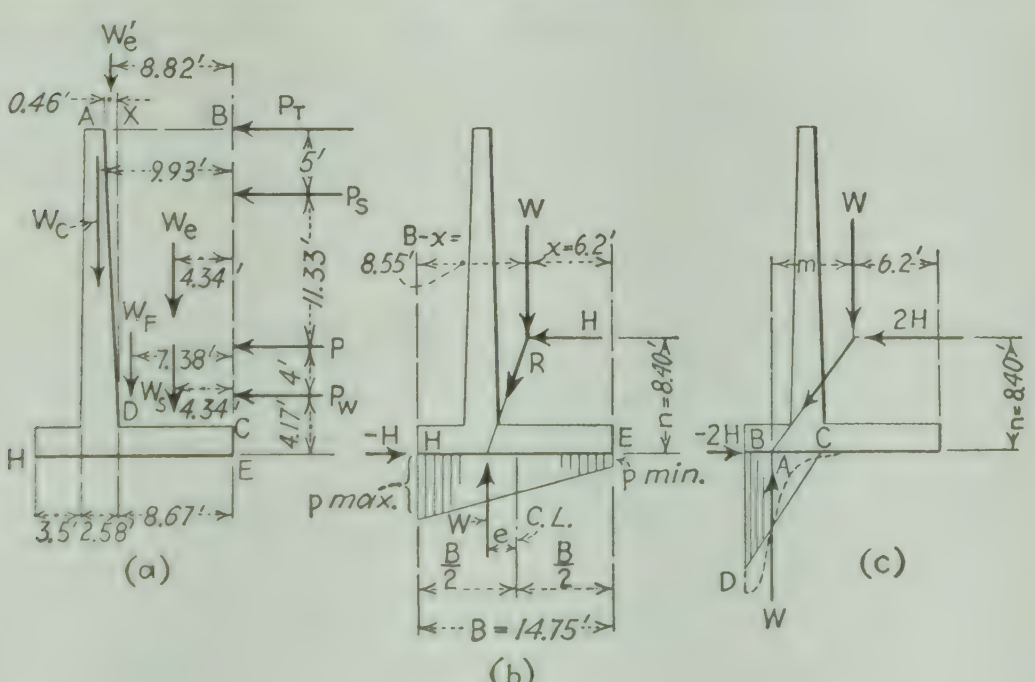


FIG. 8-23.

of the resultant of W and H with respect to the center of the base is found by taking moments about H of Fig. 8-23(b). Thus,

$$e = \frac{B}{2} - \left[\frac{W(B - x) - Hn}{W}\right]$$

$$e = 7.38 - \frac{283{,}500 - 115{,}100}{33{,}160} = 2.3 \text{ ft}$$

This is less than $B/6 = 2.46$ ft. Therefore, the pressures on the foundation are

$$\max p = \frac{W}{B}\left(1 + \frac{6e}{B}\right) = \frac{33{,}160}{14.75}\left(1 + \frac{6 \times 2.30}{14.75}\right) = 4{,}350 \text{ psf}$$

$$\min p = \frac{W}{B}\left(1 - \frac{6e}{B}\right) = 144 \text{ psf}$$

The maximum is quite conservative.

The safety factor against overturning must now be tested. Assume the overturning moment Hn to be doubled. In this case, let n remain unchanged, and replace H by $2H$ as in Fig. 8-23(c). Therefore $\Sigma M = 13{,}700 \times 2 \times 8.40 = 230{,}200$ ft-lb. Take moments about A [Fig. 8-23(c)].

$$m = \frac{2Hn}{W} = \frac{230{,}200}{33{,}160} = 6.94 \text{ ft}$$

$BA = 14.75 - 6.2 - 6.94 = 1.61$ ft. The point A is thus inside the toe B, but the pressure upon the foundation must be investigated. In Fig. 8-23(c), assume a triangular pressure diagram with the resultant W at its center of gravity. Therefore, $AC = 2(BA) = 3.22$ ft, and the maximum pressure is

$$W = \frac{BD \times BC}{2}$$

or

$$BD = \frac{2W}{BC} = \frac{2 \times 33{,}160}{4.83} = 13{,}700 \text{ lb}$$

This value is less than twice the permissible pressure of 7,000 psf which was one of the conditions of the problem. If BD had exceeded the capacity of the earth, the latter would yield, and the pressure diagram might look like the dotted line in Fig. 8-23(c).

Remembering that the thrust due to the effects of temperature P_T is to be omitted when one computes the forces that cause sliding, the required coefficient of friction is

$$f = \frac{H - P_T}{W} = \frac{13{,}000}{33{,}160} = 0.39$$

This seems to give a safety factor of the permissible coefficient divided by the required one, $0.45 \div 0.39 = 1.15$, which is far below the desired value. However, assume that the passive resistance of the soil in front of the wall is 10 times its active pressure so that $P = 10 \times 15h_1^2$.* Then, for a depth of 4.5 ft, this gives 3,040 lb extra resistance per ft of wall which increases the safety factor to

$$0.45 \div (13{,}000 - 3{,}040)/33{,}160 = \text{about } 1.5$$

The bottom of the toe will be sloped upward as pictured in Fig. 8-18 in order to take advantage of the fact that the coefficient of shearing friction of earth on earth was assumed to equal 0.55 at the start of the problem. Since Fig. 8-23(c) shows that, before failure, most of the pressure diagram will be acting upon the toe, this higher coefficient will be used. Then the new safety factor is

$$0.55 \div (13{,}000 - 3{,}040)/33{,}160 = 1.83$$

This is not quite up to the original requirements, but it will be accepted.[1]

* An assumption that approximates Rankine's theory.

[1] Earth-borne retaining walls generally must be heavier than those which are on rock in order to prevent sliding. However, when one considers all probable forces, as has been done here, a safety factor of 2 is very conservative. If P_T is added to the vertical forces, the situation is improved.

8-13. Design of the heel. The footing, as it has been assumed in the preceding article ($D = 30$ in.), will now be designed for strength.

From Fig. 8-23(a), it is apparent that the weight of the earth and the heel and also the pressure from the surcharge are the forces that will affect the heel when the wall tends to tip over, causing the heel to bend as shown in Fig. 8-24(a). The pressure upon the foundation tends to relieve this bending. Figure 8-24(b) indicates the portion of the pressure diagram that acts in this way.

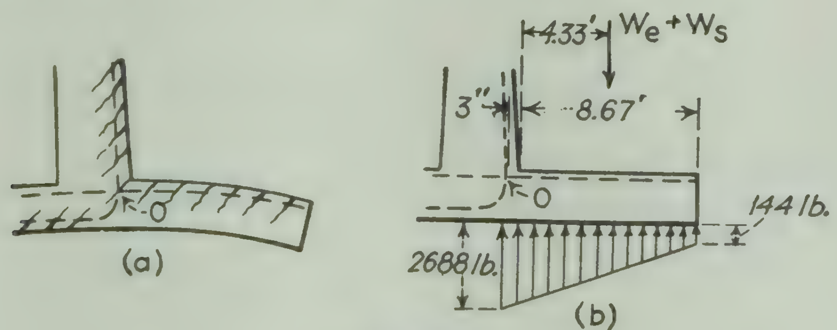


FIG. 8-24.

The heel is a cantilever beam which projects from the line of resistance, the tensile steel in the stem. Therefore, taking moments about O,

$$M = (W_e + W_s) \times \text{lever arm} + \text{weight of heel} \times \text{lever arm} - \text{area of pressure diagram} \times \text{lever arm}$$

$$M = (19{,}100 + 390)(4.33 + 0.25) + \frac{2.5 \times 8.92^2}{2} \times 150 - \frac{144 \times 8.92^2}{2} - \frac{2{,}544 \times 8.92}{2} \times \frac{8.92}{3} = 64{,}700 \text{ ft-lb}$$

To find a trial value for the reinforcement, assume that $d = 27$ in. and $j = 0.87$. Then

$$A_s = \frac{M}{f_s jd} = \frac{64{,}700 \times 12}{18{,}000 \times 0.87 \times 27} = 1.84 \text{ in.}^2$$

The rods that are used should be spaced in multiples of $4\frac{1}{2}$ in. so as to match those coming down from the stem. Using No. 8 rods at $4\frac{1}{2}$ in. c.c. gives

$$A_s = \frac{0.79 \times 12}{4.5} = 2.11 \text{ in.}^2$$

Therefore,

$$p = \frac{A_s}{bd} = \frac{2.11}{12 \times 27} = 0.0065 \qquad \text{and} \qquad pn = 0.0065 \times 12 = 0.078$$

$$k = \sqrt{2pn + (pn)^2} - pn = 0.324 \qquad \text{(see Fig. 10, Appendix)}$$

$$j = 1 - \frac{0.324}{3} = 0.89$$

$$f_s = \frac{M}{A_s jd} = \frac{64{,}700 \times 12}{2.11 \times 0.89 \times 27} = 15{,}300 \text{ psi}$$

$$f_c = \frac{2M}{kjbd^2} = \frac{2 \times 64{,}700 \times 12}{0.324 \times 0.89 \times 12 \times 27^2} = 616 \text{ psi}$$

$$V = W_e + W_s + \text{heel} - \text{pressure diagram}$$

Therefore,

$$V = 19{,}100 + 390 + 2.5 \times 8.92 \times 150 - 144 \times 8.92 - \frac{2{,}544 \times 8.92}{2} = 10{,}200 \text{ lb}$$

$$v_L = \frac{V}{bjd} = \frac{10{,}200}{12 \times 0.89 \times 27} = 35 \text{ psi}$$

$$u = \frac{V}{(\Sigma o)jd} = \frac{10{,}200}{3.14 \times 12/4.5 \times 0.89 \times 27} = 51 \text{ psi}$$

Before calling these results satisfactory, the heel must be checked to see that, if the wall tips so as to relieve the upward foundation pressures under the heel, the

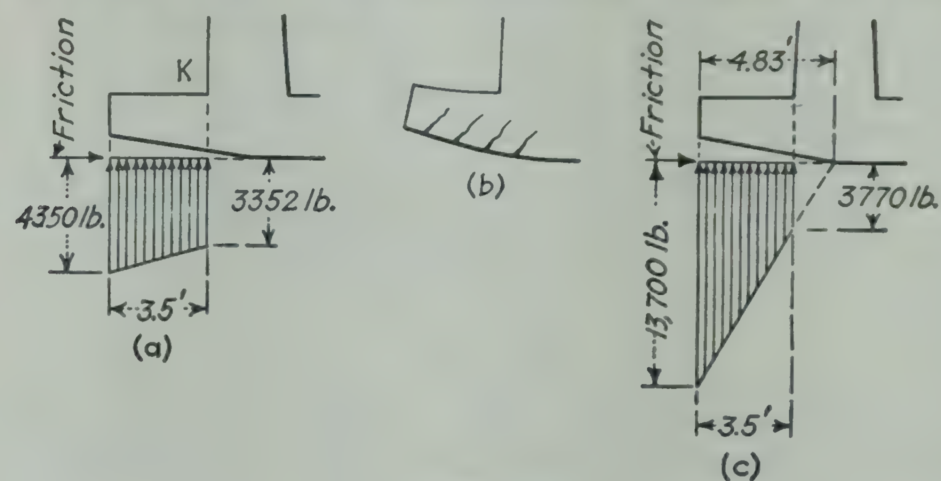


FIG. 8-25.

resultant stresses will not exceed the elastic limit of the steel—$f_s = 36{,}000$ psi—or the ultimate strength of the concrete. For this case,

$$M = (W_e + W_s) \times \text{lever arm} + \text{weight of heel} \times \text{lever arm}$$

Therefore,

$$M = (19{,}100 + 390)(4.33 + 0.25) + \frac{2.5 \times 8.92^2 \times 150}{2} = 104{,}200 \text{ ft-lb}$$

Since this does not exceed twice the 64,700 ft-lb previously calculated, the rods in the heel will not be overstressed.

The heel will be sloped on the top from $D = 30$ in. at the stem to 18 in. at the back edge so as to save concrete, but the stresses will not be recomputed. Alternate rods may be cut off at about 7 ft from the stem, but the others will be extended full length and will be hooked as pictured in Fig. 8-29.

8-14. Design of the toe. Part of the pressure diagram of Fig. 8-23(*b*) is reproduced in Fig. 8-25(*a*). Under its influence the toe will bend as shown in Fig. 8-25(*b*), but the weight of the concrete of the toe will be counted upon to counteract some of the bending moment. Technically, the frictional resistance along the base and under the toe will also tend to annul part of the bending moment, but it is too uncertain and too theoretical to rely upon. Then

$$M = (3{,}352 - 2.0 \times 150)\,\frac{3.5^2}{2} + 0.67 \times 998 \times \frac{3.5^2}{2} = 22{,}800 \text{ ft-lb}$$

A trial area of the rods is, assuming $d = 24$ in., because of the sloping bottom of the toe, and $j = 0.87$,

$$A_s = \frac{M}{f_s jd} = \frac{22{,}800 \times 12}{18{,}000 \times 0.87 \times 24} = 0.73 \text{ in.}^2$$

Since the stem reinforcement is No. 8 bars 4½ in. c.c., assume that every other rod from the stem is bent around into the bottom of the toe. Then

$$A_s = 0.79 \times \frac{12}{9} = 1.05 \text{ in.}^2$$

Neglect the rods in the top of the toe.

$$p = \frac{A_s}{bd} = \frac{1.05}{12 \times 24} = 0.0036 \qquad pn = 0.0036 \times 12 = 0.043$$

$$k = \sqrt{2pn + (pn)^2} - pn = 0.253 \qquad j = 0.92$$

$$f_s = \frac{M}{A_s jd} = \frac{22{,}800 \times 12}{1.05 \times 0.92 \times 24} = 11{,}800 \text{ psi}$$

$$f_c = \frac{2M}{kjbd^2} = \frac{2 \times 22{,}800 \times 12}{0.253 \times 0.92 \times 12 \times 24^2} = 340 \text{ psi}$$

$$v_L = \frac{V}{bjd} = \frac{0.5(4{,}350 + 3{,}352)3.5 - 300 \times 3.5}{12 \times 0.92 \times 24} = 47 \text{ psi}$$

$$u = \frac{V}{(\Sigma o)jd} = \frac{12{,}400}{3.14 \times \frac{12}{9} \times 0.92 \times 24} = 134 \text{ psi}$$

These stresses will be accepted.

The magnitudes of v_L and u are conservative, as they should be. Web reinforcement in such members is not desirable. The rods should be hooked to guarantee good anchorage. The toe will be shaped, and the reinforcement will be arranged as shown in Fig. 8-29.

If the safety factor is tested by doubling H, and if the resultant pressure diagram of Fig. 8-23(c) is applied to the toe, the upward pressures will be as shown in Fig. 8-25(c). Then

$$M = (3{,}770 - 300) \times \frac{3.5^2}{2} + 0.67 \times 9{,}930 \times \frac{3.5^2}{2} = 62{,}000 \text{ ft-lb}$$

$$f_s = \frac{62{,}000 \times 12}{1.05 \times 0.92 \times 24} = 32{,}000 \text{ psi}$$

$$f_c = \frac{2 \times 62{,}000 \times 12}{0.253 \times 0.92 \times 12 \times 24^2} = 930 \text{ psi}$$

These two stresses are within the allowable values.

$$v_L = \left(\frac{13{,}700 + 3{,}770}{2} - 300\right) \frac{3.5}{12 \times 0.92 \times 24} = 111 \text{ psi}$$

$$u = \frac{29{,}500}{3.14 \times {}^{12}\!/_{9} \times 0.92 \times 24} = 319 \text{ psi}$$

The shearing stress and bond are satisfactory since v_L can be

$$0.03 \times 2{,}500 \times 2 = 150 \text{ psi}$$

and u can be $0.07 \times 2{,}500 \times 2 = 350$ psi. Rods E of Fig. 8-29 are added for the reasons explained in Art. 8-16.

8-15. Design of the parapet. The parapet at the top of the wall must be made sufficiently strong to withstand the effect of a vehicle colliding with it. The force of such a blow is problematical but, since it may be spread over a considerable length of the parapet, it will be assumed to be 500 lb per ft of wall, applied as shown in Fig. 8-26. It will not be considered in the design of the main wall because of the latter's safety factor and inertia.

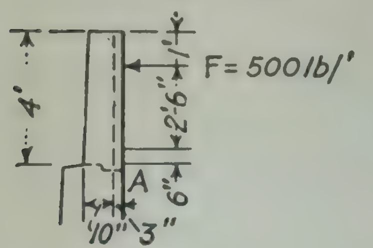


FIG. 8-26.

Assuming $j = 0.9$,

$$A_s = \frac{M}{f_s jd} = \frac{500 \times 36}{18{,}000 \times 0.9 \times 10} = 0.11 \text{ in.}^2$$

Using No. 4 rods 12 in. c.c., $A_s = 0.2$ in.². These rods will be adopted and no further analysis is necessary.

8-16. Arrangement of reinforcement and other practical details. Figure 8-27 is a photograph of the lower portion of the stem of a 35-ft

FIG. 8-27. Partially completed retaining wall, Exit Plaza of the Lincoln Tunnel, New York City.

retaining wall which is located on the south side of one of the plazas of the Lincoln Tunnel at New York City. The work has been stopped at a horizontal construction joint. It shows many of the features that have been or will be discussed, such as the division into sections by expansion joints, keyways, form ledges, dowels, flashing at expansion joints, and extensions of rods for the next pour.

Expansion (or contraction) joints should be located about 30 to 40 ft apart so as to eliminate shrinkage and temperature cracks. Special horizontal reinforcement will be placed in the stem of each section to hold it together as a unit. For this purpose, an area of steel equal to 0.2 per cent of the cross section of the stem will be used with about two-thirds

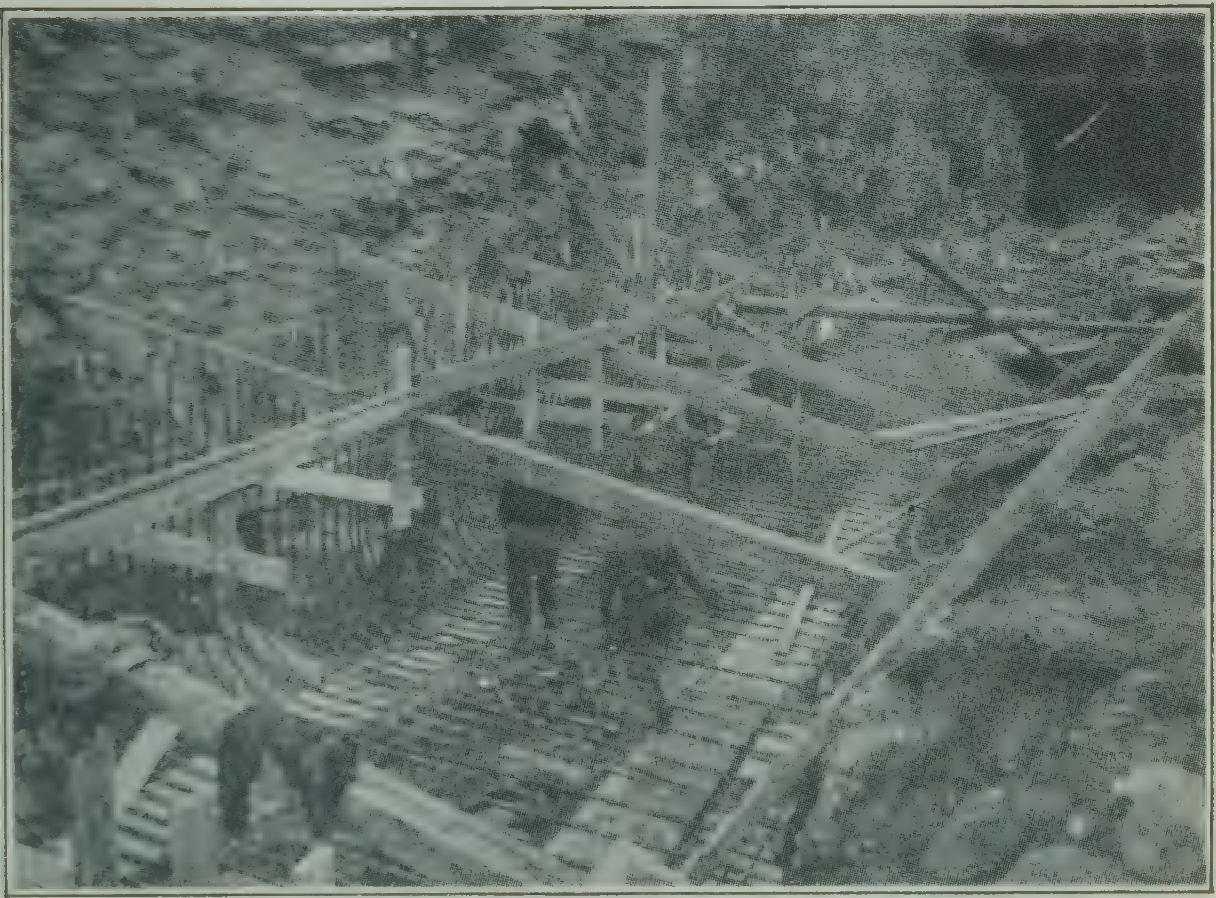


FIG. 8-28. Placing reinforcement in the footing of a retaining wall, Lincoln Tunnel, New York City.

of the steel near the front face because of its greater exposure. The parapet will have reinforcement equal to 0.3 per cent. Then

$$A_s = 0.002\,\frac{(20 + 31)}{2} \times 12 = 0.61 \text{ in.}^2 \text{ per ft of height of stem}$$

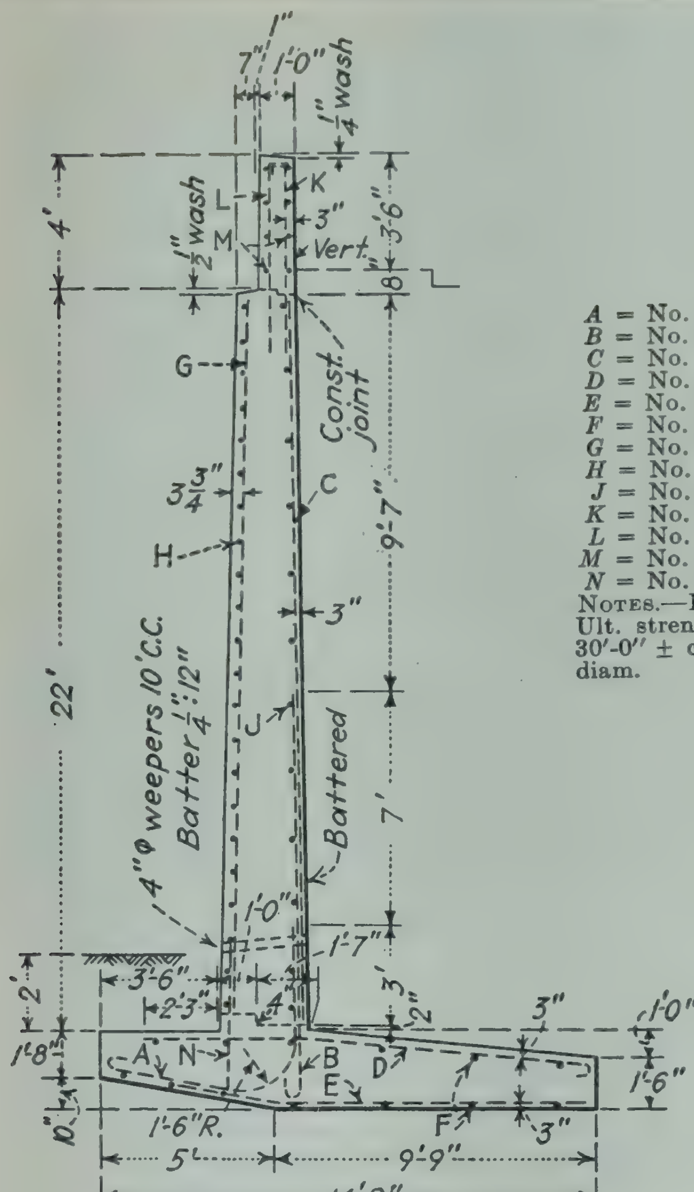
Use No. 6 rods 12 in. c.c. at front and 24 in. c.c. at back ($A_s = 0.66$ in.^{2}).

$$A_s = 0.003 \times 12.5 \times 12 = 0.45 \text{ in.}^2 \text{ per ft of height of parapet}$$

Use No. 4 rods 10 in. c.c. front and back ($A_s = 0.48$ in.^{2}).

The longitudinal rods in the footing itself will be designed upon the basis of their use as ties; for a wall that is on earth, they should be 0.1 to 0.2 per cent of the cross section of the base in order to serve as adequate temperature reinforcement. Of course, when a footing is on rock, these rods are needed only as ties because the keying effect of the rough rock will not let the wall move.

Figure 8-28 is a picture at the base of the wall of Fig. 8-27—the small-heel and large-toe type. Note the way the toe reinforcement is bent up to serve as dowels for the stem, the cutting off of alternate rods in the toe, the tie rods, the heel reinforcement which the men are placing, the strips of concrete which will support the future chairs or spacers to be



Schedule of reinforcement

A = No. 8, 9" c.c.
B = No. 8, 9" c.c.
C = No. 8, 9" c.c., alt. rods 15'-3" lg.
D = No. 8, 4½" c.c., alt. rods 11'-10" lg.
E = No. 6, 18" c.c.
F = No. 6, F.L. (ull length)
G = No. 6, 27" c.c.
H = No. 6, 12" c.c., F.L.
J = No. 6, 24" c.c., F.L.
K = No. 4, 12" c.c.
L = No. 4, 24" c.c.
M = No. 4, 10" c.c., F.L.
N = No. 4, 27" c.c., 5'-0" lg.

Notes.—Exposed surfaces to be smooth finish. Ult. strength of concrete = 2,500 psi. Exp. joints 30'-0" ± c.c. Min. lap = 36 diam. Min. hooks = 8 diam.

FIG. 8-29. Dimensions and reinforcement.

placed under the rods, and the timbers in the background for making the keys and form-supporting strips which are shown completed in Fig. 8-27.

A final sketch of the wall pictured in Fig. 8-18, as it would be made for contract purposes, is shown in Fig. 8-29. The explanations for certain practical details that have been shown in the sketch are as follows:

1. Rods A are extended as dowels for the stem, splicing rods C which are placed on the concrete of the construction joint after the base is set. (Rods should not be "hung in the air" if it is avoidable.) If the spacing

of the bars is small, rods A should be placed farther from the back so that rods C can be set at their rear instead of beside them, thus avoiding a "screening effect" which might seriously injure the development of bond on the backs of these rods. The radius of the bend in rods A should be reasonable but not excessive.

2. Rods B are hooked into the footing and extended into the stem as far as they are required in order to avoid unnecessary laps. However, these rods must not be too long, or they will become difficult to hold in place. They are also useful in resisting the effect of the downward shear which comes from the loads on the heel. If the heel is large and all the rods from the stem are bent like rods A, there is no reinforcement that extends directly into the region of the compressive forces caused by the cantilever action of the heel.

3. Rods D are extended into the toe to develop them beyond the neutral axis of the stem. The designer should notice that there is considerable question regarding the bond stresses along rods D in front of the stem. A glance at Fig. 8-1(c) shows the fundamental actions of the parts. Both the heel and the toe tend to rotate in the same direction. A substantial fillet at the junction of the toe and the stem might help to develop the bond under the compressive side of the stem, but it would be somewhat troublesome to build. In fact, rods A are also a possible source of weakness in bond, but one may look upon them as "cables" carried through the concrete to prevent closing of the angle between the top of the toe and the front of the stem.

4. Rods E are added arbitrarily in the footing to care for the bending stresses caused by the weight of the stem before the backfill is placed.

5. Rods F are below the main rods so as to simplify the supporting of the latter.

6. Rods G are set on the construction joint so as to support rods H which serve as temperature reinforcement.

7. Rods J are in front of A, B, and C because it is assumed that the back form for the stem will be erected first; then C will be placed. Rods G will be set next; then rods H will be wired to them; after which the front forms will be erected.

8. Rods K are bent to knit the top of the parapet together and to develop them.

9. Rods L are stuck into the wet concrete and serve as ties for rods M, the latter being on the outside because they are placed later. They might be placed inside of K and L.

10. Rods N are anchors to hold rods G and H so that they will not be displaced during the concreting.

8-17. Stone-faced retaining walls. A buttressed masonry retaining wall is pictured in Fig. 8-30. Such a wall must be more or less of the

FIG. 8-30. A buttressed masonry retaining wall, Weehawken, N.J.

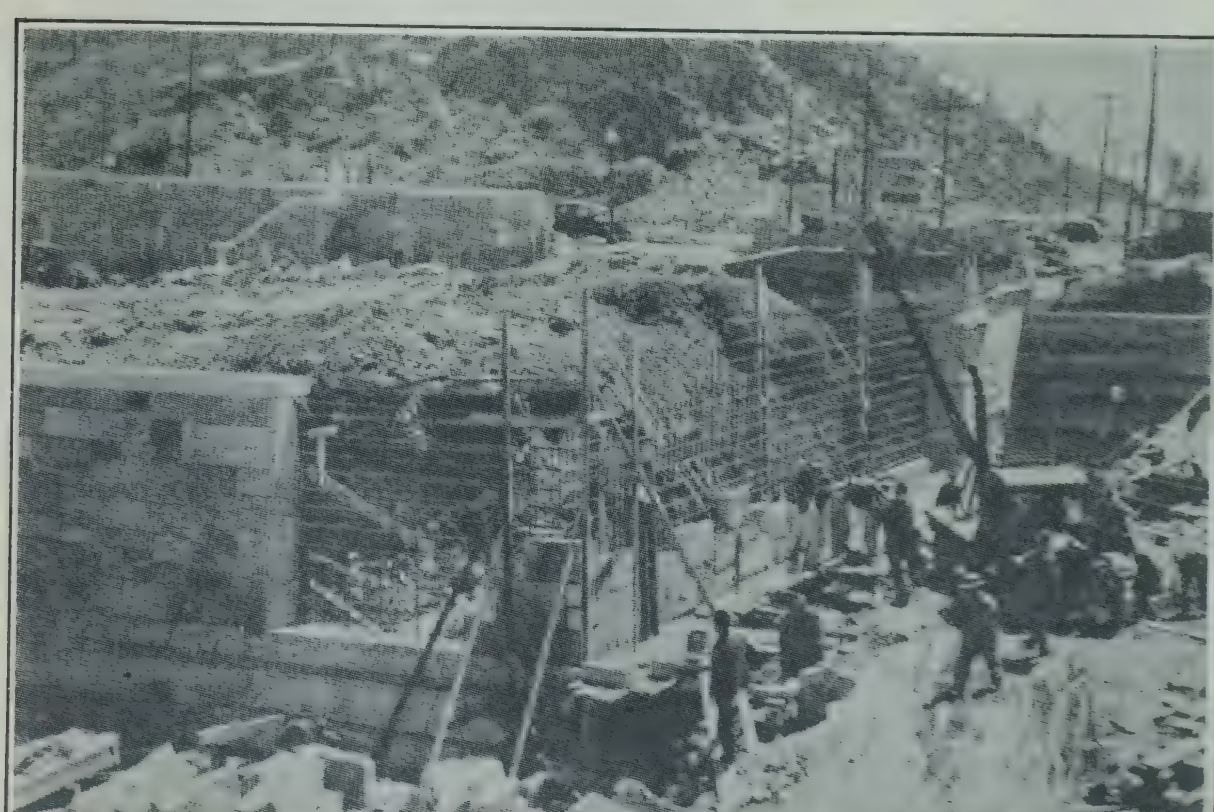


FIG. 8-31. Construction of a stone-faced retaining wall, New Jersey plaza of the Lincoln Tunnel.

gravity type. However, walls of similar appearance may be built with stone facing which is backed by reinforced concrete as shown in Fig. 8-31.

If the masonry is applied on the face of the concrete and anchored thereto after the latter is set, the wall should be designed as an ordinary reinforced-concrete structure, neglecting the facing. However, if the stone facing is laid first, as it should be, a few feet at a time, then backed up with concrete which bonds to it thoroughly, the stones should not be disregarded in the design. Bearing in mind that stress and strain are coexistent, it is not correct to assume that the stones carry no load. Therefore, some fundamental principles will be explained, and a method for the design of such walls will be outlined.

What is the distribution of the stresses upon a cross section of the wall of Fig. 8-32(a)? If the section is taken through the mortar—E = about

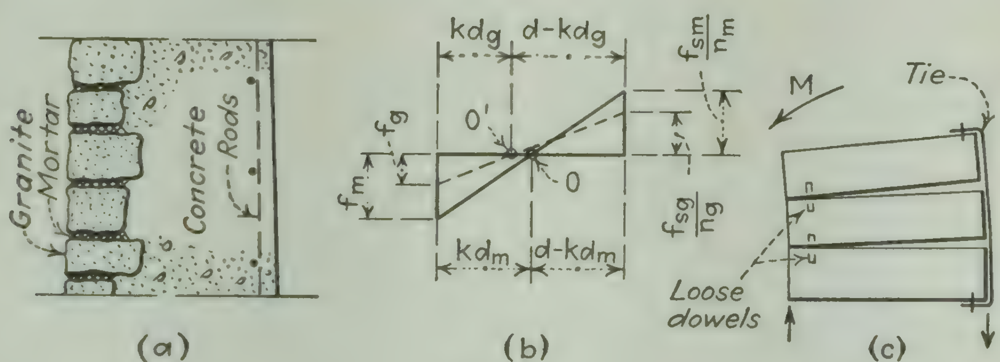


FIG. 8-32.

1,500,000 psi, for cement and lime, f'_c = about 1,500 psi—the location of the neutral axis will depend partly upon n, which may be 20. Theoretically, the situation will be as shown by the solid lines in Fig. 8-32(b). If the section passes through the granite—E = say, 6,000,000 psi and for solid granite f'_c = about 12,000 psi—the theoretical position of the neutral axis will be very different because n may be 5. The dotted lines of Fig. 8-32(b) show this latter condition. Obviously, the neutral axis will not jump from O to O' and back again at each stone; the unit stress in the mortar cannot be one thing and that in the stone something else; the stress in the steel opposite the mortar cannot be thousands of pounds different from what it is an inch or two away, opposite the stone. The stresses must be consistent and reasonable.

If the wall is assumed to be a set of blocks as shown in Fig. 8-32(c), with a rubber band tied to their backs and with shear dowels near their fronts, and if a moment is applied to the set, they will distort as pictured. The compressive stresses will be concentrated at the front edges of the blocks. The bands will stretch. Then $k = 0$, and $j = 1$. This will approximate the action of the granite and the steel alone if the wall is entirely granite and steel with no concrete. On the other hand, if the

wall is composed of concrete only, k may be somewhere around 0.25 to 0.4, and j may be 0.92 to 0.87. Furthermore, the mortar in the joints, being weaker than the granite, will control the allowable magnitudes of the compressive stresses.

In general, the values of Table 8-2 may be used, assuming the best mortar and good workmanship:

TABLE 8-2. Properties of Stone Masonry

Kind of masonry	Weight, pcf	E, psi	n	Allowable compressive stress, psi, f_c	
				Cement-lime mortar	Cement mortar
Best granite ashlar..................	165	4,000,000	7.5	650	800
Medium granite ashlar...............	160	4,000,000	7.5	500	700
Rubble..........................	150	2,000,000	15	250	350

The last column in this table may seem to indicate high values for f_c, but they will almost surely occur unless the wall is made unreasonably thick or the unit stress in the steel is kept very small. Unless the facing is relatively thin, the fact that some of the concrete is in compression may be neglected.

Therefore, in the design of masonry-faced walls, the following procedure is recommended:

1. Deduct at least 1 in. from the front face because of the weathering of joints. Then find d accordingly.
2. Use f_c and n as given in Table 8-2 for the materials and workmanship that are applicable.
3. Allow the usual maximum stress in the steel—18,000 psi—but the designer should notice that f_s will be low when a poor quality of masonry is used, because f_c must be low.
4. Design the wall as though it were concrete, but use the value of n from Table 8-2.
5. Be sure that the stones are well bonded into the concrete so as to resist the longitudinal shear.
6. Use expansion joints as for concrete walls.
7. Generally, unless the wall is very thick, use no front layer of steel because it will interfere with the work. Place all the temperature steel near the back.

A stone-faced wall supporting a side hill above a roadway cut is pictured in Fig. 8-33. The following features should be noticed:

1. The back of the stem is covered with membrane and asphalt waterproofing in order to avoid staining of the front by the leakage of water. This waterproofing is protected from injury by covering it with a 4-in. layer of concrete. (Sometimes bricks or 2-in. precast concrete blocks set in mortar are used.)

2. Since the wall rests upon rock, seepage water cannot readily pass under it. Weepers discharging onto the sidewalk will be objectionable; therefore a longitudinal drainage pipe with loose joints and surrounding gravel cover is placed on the heel. It has occasional outlets through the wall to a main drain under the roadway.

3. The toe is large and the heel is short so as to minimize excavation, but the latter must provide sufficient space for men to work in—about 21 in. clear.

4. The term *net line* means the theoretical line inside which no rock will be allowed to project. These lines are used as the theoretical dimensions of the base. The bottoms of the heel and toe are sloped upward in order to reduce excavation costs and because it is impossible and useless to require anyone to blast out rock so as to provide sharp reentrant angles or corners. A trench may look very nice on a drawing, but when the designer looks for it in the field he will probably see just a sort of gully. That is why it is usually customary to establish *payment lines* for excavation and concrete, these lines being beyond the net lines. However, the foundation must be clean and free from loose material. The concrete must be poured to sound bedrock as indicated in Fig. 8-33.

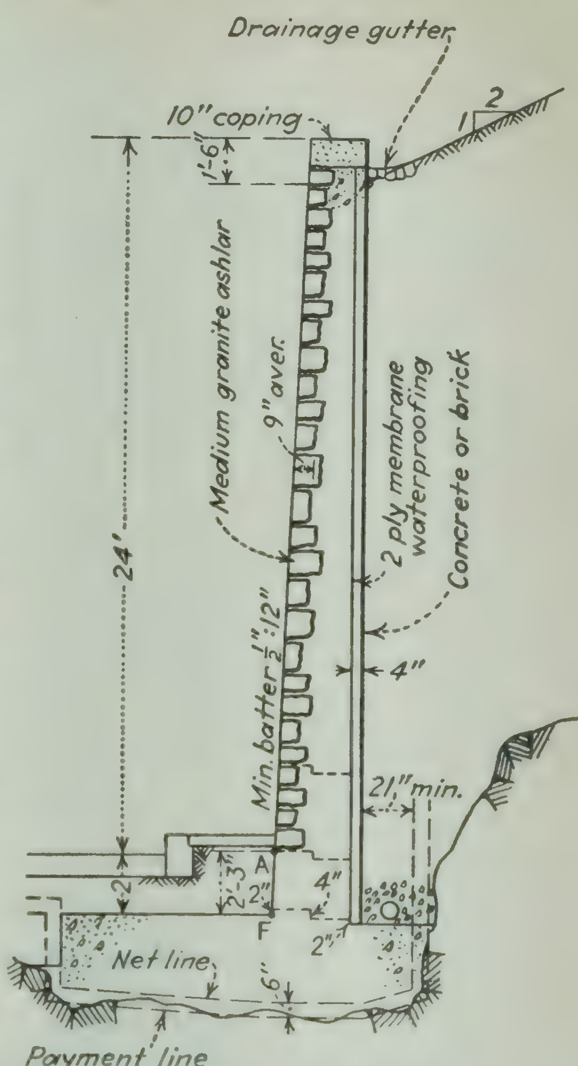


FIG. 8-33. A masonry-faced wall with membrane waterproofing.

5. The batter on the front face of the wall is used to avoid the optical illusion of leaning forward—an effect that may result if a vertical face is viewed against the background of the sloping hillside behind it.

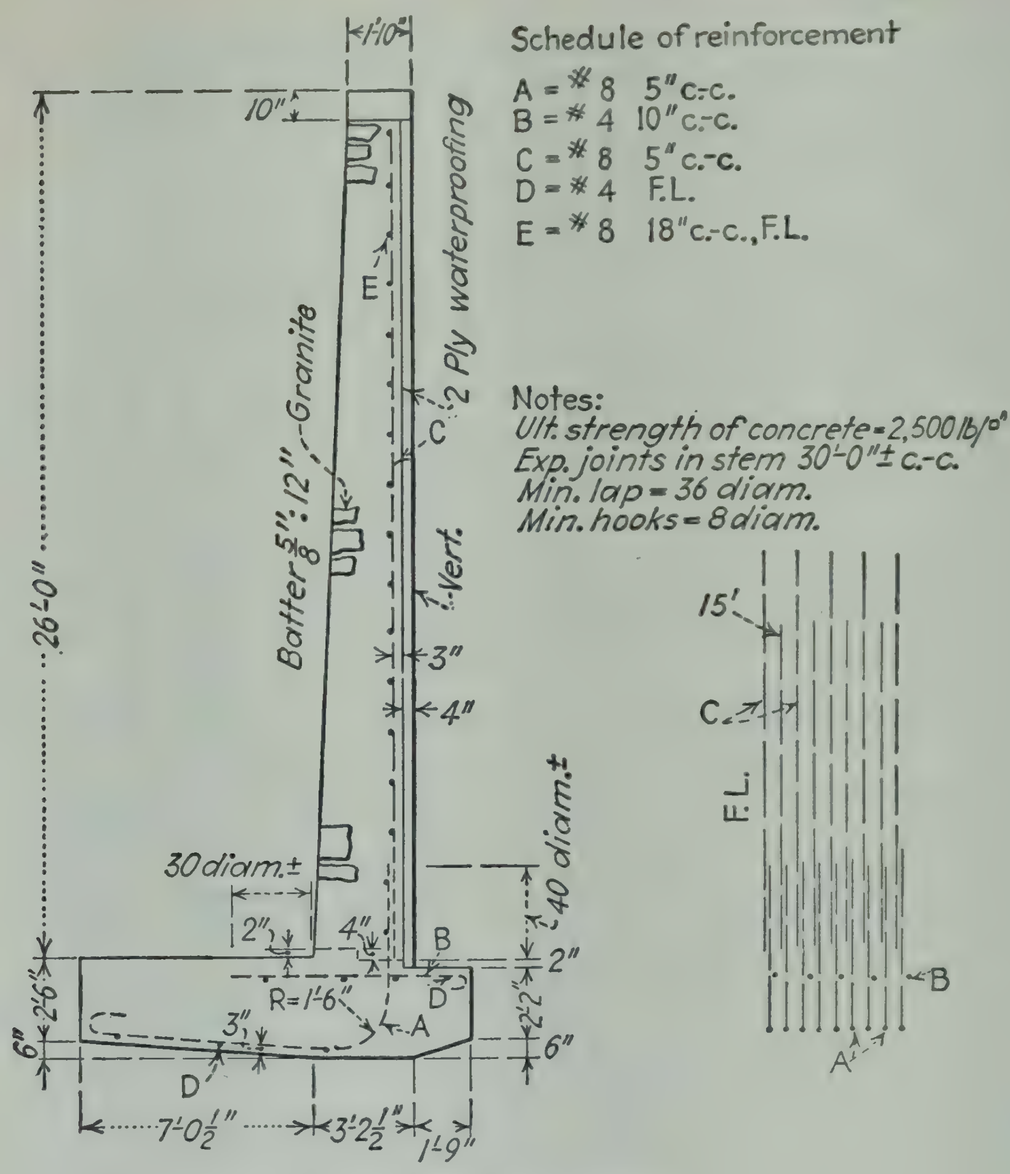
6. The toe is depressed below the curb and sidewalk in order to provide space for ducts and pipes and to get below frost depth.

7. The coping seals over the top of the masonry facing and the mem-

brane waterproofing. Both are thus protected against the penetration of water and loosening of the materials.

The reinforcement used in this wall is shown in Fig. 8-34.

8-18. Miscellaneous data. The wall in Fig. 8-27 appears to be tapered—and it really is—becoming shallower in the background. This



Dimensions and Reinforcement
FIG. 8-34.

varying of height is a common occurrence, and it introduces some problems in detail. These problems vary with the particular situation, but some suggestions that may be helpful are the following:

1. Design the wall for a series of heights varying by 4 or 5 ft; then interpolate for intermediate heights if necessary.

2. Fix the alignment of the wall with the front edge of the top or coping as the controlling line.

3. Use a constant width of coping and top thickness of the stem unless

the range of variation of height is too great. In such a case, make one or two definite breaks in the back of the coping and wall at contraction or expansion joints in order to decrease the thickness of the top.

4. Maintain a constant depth of coping or uniform top marking arrangement as shown in Fig. 14-5.

5. If the top of the wall is level whereas the bottom varies, keep intermediate markings and joints level also; if the top slopes and the bottom is level, keep them parallel to the bottom. In neither case should markings or joints die out to feather edges. It is better to make distinct breaks at vertical construction joints. In the case of Fig. 14-5,

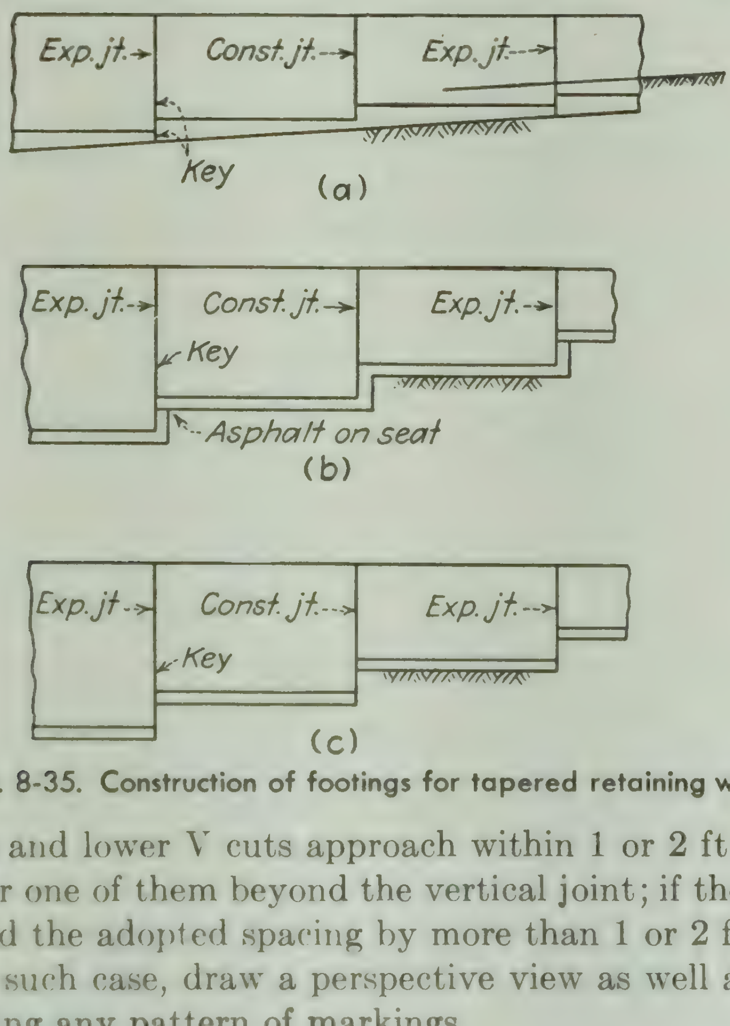


FIG. 8-35. Construction of footings for tapered retaining walls.

when the top and lower V cuts approach within 1 or 2 ft of each other, omit the lower one of them beyond the vertical joint; if they are separating and exceed the adopted spacing by more than 1 or 2 ft, add another one. In any such case, draw a perspective view as well as an elevation before accepting any pattern of markings.

6. Use a constant batter for the front and rear faces, starting from the top, and let the width of the stem vary at the bottom.

7. If the rate of slope of the bottom is small, the lower side of the footing may be set parallel to the grade whereas its top, if buried, may be in level steps as pictured in Fig. 8-35(*a*). This uses more concrete but it simplifies the formwork and the reinforcing.

8. If the rate of slope of the bottom is large, the lower side of the footing may have to be stepped as in Fig. 8-35(*b*). Note particularly the filler wall at the offset and the relation to the vertical joint. The foot-

ings should not be made as shown in Fig. 8-35(*c*) because the differences in action at the junction of the higher and lower portions will almost certainly crack the wall. Furthermore, be sure that the excavation can be stepped safely without weakening the bearing value of the soil too much. It may be advisable to excavate as near the desired shapes as possible, then pour the footing directly on the soil, wherever it is.

9. Endeavor to use a uniform style of reinforcement and size of rods in the wall, varying the spacing in sections in order to simplify the work. Do not attempt to vary the spaces by less than 1 in. However, make a complete change when necessary; *e.g.*, do not try to use No. 11 rods in 6-ft walls.

Practice Problems

8-1. Assume the concrete retaining wall shown in Fig. 8-36(*a*). Let w, the weight of earth, = 100 pcf; ϕ, the angle of internal friction = 34°. Using these data find the following:

a. If the earth is level and flush with point *A*, find the bending moment at the level of *K* for the earth pressure alone.

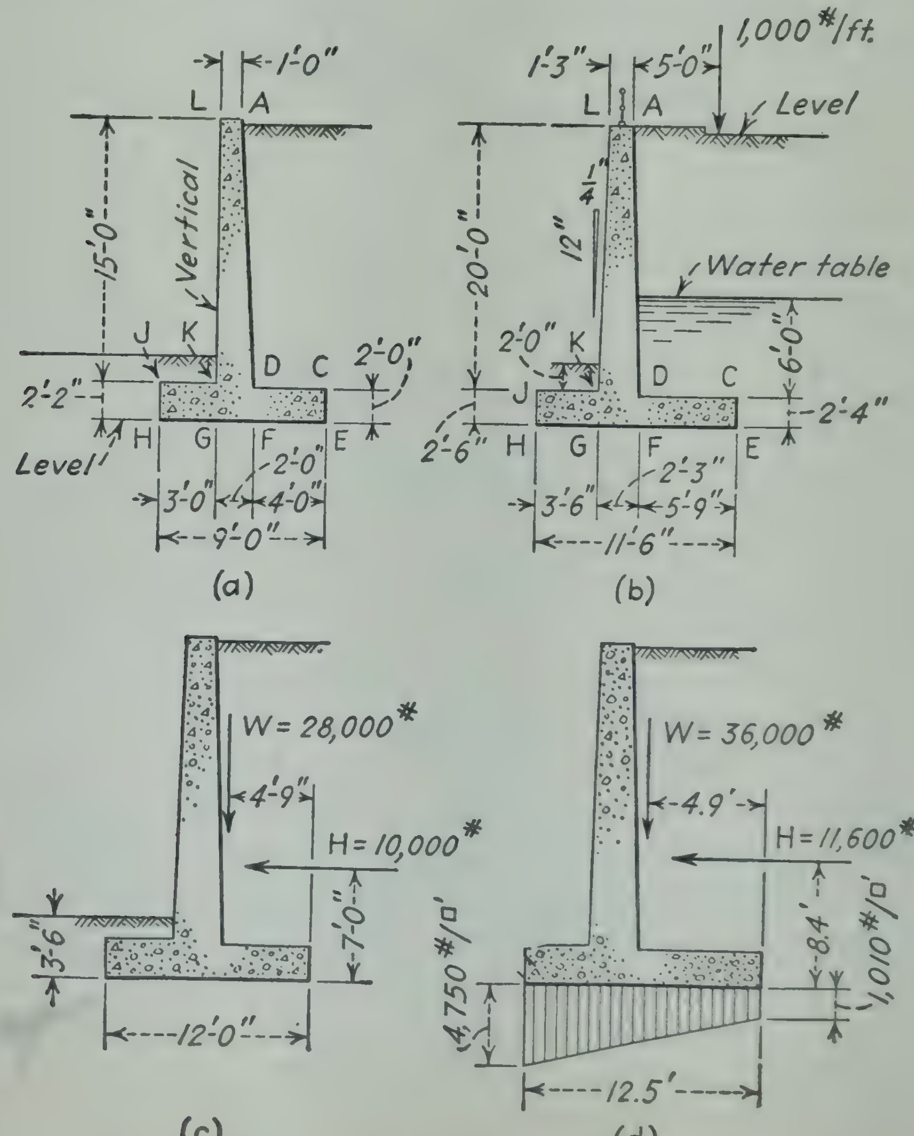


FIG. 8-36.

Discussion. Use Eq. (8-6) with $\delta = 0$ to find P, which is inclined at the angle ϕ; then find the horizontal component of P in order to compute the bending moment.

Ans. $M = 12{,}000$ ft-lb.

b. Repeat part *a*, but include the effect of water pressure for an 8-ft head above K at 45 psf per ft of depth, plus ice pressure of 700 plf at A.

c. How much bending moment is caused at K by the surcharge effect of 1,000 lb per ft of wall at a point 5 ft behind A? Combine this with *b* and *a*, but assume that the full magnitude of P is to act horizontally.

Discussion. Use the data in Figs. 8-13 and 8-14 to find the magnitude and position of the lateral thrust.

8-2. For the retaining wall in Fig. 8-36(*a*) determine the safety factor against overturning and sliding, also draw the pressure diagram under the footing. Assume

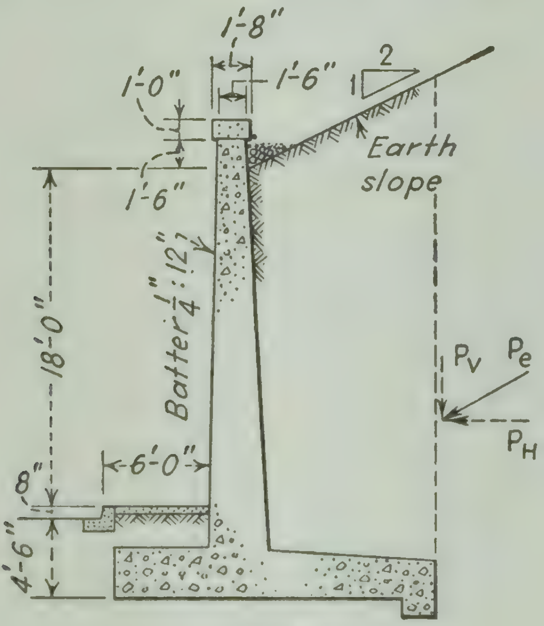


FIG. 8-37.

$w = 110$ pcf, $\phi = 35°$, allowable soil pressure $= 7{,}000$ psf, and the coefficient of friction $= 0.6$. Include earth pressure, also water pressure for a head 10 ft above HE. Neglect the effect of the earth on top of the toe. Also, neglect the abutting resistance of the earth in front of the wall. The earth pressure is inclined at 35°.

Ans. Safety factor against overturning about $H = 3.65$. Safety factor against sliding $= 1.73$. Soil pressure at $H = 1{,}920$ psf and at $E = 1{,}640$ psf.

Data for Probs. 8-3 to 8-10, inclusive (assume the earth pressure to be horizontal):

$w = 100$ pcf
$\phi = 30°$
Allowable pressure on soil $= 6{,}000$ psf
Coefficient of sliding friction of soil $= 0.6$
$f'_c = 3{,}000$ psi $\quad f_c = 1{,}000$ psi $\quad f_s = 20{,}000$ psi
$v'_L = 90$ psi $\quad u = 210$ psi
Water pressure $= 45$ psf per ft of depth
Determine surcharge as in Art. 8-7.

Consider the weight of earth on the toe. Also include the abutting resistance of the soil in front of the wall, assuming this resisting pressure $= 300$ psf per ft of depth.

8-3. Design and detail the reinforcement for the stem of the retaining wall shown in Fig. 8-36(*b*). Consider pressures caused by earth, water, and surcharge as shown.

8-4. Determine the total vertical load W of the wall of Fig. 8-36(b), and compute the horizontal forces acting on it. Then locate the position of their resultant at the base HE and find its eccentricity.

Ans. W = 22,500 lb 4.66 ft from E. H = 9,700 lb 7.25 ft above E. e = 2 ft on the left side of the center line.

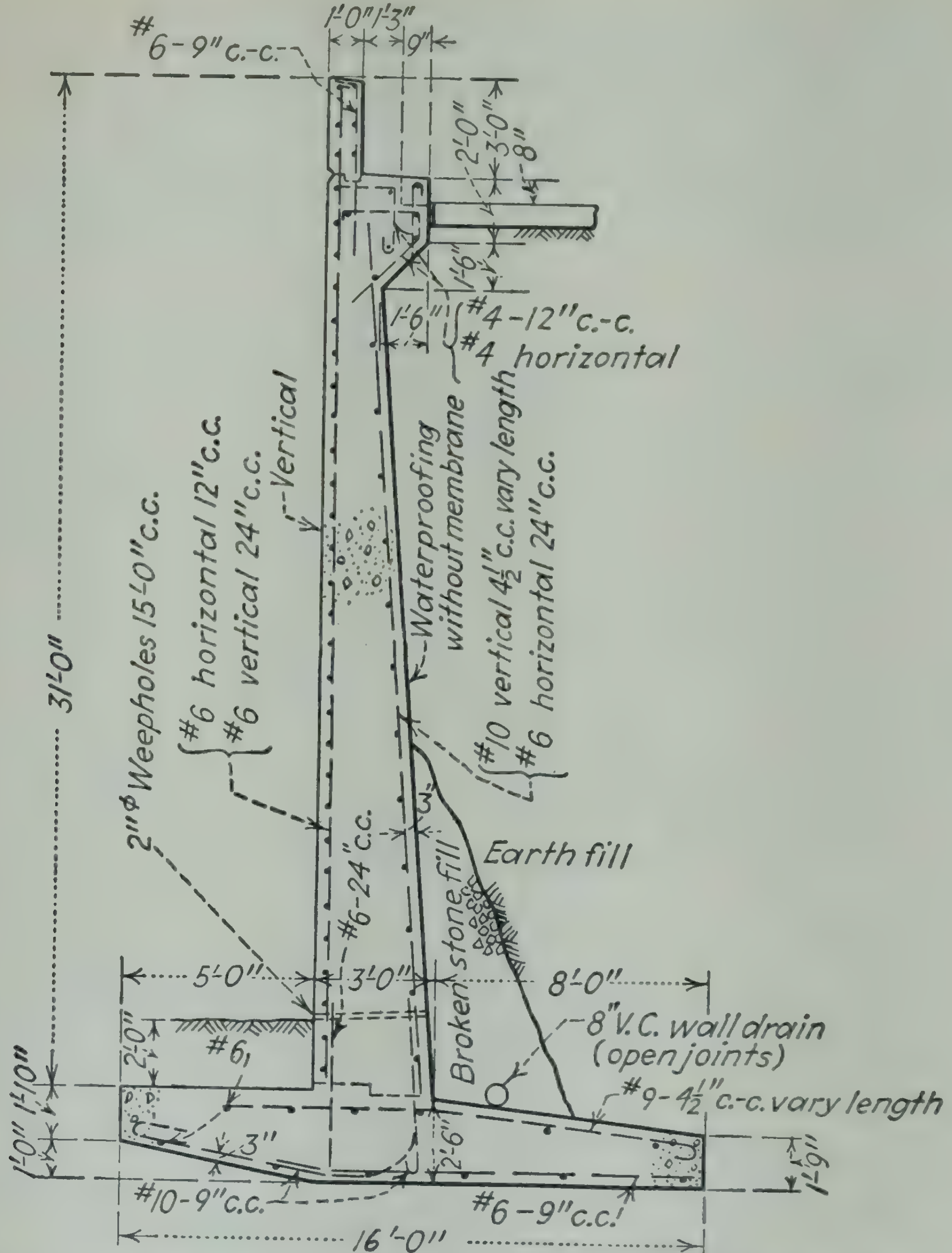


FIG. 8-38. Retaining wall designed for the connection to the Harlem River Speedway, Highbridge Park, New York City.

8-5. Recompute Prob. 8-4 if the height of the stem is reduced from 20 to 18 ft and the heel FE is 5 ft long.

8-6. With the resultant forces shown in Fig. 8-36(c), determine the pressure diagram under the footing.

8-7. Using the resultant forces shown in Fig. 8-36(c), determine the safety factor against overturning about a point 6 in. inside the edge of the toe. Also determine the safety factor against sliding, including the effect of the soil in front of the wall.

8-8. Using the data shown in Fig. 8-36(*d*), design the reinforcement in the heel and toe. Heel = 6 ft; toe = 4 ft; d = 24 in., h_1 = 25.2 ft.

8-9. Using W and H as given in Fig. 8-36(*d*), see if this wall has a safety factor of 2 for overturning, sliding, and soil pressure.

8-10. Design a retaining wall for the conditions shown in Fig. 8-37. Assume that the earth pressure is inclined at the angle ϕ as indicated. Assume that its components act as shown by the dotted lines.

8-11. Assume the heavy concrete retaining wall of Fig. 8-38. Analyze it completely if the following data are to be used: w = 100 pcf, ϕ = 34°, δ = 0; P is to act horizontally with its full magnitude (because of vibrations, its vertical component being assumed = 0); surcharge is to be as given in Figs. 8-13, 8-14, and 8-15 for W' = 1,000 lb per ft of wall, the wheel load being applied 1 ft from the curb; ice pressure = 500 plf; water pressure = 45 psf per ft of depth with a head of 8 ft above the top of the toe; f'_c = 3,000 psi, n = 10; the maximum coefficient of friction = 0.6; the safety factor = 1.75.

9

FOOTINGS

9-1. Introduction. A building or a bridge is generally considered to have two main portions—the superstructure and the substructure. The latter is often called the *foundation*. It supports the superstructure, but it may contain various parts or units of its own. There are many special kinds of foundations for which concrete is used, but this chapter will be confined mostly to reinforced-concrete footings.

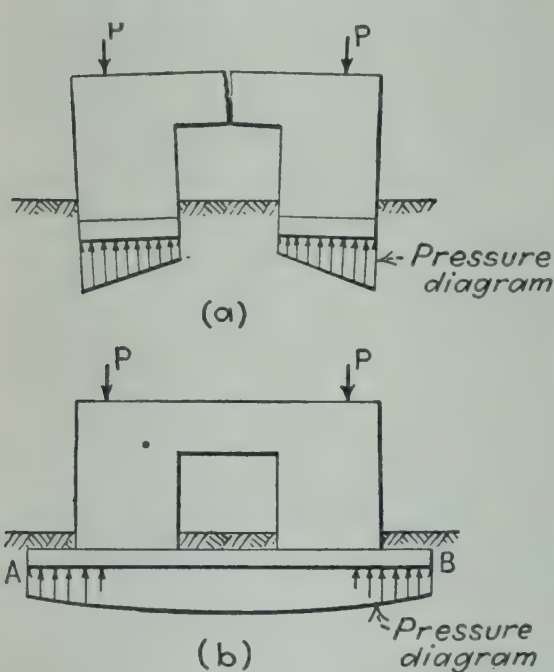


FIG. 9-1.

The term *foundation* generally includes the entire supporting structure. Sometimes, as in the discussion of retaining walls, it is used to designate the material upon which the wall is supported. It must not be confused with the word *footing*, which generally applies only to that portion of the structure which delivers the load to the earth, as illustrated by AB of Fig. 9-1(b). These are called *spread footings* because they distribute the concentrated load over a large area which has a low intensity of pressure. When a soil under a building is so poor that the footings are all joined together in one large slab or floor under the whole structure, the resultant footing is sometimes called a *reinforced-concrete mat*.

An *isolated footing*, which may also be called a spread footing, is an individual member that is used generally to distribute the load from the base of a column. If small, it may sometimes be made of plain concrete as in Fig. 9-3; if large, it may be reinforced like one of those shown in Fig. 9-4.

Footings are also used to support heavy walls, piers, abutments,

machines, and many other structures whose large loads have to be spread over a considerable area of soil.

When concrete rests directly upon sound rock, the latter often has a strength equal to or greater than that of the concrete itself so that there is no difficulty in obtaining sufficient support for the superstructure. Sometimes, concrete caissons or piles are used to carry the loads through inadequate material to rock or some other suitable stratum at a lower level. The illustrations here will be limited to footings that transmit the loads directly to earth or piles.

9-2. Fundamental principles. There are certain fundamental principles to be considered in the design of earth-borne footings. When a load is placed upon earth, the latter is compressed somewhat. The amount of this deformation depends upon the intensity of the load, the loaded area, the nature of the soil, the depth at which the load is applied, and similar matters. These settlements will not be investigated here, but they must be kept in mind because it is essential to plan any foundation so that the entire structure will settle equally. If the unit bearing pressures under the footings vary greatly, the settlements of different parts of the structure are likely to vary also, causing cracks to appear. For instance, if a wall is loaded as shown in Fig. 9-1(a), the ends tend to settle and cause a crack in the top. It is best to have the pressure uniformly distributed; but if this cannot be done, it is desirable to have the greater deformations near the center so that there will be a tendency for the structure to compress near the top rather than to open up. Of course, cracks at the bottom should also be prevented.

No footing should tilt harmfully. The resultant of the resisting pressures under the footing will pass through the center of gravity of the applied loads, including those of the footing itself. However, if the resultant is offside the center of gravity of the bearing area, the side having the higher pressures may settle more and cause the structure to lean in that direction.

The effects of live loads and impacts are usually omitted in the study of the settlements of footings unless the structure is a warehouse or a building which may be subjected to large live loads for long periods of time. The reason for this is the fact that the soil will not usually move or squeeze out quickly. One-half or more of the live load may be considered in the case of warehouses, but the uniformity of the distribution of the foundation pressures for dead load alone must be investigated also. However, the live load and impact must be included in the loads for which the individual footings are designed.

If the foundation of a building is partly upon rock and partly upon earth, the situation is dangerous. This is obvious. In such a case, one should use caissons or piles down to the rock under the portion of the

structure that does not rest directly upon it, isolate the two portions of the structure so that the settlement will not cause trouble, or, if the condition is unavoidable, use a very low intensity of pressure upon the soil.

It is advisable to place the bottoms of all footings below the frost line, usually 4 to 5 ft in cold climates.

The safe bearing value of any soil is a difficult thing to ascertain. Borings and loading tests should be made at the site before any important structure is built. However, Table 8-1 may be used as a general guide. When a foundation goes to a depth of 10 ft or more below the surface or any adjacent excavation in natural undisturbed granular soil, these specified soil pressures may be increased to some extent.

The vertical component of hydrostatic pressure under a foundation is not considered as an additional load in the design of the footings. Since $\Sigma V = 0$, it makes no difference if the structure tends to float like a boat as a result of hydrostatic uplift, because the total pressure is dependent upon the weight of the entire structure. However, the side walls of basements in such special cases must be designed to withstand lateral pressures, and the basement floor will have to be of the mat type—or partially so.

As stated in connection with retaining walls, the pressure upon the soil should be computed by including the weight of the footing, or at least the excess weight of the footing over that of the earth originally above the bearing area, but the loads for which the footing itself must be designed may not include the weight of the footing itself because the wet concrete is already supported by the earth before it sets. Backfill is also neglected when computing shears and bending moments in footings because its weight is generally fairly uniform over the footing and its direction is opposite to that of the bearing pressure. This reduced intensity of load may be called the "net" pressure upon the footing.

Besides Table 1-8, the following are some of the requirements of the Code regarding isolated footings and wall footings on soil:

1. For concentric loads without accompanying overturning tendencies, assume the load to be uniformly distributed over the bearing area of the column, pedestal, wall, or metallic column base and footing, and also over the soil or piles underneath.

2. Reactions from piles are assumed to be concentrated at their centers.

3. Sloped and stepped footings must be so proportioned that the allowable stresses are not exceeded at any point.

4. The external bending moment on any section is to be determined on the basis of a vertical plane extending clear across the footing. Com-

pute the moment of all the forces acting on one side of this plane.[1] The location of this plane shall be

a. At the face of a concrete column, pedestal, or wall.

b. Halfway between the middle and edge of masonry walls.

c. Halfway between the face (or tips of flanges) of a steel column and the edge of the metallic base.

5. The width resisting compression at any section is to be assumed as the entire width of the top of a flat-topped footing at that section, and the area resisting compression is the area above the neutral axis of the section.

6. Calculations for the maximum shear to be used in the equation for bond, $u = V/(\Sigma o)jd$, are to be based upon the same section and loading as for bending moment. Points of change of section or of reinforcement are also to be investigated similarly.

7. The critical section for shear to be used in computing diagonal tension by the equation $v_L = V/bjd$ shall be assumed as a vertical section obtained by passing vertical planes through the footing as shown by BA and QR in Fig. 9-6. Each plane is parallel to a corresponding face of the column, pedestal, or wall and located a distance d therefrom.

The straight-line theory will be used here for the analysis of footings. Such structures are usually made with thick sections, and they are generally underreinforced.

9-3. Concrete pedestals. Even when a structure rests upon rock it is customary to use some sort of footing under each column. Thick chunky members like the footing in Fig. 9-3(a) are sometimes called *pedestals*. This term is also used to describe the upper part of the stepped footing of Fig. 9-4(a), and it will be used in this sense hereafter.

It must be remembered that the reinforcement in concrete cannot be effective in its ordinary function unless the concrete can elongate or bend. Some footings are primarily blocks rather than members which act as beams. For instance, suppose that a column is supported by a concrete footing which rests upon rock as pictured in Fig. 9-2(a). The reinforcement which is shown at the bottom is almost perfectly useless. If the rock is properly prepared and roughened, the footing cannot possibly spread sideways so as to open up cracks and to stretch the reinforcement because of the shearing resistance of the rock. Then, since there is no strain in the direction of the rods, there can be no stress in them.

The lines of the forces in a thick base are somewhat as pictured in Fig. 9-2(b), a combination of direction compression and shear. The only rein-

[1] This is apparently in accordance with the tests reported by Frank E. Richart, Reinforced Concrete Wall and Column Footings, *J. ACI*, October and November, 1948.

forcement that will do much good in such a case is that which will prevent the formation of cracks at the top of the footing, such as those near the upper corners in Fig. 9-2(*b*). These cracks are not likely to occur if the footing or pedestal has sufficient area.

The Code states that the bearing pressure on the top of a footing or pedestal under a concrete column or the base plate of a steel column should not exceed $0.25f'_c$ if the full area is loaded; $0.375f'_c$ if one-third of the area is loaded; and proportionately between these values when the loaded area is less than the full area but more than one-third of that

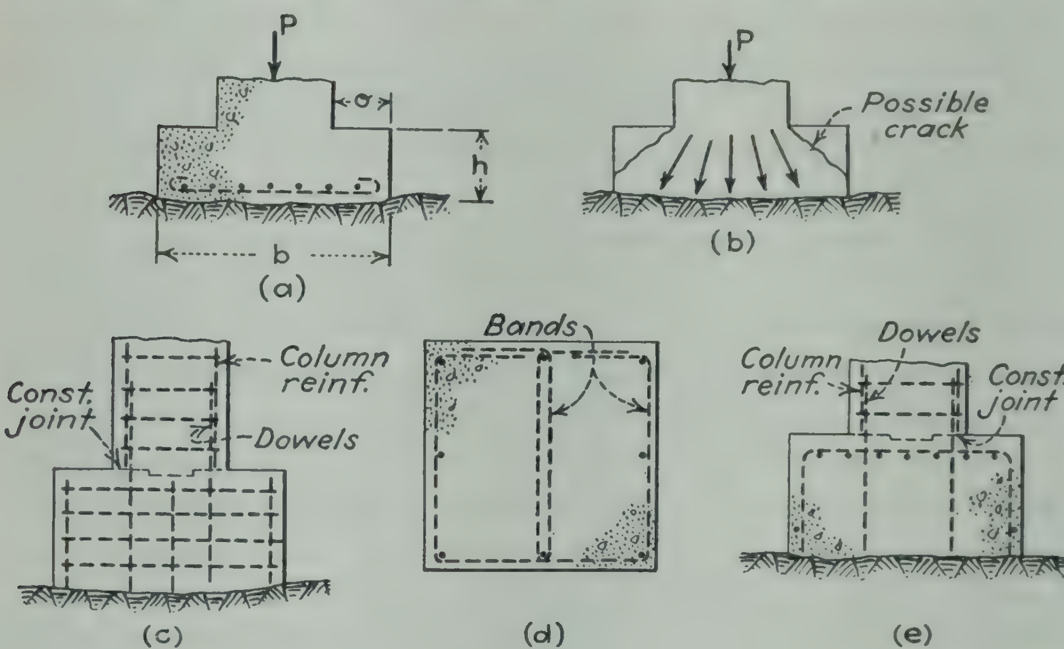


FIG. 9-2.

area. If the bearing stress exceeds these limits the base should be reinforced as a column.

Figures 9-2(*c*), (*d*), and (*e*) show some ways of arranging the reinforcement in pedestals and in thick bases resting upon rock. The first two picture hoops or bands with a few supporting ties, alternate bands being reversed in direction. The last shows a two-way mat with the rods bent down. Both schemes are to prevent bulging and failure like that of Fig. 9-2(*b*).

Whenever a heavily reinforced column rests upon a concrete pedestal (or footing), the stresses in the longitudinal rods cannot suddenly vanish. If the steel is needed in the column shaft, it, or its equivalent, is also needed where the column joins the pedestal. It is therefore necessary to extend the bars down or to provide dowels that have at least the same area as the main rods and that extend up into the column and down into the pedestal sufficiently to develop the full working stress by means of bond. Such dowels are shown in Fig. 9-2(*c*). Spirals are not often used in the pedestal. It is possible to have the ends of the longitudinal rods

in the column squared off and made to bear directly upon a steel slab which rests upon the concrete, although this is not customary.

The dimensions of a pedestal or a footing like that of Figs. 9-2(*a*) and (*c*) are largely a matter of judgment. If founded upon rock the distance *o* should not be less than 4 or 6 in. so that the forms for the column can be supported upon the pedestal. The depth *h* must be sufficient to develop the dowels. In no case should an unreinforced-concrete pedestal have a depth that is less than the offset *o*.

When a plain concrete pedestal is on earth, it must be large enough to spread the load without exceeding the allowable pressure upon the soil. In such a case, too, a certain amount of bending may be set up in the pedestal itself which then becomes a short thick cantilever beam acting in two directions. It can be designed as a beam of homogeneous material, using a limiting tensile stress of $0.03f'_c$ and a maximum diagonal tensile stress of $0.02f'_c$, according to the Code. The principles will be illustrated by direct application to a problem.

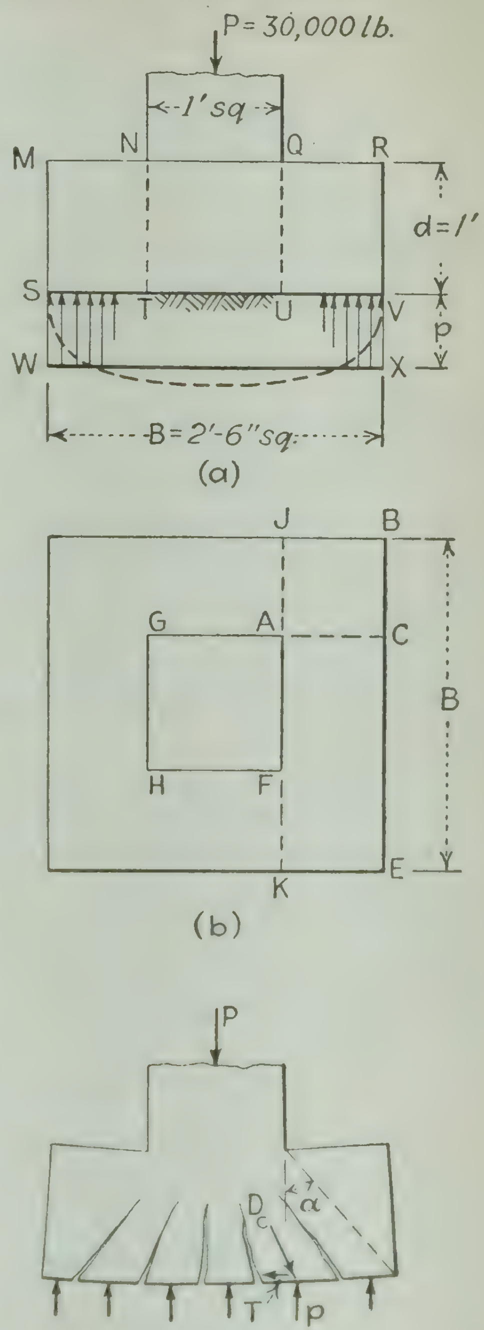


FIG. 9-3.

Example 9-1. Find the stresses in the plain concrete pedestal of Fig. 9-3(*a*) if $f'_c = 2{,}500$ psi.

Although the intensities of the soil pressures may vary somewhat as shown by the dotted diagram in Fig. 9-3(*a*), it is customary and satisfactory to assume that they are practically equal at all points. The rectangle *SVXW* will therefore represent these pressures. Since the weight of the footing itself is excluded, the net upward pressure

$$p = \frac{P}{B^2} = \frac{30{,}000}{2.5^2} = 4{,}800 \text{ psf}$$

At *NQ*,

$$f_c = \frac{P}{A} = \frac{30{,}000}{144} = 208 \text{ psi}$$

This is safe because the area across *MR* is more than three times that of *NQ*, and f_c is much less than $0.25f'_c$. Therefore, for a small load like this, no reinforcement is needed as far as the pressures at the top of the pedestal are concerned.

The upward pressures upon the bottom of the pedestal tend to cause compression in its top and tension at the bottom so as to split it apart, as shown in exaggerated fashion

in Fig. 9-3(c). The concrete, since it is unreinforced, must act as a beam of homogeneous material in resisting this tension.

The footing under a continuous wall acts as a pair of cantilever beams which bend one way, but this pedestal must bend in two directions. Therefore, as a typical case, assume that the portion of the base that is outside the column area [$AFHG$ of Fig. 9-3(b)] is cut by the plane JK in Sketch (b). The portion $JKEB$ acts as a cantilever about face AF in resisting the bending moment caused by the pressures from U to V of Sketch (a). Therefore,

$$M = p \times JK \times AC \times \frac{AC}{2}$$

$$M = 4{,}800 \times 2.5 \times 0.75 \times \frac{0.75}{2} = 3{,}370 \text{ ft-lb, or } 40{,}500 \text{ in.-lb}$$

Taking the full section JK as effective in resisting this moment,

$$f_t = M \div \frac{Bd^2}{6} = \frac{40{,}500 \times 6}{30 \times 12^2} = 56 \text{ psi}$$

This is less than $0.03 \times 2{,}500$ and will be accepted.

In calculating the shearing stresses in a pedestal or footing, the punching shear around the perimeter of the column is sometimes considered. This is supposed to be a pure shear without any combination with tension. However, it is difficult to see how the uncracked pedestal can fail in this way. If the angle α of Fig. 9-3(c) is less than 45°, a part of the effect of the upward pressures p will be a diagonal compression D_c which transmits them directly to the column. There will be also a resisting tension T. It is apparent that this condition is not one of pure shear or pure flexure. Therefore, in this case, no diagonal tensile stress needs to be computed, since α is considerably less than 45°.

If the pedestal is so wide that 45° slope lines from N and Q of Fig. 9-3(a) fall inside S and V, the footing generally ought to be reinforced.

The depth QU of Fig. 9-3(a) should be able to develop the dowels extending into the column in ordinary cases. If these are assumed to be No. 5 bars, if any hooks or right-angle bends at their bottoms are neglected (as they should be when in compression), if there is a cover of 3 in. at the bottom, and if the steel stress is 12,800 psi and the allowable bond stress is 175 psi, then the length of bar required below NQ is

$$L_s = \frac{12{,}800 \times 0.31}{1.96 \times 175} = 11.6 \text{ in.}$$

Therefore, the pedestal should be made at least 15 in. deep, or a sufficiently large number of smaller dowels should be used. However, this column load is obviously very small and the column is so lowly stressed that the specifications do not really mean much in this case.

A plain-concrete footing may be used under a concrete wall if its depth exceeds twice the projection beyond the face of the wall. It can be analyzed as previously described by considering a rectangular piece 1 ft wide. However, one should question its use if the loads are very heavy and the structure is important.

9-4. Reinforced-concrete footing for a column. When a column load is large and it must be spread over a considerable area of earth, a

plain concrete pedestal will not be strong enough unless it is very thick. It is then economical to use a reinforced-concrete footing. Such a footing may be made as one large flat slab, it may be stepped as in Fig. 9-4(*a*) in order to save concrete, or it may be sloped on the top as pictured in Fig. 9-4(*b*).

The design of a footing is not a very exact affair. The members are short and thick. They do not curve in one direction like an ordinary cantilever beam but tend to curve like a saucer; hence they are structurally indeterminate. Shearing and bond stresses may be relatively high.

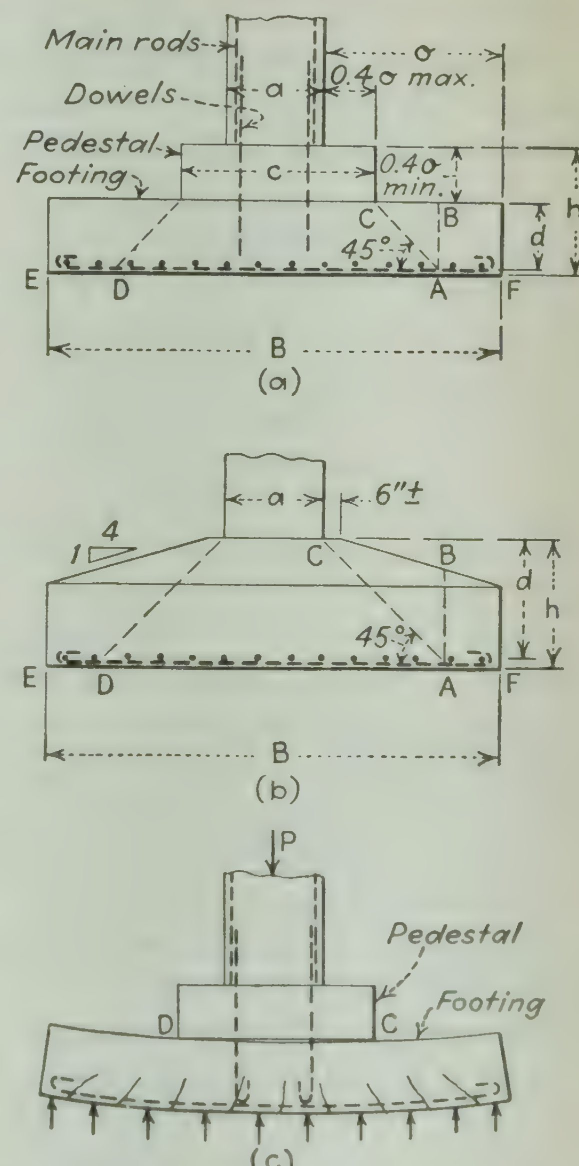


FIG. 9-4.

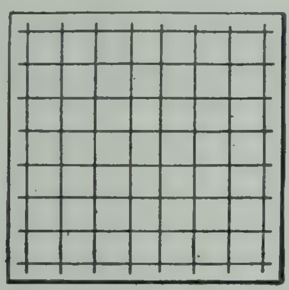
The upper part, or pedestal, of a stepped footing should be poured monolithically with the main footing so as to avoid failure through longitudinal shear at the junction between the two. This causes trouble in the formwork and sometimes in slumping of the concrete from the higher to the lower level. However, the pedestal is useful in reducing the stresses in the footing and in providing embedment to develop the dowels. It should not project too much—possibly not more than 0.3 or 0.4 of the offset *o* —and its depth should not be less than this projection [Fig. 9-4(*a*)] for proper proportioning.

The action of the pedestal of a stepped footing will be more clearly understood by examining Fig. 9-4(*c*). If the footing bends under the load, there will be compression in the concrete between *D* and *C*. The curvature of the footing, if it could be so extreme, would cause a crack between the footing and the pedestal, with the latter receiving a concentrated compressive load near *D* and *C*. These loads, in turn, might tend to cause tension in the bottom of the pedestal, but it is impossible to have the footing shorten from *D* to *C* while the pedestal elongates at the same point unless the concrete fails along the junction. However, it is sufficient to assume that the pedestal is an unyielding support for

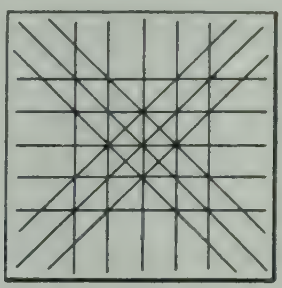
the footing and that the critical bending moments will be in the cantilevered portions of the footing.

The reinforcement for the footing may be arranged in two layers as shown in Fig. 9-5(*a*), or it can be placed in two normal and two diagonal bands as pictured in Fig. 9-5(*b*). The latter is not worth while in ordinary simple footings because of the complication of the details of the reinforcement and the packing up of the four layers at the center.

For both types of footing shown in Figs. 9-4(*a*) and (*b*), it is customary to assume that there will be a sort of cone of compression which will act downward from the pedestal or column, the outer slope being inclined at



(a)



(b)

FIG. 9-5.

45° and shown by *CA* in the figures. This cone intersects the rods at *A* and *D*. The pressure upon the portion *AF* will cause a shear along section *AB*. This is assumed to be the condition that produces the critical diagonal tension. Since all four sides are made the same in this case, the effective area of this section, as shown in Fig. 9-6, is the depth *d* at section *AB* times the width *QR* between the 45° diagonals *MO* and *NP*.

For determining the tensile reinforcement in two-way footings, the Code permits the use of 85 per cent of the theoretical bending moment at the vertical section as computed by statics. This is supposedly because of the two-way action. However, it does not agree with Richart's conclusions from his tests; but, admittedly, the magnitudes of the stresses in an isolated footing at working loads are very uncertain. In general, the author believes, after studying the report of Richart's tests, that it may be advisable to use the full static moment in order to be sure that no harm will result, because the extra cost is so small compared with the importance of the footings. On the other hand, he does not know of any properly designed footings that have failed under ordinary usage. Probably the soil is the weak partner of the combination, and it will be likely to fail before the footing itself does since one will seldom get such a large overload that the steel will fail in tension or the concrete in compression. Diagonal tension is likely to be more critical for the footing itself. Therefore, the Code will be followed here.

The Code permits the tensile reinforcement of isolated footings to be

spread uniformly across the full width. The author prefers to place most of the bars within the 45° slope intersections shown by DA in Fig. 9-4, with only a few in the edge spaces ED and AF.

The full width of the footing is to be used when computing the compression in the concrete of stepped footings. In the case of those with sloped tops like Fig. 9-4(b), the best way is to analyze the member by means of the transformed-section method. However, this is laborious, and the compressive stresses are usually relatively small. A very rough approximation may be made by assuming a rectangular section of depth d and width equal to $\frac{1}{2}[(a + 2 \times 6) + DA]$ of Sketch (b).

For testing bond, the shear should be computed by using the same forces as those used in calculating the bending moment, then using 85 per cent of this shear in the formula $u = V/(\Sigma o)jd$ for two-way footings. Because of the action of reinforced concrete under the cracked condition, as explained in Chap. 3, it seems to be desirable to provide some kind of mechanical anchorage at the ends of the bars. Instead of a standard hook, the author often uses an upstanding 90° bend so that he avoids the interference of hooks at the corners of a footing. Footings are so deep, the tensile stress must by developed so fast, and the cost of guaranteed safety is so small that it seems unwise to eliminate the anchorages of the main bars of heavily loaded footings.

In two-way rectangular footings, the bars in the long direction may be spaced uniformly across the full width, the bending moment being computed on the basis of the full section as for square footings. The rods in the short direction may also be determined on the basis of the bending moment on the full section. For computing the reinforcement each of these moments is to be multiplied by 0.85. Then, according to the Code, the part of this reinforcement (across the short way) shown by Eq. (9-1) should be uniformly distributed across a band width B centered with regard to the column or pedestal and having a width equal to the length of the short side of the footing, with the remainder of the bars uniformly spaced in the outer parts of the footing.

$$\frac{\text{Reinforcement in band width } B}{\text{Total reinforcement in short direction}} = \frac{2}{(S + 1)} \tag{9-1}$$

where S is the ratio of the long side of the footing to the short side.

Example 9-2. Design a square sloped-top footing for a circular column as shown in Fig. 9-6 using the following data: $P = 275{,}000$ lb, diameter of the column $= 24$ in., $f'_c = 2{,}500$ psi, $n = 12$, $f_c = 1{,}000$ psi, $f_s = 20{,}000$ psi, $v'_L = 75$ psi, $u = 200$ psi, allowable soil pressure $= 6{,}000$ psf.

In the case of a circular column, assume a square section of the same area.

$$a = \sqrt{\frac{\pi D^2}{4}} = 21.2 \text{ in., say 21 in.}$$

Assume the average depth of footing = 2 ft, and weight = 300 psf.

$$B^2 = \frac{P}{6,000 - 300} = \frac{275,000}{5,700} = 48.2 \text{ ft}^2$$
$$B = 7 \text{ ft (approx)}$$

Actually,

$$p = \frac{275,000}{49} = 5,620 \text{ psf}$$

Figures 9-6(*a*) and (*b*) picture the footing as it will be assumed.

$$CA = 1 \text{ ft } 10 \text{ in.} \qquad AE = 9\tfrac{1}{2} \text{ in.} \qquad QR = 5 \text{ ft } 5 \text{ in.}$$

Taking moments about MN, the edge of the equivalent square column, and using the entire static moment,

$$M = \left(p \times ST \times SO \times \frac{SO}{2}\right) 12 \text{ in.-lb}$$
$$M = \left(5,620 \times 7 \times 2.625 \times \frac{2.625}{2}\right) 12 = 1,625,000 \text{ in.-lb}$$

As seen from Fig. 9-6(*c*), the section through ST is not a rectangle. As a start,

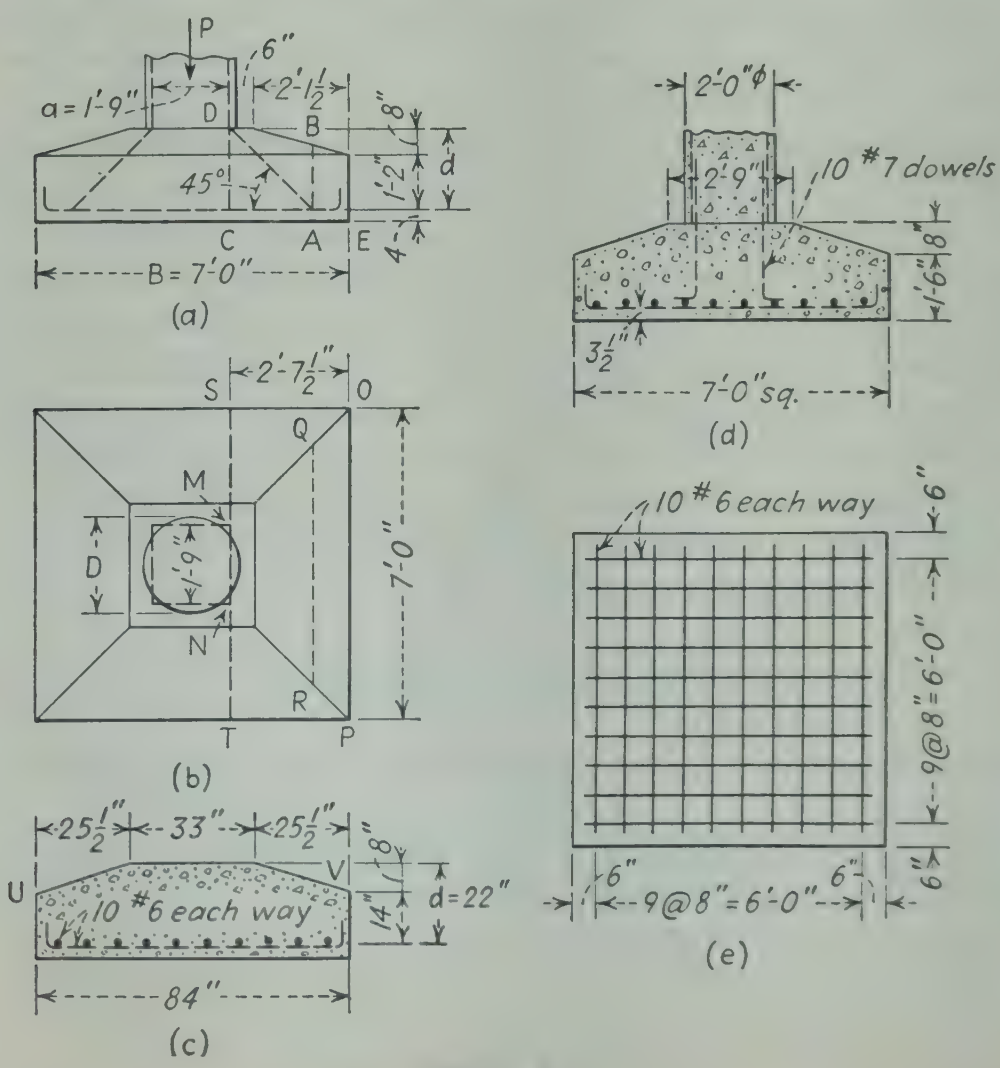


FIG. 9-6.

assume that the section is a rectangle and that $j = 0.88$. Then

$$\text{Trial } A_s = \frac{M}{f_s jd} = \frac{1{,}625{,}000}{20{,}000 \times 0.88 \times 22} = 4.2 \text{ in.}^2$$

Try 10 No. 6 bars, with $A_s = 4.4$ in.², and space them as shown in Sketch (c).

Now analyze the footing by the transformed-section method. Assume that the neutral axis is below the corners U and V. Then, taking the static moments about this neutral axis,

$$\frac{84}{2}(kd)^2 - \frac{2 \times 25.5 \times 8}{2}\left(kd - \frac{8}{3}\right) = 12 \times 4.4(22 - kd)$$

$kd = 6.03$ in. This is above the corners so that kd must be recomputed. Therefore,

$$\frac{33}{2}(kd)^2 + 2\left(\frac{kd}{2} \times \frac{25.5}{8} kd\right)\left(\frac{2}{3} kd\right) = 12 \times 4.4(22 - kd)$$

$kd = 5.5$ in. (approx) $\qquad d - kd = 16.5$ in.

$$I_c = \frac{33 \times 5.5^3}{3} + \frac{2 \times 17.55 \times 5.5^3}{36} + \frac{2 \times 17.55}{2} \times 5.5 \times 3.67^2 + 12 \times 4.4(16.5)^2$$
$$= 17{,}700 \text{ in.}^4$$
$$S_c = \frac{I_c}{kd} = \frac{17{,}700}{5.5} = 3{,}220 \text{ in.}^3$$
$$S_s = \frac{I_c}{n(d - kd)} = \frac{17{,}700}{12 \times 16.5} = 89 \text{ in.}^3$$
$$f_c = \frac{M}{S_c} = \frac{1{,}625{,}000}{3{,}220} = 505 \text{ psi}$$
$$f_s = \frac{M}{S_s} = \frac{1{,}625{,}000}{89} = 18{,}200 \text{ psi}$$

As might be expected, the compressive stress is small. The tension in the steel is satisfactory.

Using 85 per cent of the entire pressure under portion $SOPT$ of Fig. 9-6(b) as the shear in accordance with the Code, and assuming $j = 0.88$ as a conservative value, the maximum bond stress on the reinforcement is

$$u = \frac{V}{(\Sigma o)jd} = \frac{0.85 \times 5{,}620 \times 2.625 \times 7}{23.6 \times 0.88 \times 22} = 192 \text{ psi}$$

This is satisfactory. The same reinforcement will be used in both directions.

Now test the diagonal tension at section AB of Fig. 9-6(a). The assumed shear will be due to the pressure under area $QOPR$ of Sketch (b). The depth d at AB is

$$d = 22 - (22 - 6)\frac{8}{25.5} = 17 \text{ in.}$$
$$QR = 5 \text{ ft } 5 \text{ in.} \qquad AE = 9\tfrac{1}{2} \text{ in.} = 0.79 \text{ ft}$$
$$v_L = \frac{V}{bjd} = \frac{5{,}620 \times 0.79}{65 \times 0.88 \times 22}\,\frac{(5.42 + 7)}{2} = 22 \text{ psi}$$

which is very safe. However, one should be careful to keep d at the edge of such a footing at least 6 to 12 in., and the slope of the top ought not to be much more than 1 vertically to 3 horizontally because it may otherwise be difficult to finish the top without having the concrete slump too much.

The finished design is shown in Figs. 9-6(d) and (e). The dowels are bent so as to

rest upon the main reinforcing. The bottom rods are bent at right angles in order to provide a little extra length to ensure their development by bond. Although some think that this is not theoretically necessary with A 305 bars, it is good and inexpensive insurance against slippage.

Example 9-3. Design a stepped footing for the conditions shown in Fig. 9-7(*a*). Assume $f_c' = 3{,}000$ psi, $f_s = 20{,}000$ psi, and the allowable soil pressure $p = 8{,}000$ psf. Use Code unit stresses. An adjacent retaining wall limits the width of the footing to 6 ft.

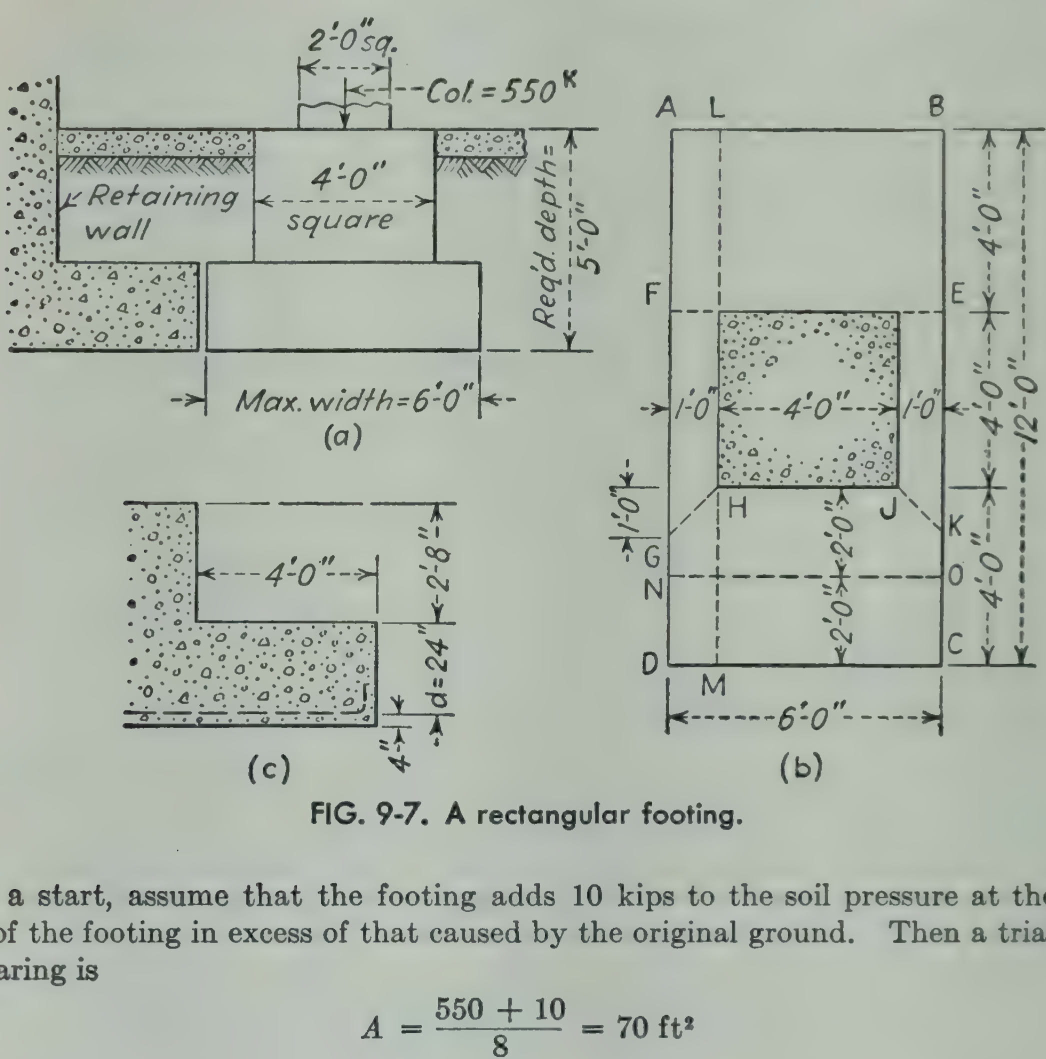


FIG. 9-7. A rectangular footing.

As a start, assume that the footing adds 10 kips to the soil pressure at the bottom of the footing in excess of that caused by the original ground. Then a trial area of bearing is

$$A = \frac{550 + 10}{8} = 70 \text{ ft}^2$$

Call the dimensions 6 by 12 ft. A plan of this footing is shown in Fig. 9-7(*b*), and a section of one long end is pictured in (*c*). Notice that *d* is deliberately made equal to one-half of the cantilever length.

Check the excess weight of this trial footing at 50 pcf.

$$W' = (6 \times 12 \times 2.33 + 4 \times 4 \times 2.67)50 = 10{,}600 \text{ lb}$$

This is near enough. The net pressure for the design of the footing is

$$p = {}^{550}\!/\!_{72} = 7{,}640 \text{ psf}$$

Bending at *FE*:

$$M = 7{,}640(6 \times 4 \times 2)12 = 4{,}400{,}000 \text{ in.-lb}$$

$$A_s = \frac{4{,}400{,}000}{20{,}000 \times 0.88 \times 24} = 10.4 \text{ in.}^2 \text{ (approx)}$$

Try 13 No. 8 rods at $5\frac{1}{2}$ in. c.c. $A_s = 10.3$, $\Sigma o = 40.8$.

$$u = \frac{7{,}640 \times 6 \times 4}{40.8 \times 0.88 \times 24} = 213 \text{ psi (approx) vs. 240 allowed}$$

$$\text{Approx } f_c = \frac{6M}{bd^2} = \frac{6 \times 4{,}400{,}000}{72 \times 24^2} = 640 \text{ psi} \qquad \text{(safe)}$$

Diagonal tension across NO:

$$V = p(ND \times NO) = 7{,}640 \times 2 \times 6 = 91{,}700 \text{ lb}$$

$$v_L = \frac{V}{bjd} = \frac{91{,}700}{72 \times 0.88 \times 24} = 60 \text{ psi vs. 75 allowed}$$

Bending at LM:

$$M = 7{,}640(12 \times 1 \times 0.5)12 = 550{,}000 \text{ in.-lb}$$

$$A_s = \frac{550{,}000}{20{,}000 \times 0.88 \times 24} = 1.3 \text{ in.}^2$$

For maximum $u = 240$ psi,

$$\Sigma\, o = \frac{V}{ujd} = \frac{7{,}640 \times 12 \times 1}{240 \times 0.88 \times 24} = 18.1 \text{ in.}^2 \text{ needed}$$

Use 12 No. 4 bars, the smallest size desired. $A_s = 2.40$, $\Sigma o = 18.8$. Inspection shows that it is not necessary to compute v_L and f_c for the long direction. From Eq. (9-1),

$$\frac{2}{S + 1} = \frac{2}{{}^{12}\!/_{6} + 1} = \frac{2}{3}$$

Therefore, place eight of these No. 4 bars in a 6-ft width centered on the pedestal, with two near each end at about 16-in. spacing.

Example 9-4. Compute the maximum bending moment and diagonal tension in the footing of Fig. 9-8(a). The load P is supported by 25 wooden piles.

Piles can seldom be driven in exactly the positions shown on the drawings; hence conservatism in design is necessary. For example, there are five piles in rows 4 and 5, each having a net load of

$$p = \frac{1{,}100}{25} = 44 \text{ kips}$$

The theoretical bending moment about a plane through AE is

$$M = 5 \times 44(1 + 4) = 1{,}100 \text{ ft-k}$$

But suppose that these piles are 6 in. too far to the right. The bending moment would be $5 \times 44(1.5 + 4.5) = 1{,}320$ ft-k, an increase of 20 per cent. This is not at all improbable. Therefore, it seems that the 0.85 factor in reducing M to determine the reinforcement should not be used. Even though the safety factor might cover it, this conservatism is desirable. The same statement applies to computations for bond.

The Code states that the critical section for computing diagonal tension in footings on piles shall be $\frac{1}{2}d$ from the pedestal or column, as shown by EC in Fig. 9-8(a). A pile is considered to affect the section when its center is at a distance of 6 in. outside the section, C in this case. If its center is 6 in. inside of C, the "effect" is called zero. Between these positions, the effect of the pile reaction on the shear is assumed to vary directly between these limits in proportion to the distances concerned. This is a way of providing something for inaccuracies of driving. Laterally, the positions with

respect to the 45° diagonals in Fig. 9-8(*b*) must be considered. The pile reaction is assumed to be at the center of the pile. In this case, assume

$$V = 3 \text{ piles No. 5 plus } \tfrac{1}{2} \text{ (2 piles No. 5 at the corners) plus } \tfrac{1}{2} \text{ (3 piles No. 4)}$$

or

$$V = 3 \times 44 + 2\tfrac{1}{2} \times 44 = 242 \text{ kips}$$

Then, if b is taken as GF of Fig. 9-8(*b*),

$$v_L = \frac{V}{bjd} = \frac{242{,}000}{(6 \times 12) \times 0.88 \times 24} = 159 \text{ psi (too high)}$$

Notice that the pile tops are embedded and the reinforcement is 3 in. above them for adequate cover. Increase d or the pedestal. Redesign the footing.

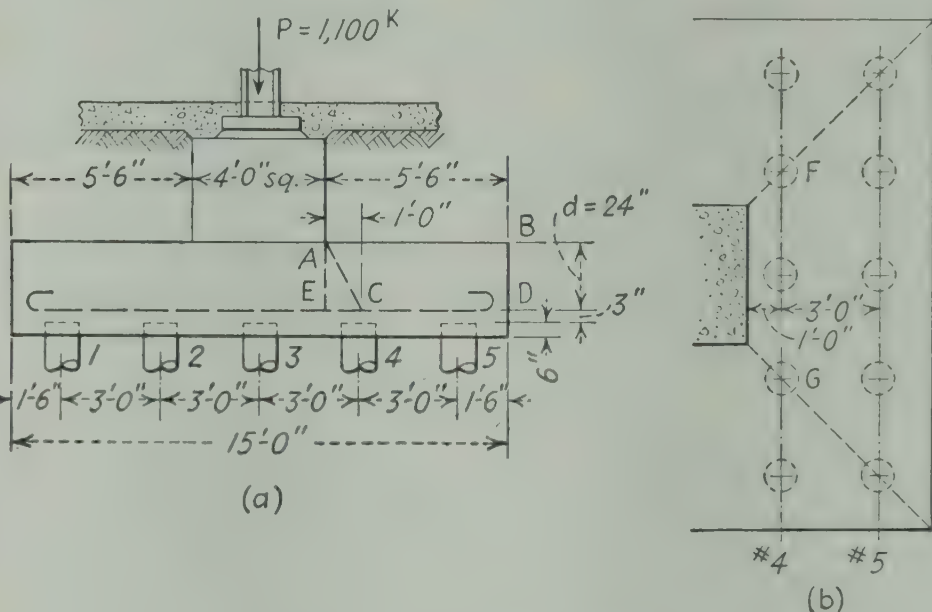


FIG. 9-8. Footing on piles.

The design of footings for walls is similar to that described for column footings except that the bending moment to be used in computing the reinforcement needed is to be the entire static value about the proper center of moments. The entire shear on one side of the section is also to be used when computing bond. A 1-ft strip is generally used for simplicity of calculations. The diagonal tension is computed on the basis of the distance d for footings on ground and $d/2$ for footings on piles.

9-5. Column footings subjected to overturning moments. In practical work, one frequently must design footings for columns carrying direct loads and bending, the latter being due to such things as wind pressures and lateral crane loads when the columns are fixed at their bases; sometimes footings support direct loads from columns plus horizontal shears delivered by them to the tops of the pedestals, as when the columns bend because of wind but are not fixed at their bases or when they have bracing members connecting near their bottoms; occasionally the

moment may be caused by the fact that the column is located eccentrically on the footing. In such cases, the soil pressures are not uniformly distributed as in Fig. 9-3(a) but they may be assumed to vary uniformly across the footing. When these overturning moments always act parallel to one axis of the footing, it may be economical to make the footing rectangular with its long side in the direction of the overturning tendencies.

Example 9-5. Design a spread footing for the steel columns shown in Fig. 9-9, using one of the typical columns along the north (right-hand) side of the concentrator.

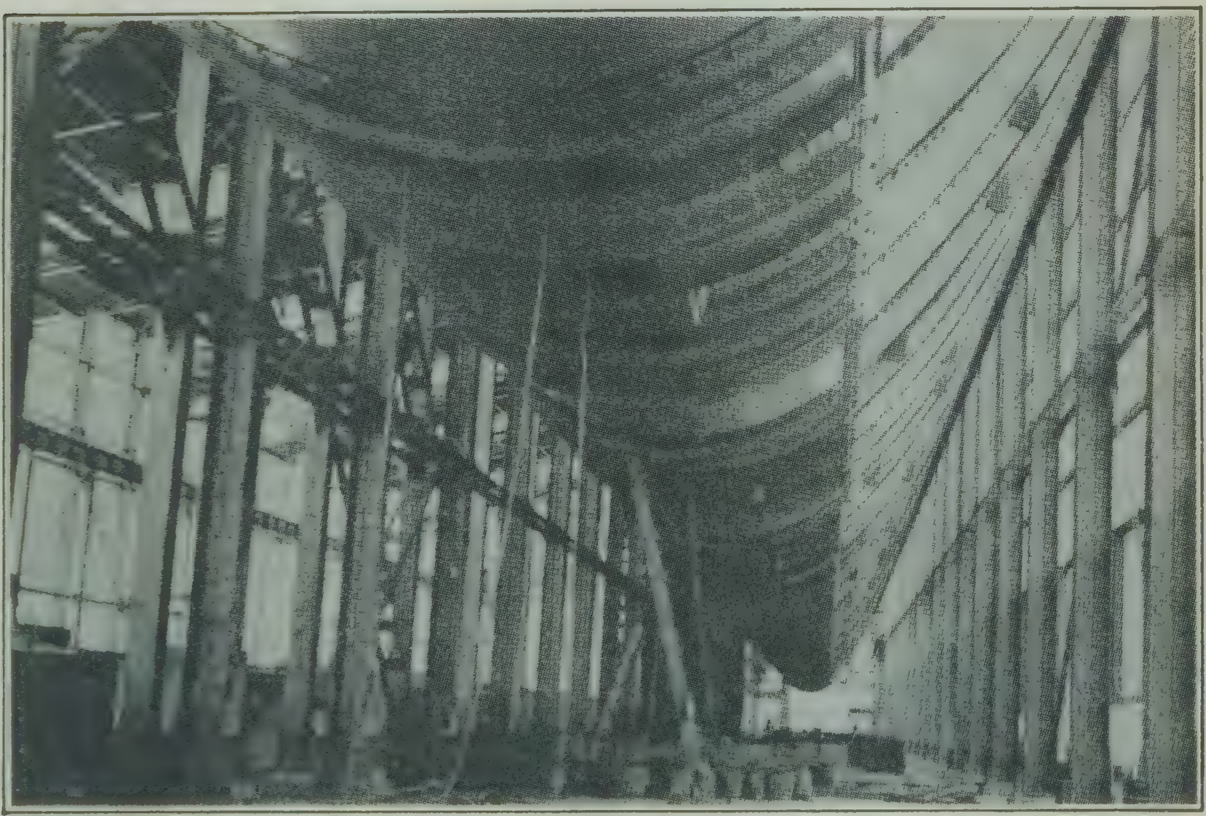


FIG. 9-9. Suspended ore bin along north side of concentrator at the Morenci Reduction Works, Phelps Dodge Corporation, Morenci, Ariz. View looking west.

It supports one side of a suspended bunker about 30 ft deep; the moment is due to wind on the outside and to crane forces from the inside of the building, and of course it is reversible. The soil is caliche, having an allowable bearing value of 5 tons per ft². The allowable f_s, f_c, v_L', and u are 18,000, 1,200, 90, and 150 psi, respectively; $n = 10$. Since the bin may be full or empty when the wind blows, the combination of wind and maximum vertical load produces the critical bending moment in the footing and pressure on the soil, but when the bin is empty, the wind may overturn the footing. Both cases must be investigated, the design data being the following:

$$\max P + M = 725{,}000 \text{ lb} + 300{,}000 \text{ ft-lb}$$
$$\min P + M = 40{,}000 \text{ lb} + 300{,}000 \text{ ft-lb}$$
$$S = 15{,}600 \text{ lb}$$

where P = vertical load, M = moment, and S = horizontal shear, all at the bottom of the steel.

For a combination of wind and live loads, assume that the allowable bearing pressure and unit stresses may be increased 30 per cent.

When starting such a problem, the designer should see what limiting factors affect the case—interferences with width and length, use or omission of a pedestal, size of pedestal if used, depth to bottom of footing, etc. He should choose the type of footing that he thinks is best, then see if he can use it; if something interferes, then the design should be modified as necessary.

One way to obtain trial dimensions of such a rectangular footing is to find the minimum area for direct load only, adding an estimate of the weight of the footing and soil or flooring over it. Thus, call $P = 775{,}000$ lb.

$$A = \frac{775{,}000}{10{,}000} = 77.5 \text{ ft}^2 \text{ (say } 8 \times 10 \text{ ft)}$$

Next assume that, for direct loads and overturning, the pressures under the footing vary uniformly. Then the maximum pressure under the footing is

$$p = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{BH} + \frac{6M}{BH^2} \tag{9-2}$$

where B and H are, respectively, the width and length of the footing, the latter being in the direction of the rotational tendency. Then, assume H and solve for B.

Assuming $H = 10$ ft, and using M at the bottom of the footing as

$$300{,}000 + S \times 5 = 378{,}000 \text{ ft-lb}$$

$$10{,}000 \times 1.3 = \frac{775{,}000}{10B} + \frac{6 \times 378{,}000}{100B} \qquad B = 7.7 \text{ ft}$$

This could be used, but test out a longer narrower footing 6.5 by 13 ft. Its area will be about the same as the broader one. Therefore, it will be adopted because it is more effective in resisting the overturning.

Before proceeding further, test the pressure for the case of overturning with the minimum vertical load $P = 40{,}000$ lb + 50,000 lb for the footing, etc., $M = 378{,}000$ ft-lb.

$$\text{Eccentricity} = e = \frac{378{,}000}{90{,}000} = 4.2 \text{ ft}$$

The pressure diagram is therefore as shown in Fig. 9-10(d).

$$\left(p \times \frac{6.9}{2}\right) 6.5 = 90{,}000 \qquad p = 4{,}000 \text{ psf}$$

This is satisfactory.

The trial footing is shown in Fig. 9-10. An estimate of its weight plus the floor and earth over it is 52,000 lb. The revised maximum P and M are given in Fig. 9-10(a); the pressure diagram, in Sketch (c). Since the footing is narrow, it is sloped two ways only. This makes a strong ridge clear across the top, forming a one-way footing. Because of this, the bending moment at the edge of the pedestal will be computed at the section EF of Sketch (b) for the pressure on the entire portion $EBCF$ with 600 psf deducted from the values given in (c) because of the weight of the footing and earth on it.

$$M_{EF} = 6.5\left(9{,}080 \times \frac{5^2}{2} + 1{,}620 \times \frac{5}{2} \times 0.67 \times 5\right) = 826{,}000 \text{ ft-lb}$$

Assume $j = 0.88$; $d = 39$ in.

$$A_s = \frac{826{,}000 \times 12}{(18{,}000 \times 1.3)0.88 \times 39} = 12.4 \text{ in.}^2$$

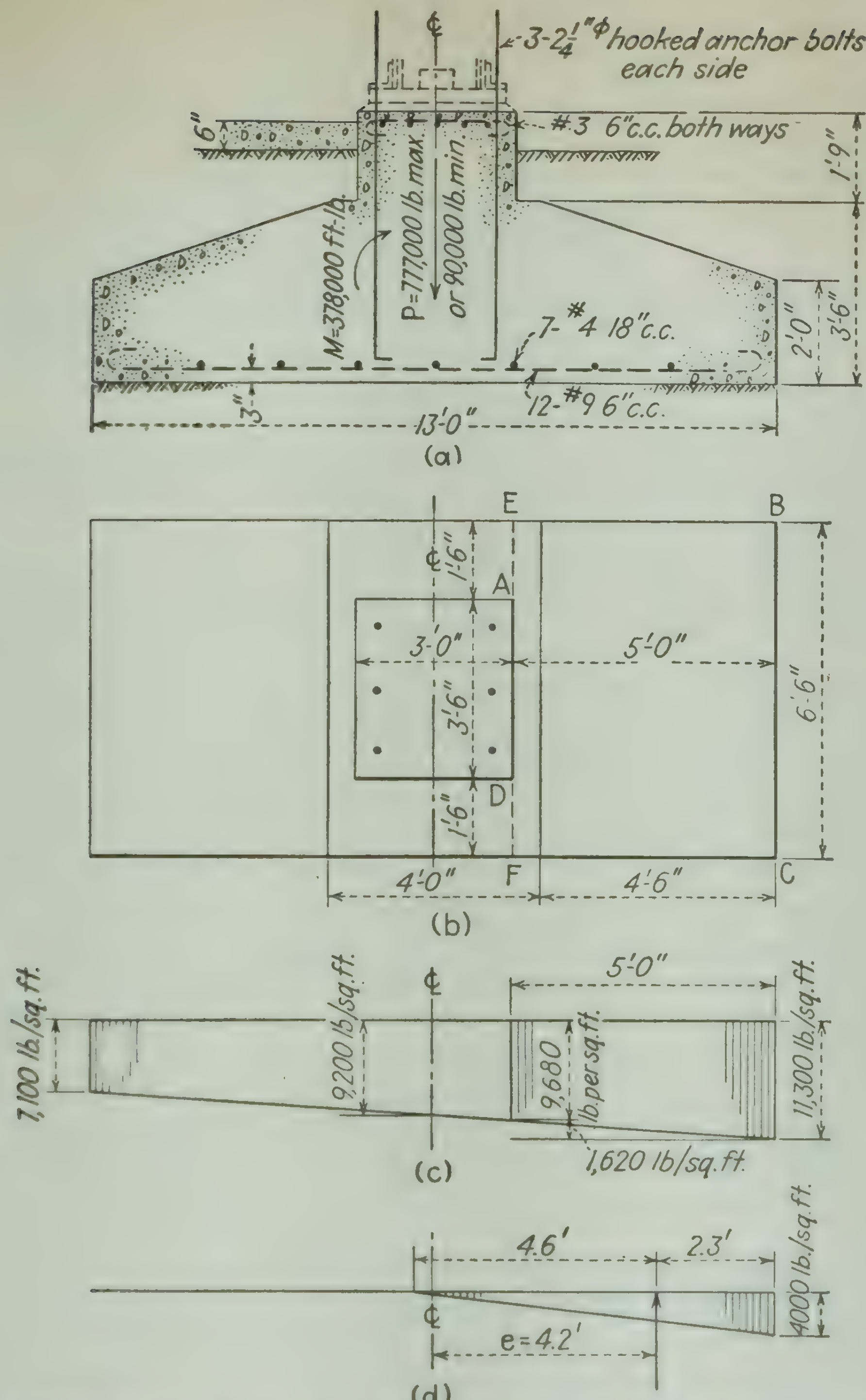


FIG. 9-10. Footing under column supporting ore bin, Morenci Reduction Works, Phelps Dodge Corporation, Morenci, Ariz.

For the maximum vertical load alone,

$$M_{EF} = 6.5\left(\frac{777,000}{84.5} - 600\right)\frac{5^2}{2} = 6.5(9,200 - 600)\frac{5^2}{2} = 699,000 \text{ ft-lb}$$

$$A_s = \frac{699,000 \times 12}{18,000 \times 0.88 \times 39} = 13.6 \text{ in.}^2$$

Therefore, use fourteen No. 9 hooked rods 5 in. c.c. longitudinally. Put in a few transverse spacer ties as shown in Fig. 9-10(a).

$$V_{EF} \text{ for Fig. 9-10}(c) = \left[\frac{(9{,}680 + 11{,}300)}{2} - 600\right] 5 \times 6.5 = 321{,}000 \text{ lb}$$

$$u = \frac{321{,}000}{56 \times 0.88 \times 39} = 167 \text{ psi} \qquad (\text{less than } 1.3 \times 150)$$

For vertical loads alone,

$$u = \frac{(9{,}200 - 600)5 \times 6.5}{56 \times 0.88 \times 39} = 145 \text{ psi}$$

The diagonal tension at the bottom of a 45° slope from *EF* of Fig. 9-10(*b*) is to be checked.

$$d = 39 - 18\left(\frac{39 - 6}{54}\right) = 28 \text{ in.}$$

$$V = \left(\frac{11{,}300 + 10{,}730}{2} - 600\right) 1.75 \times 6.5 = 118{,}000 \text{ lb}$$

$$v_L = \frac{118{,}000}{78 \times 0.88 \times 28} = 62 \text{ psi} \qquad (\text{safe})$$

$$\text{Approx } f_c = \frac{6M}{bd^2} = \frac{6 \times 914{,}000 \times 12}{78 \times 39^2} = 555 \text{ psi} \qquad (\text{very safe})$$

A test of the loading case shown in Fig. 9-10(*d*) shows that the shears and moments in the footing are less than those already considered; hence the footing is satisfactory.

If the anchor bolts shown in Fig. 9-10(*a*) did not go to the bottom to grip the footing, reinforcement might have to be used to keep the pedestal from being ripped off. The rods under the billet are used arbitrarily to prevent cracking of the top by shear.

Suppose the footing of Fig. 9-10(*a*) is subjected to overturning moments both lengthwise and crosswise of the footing. It will be sufficient to compute the bearing pressure for the vertical load alone, add to it that due to the moment in one direction ($6M/BH^2$), then add also the pressure caused by the crosswise moment ($6M'/B^2H$). The bending moments in the footing may be computed at one side of the pedestal for $P + M$ alone or at the adjacent side for $P + M'$ only.

Example 9-6. Check the footing shown in Fig. 9-11 for the loads and overturning moment given there. This footing is notched out to clear a heavy machinery foundation which must be isolated from it. The maximum permissible soil pressure is 4 tons per ft²; $n = 10$; max f_s, f_c, v_L', and u = 20,000, 1,200, 90, and 240 psi, respectively.

The weight of the footing is taken as the approximate total weight of the concrete and the earth over it, this weight being assumed to be at the center line of the column. The earth is often omitted from the calculations, especially when the footings are deep.

Since this footing is not rectangular, Eq. (9-2) must be used in its general form. An outline of a procedure yielding sufficiently satisfactory answers is the following:[1]

1. Locate the center of gravity of the assumed bearing area.
2. Compute the moment of inertia of this area about both rectangular axes.
3. Combine the moments due to eccentricity of loading with those due to other causes.
4. Compute the resultant pressure by adding that due to the direct load to those caused by the moments.
5. Find the critical portion of the footing, approximate the bending moment, and test the section.

[1] For an explanation of the use of principal axes for such a problem, see C. W. Dunham, "Foundations of Structures," McGraw-Hill Book Company, Inc., New York, 1950.

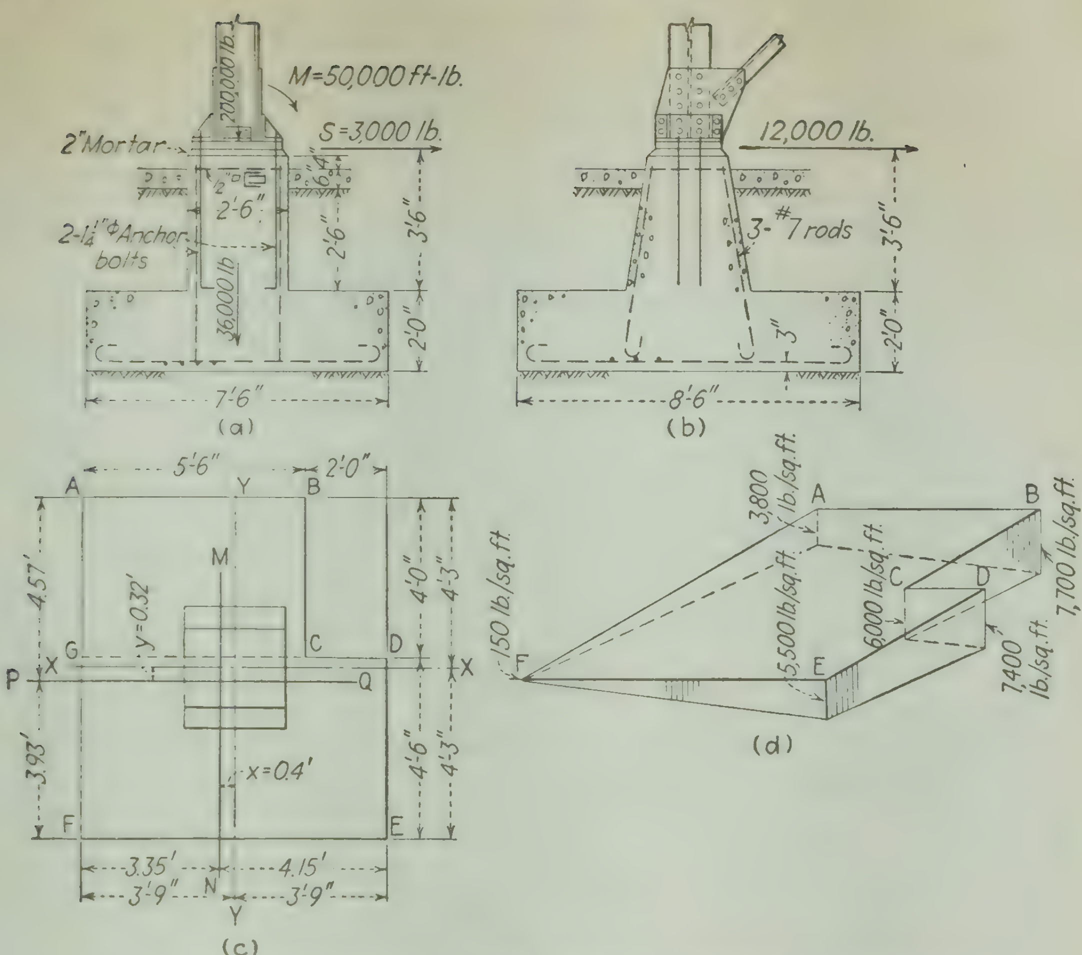


FIG. 9-11. Unsymmetrical footing for steel column in industrial plant.

Axis X-X:

Part	A	l	M	l^2	I_{CG}	Al^2	ΣI
$ABCG$	22	2.25	49.5	5.06	29.3	111.3	140.6
$GDEF$	33.8	2	−67.6	4	57	135.2	192.2
	55.8		−18.1				332.8

$$y = \frac{-18.1}{55.8} = -0.32 \text{ ft}$$

$$I_{PQ} = I_X - Ay^2 = 332.8 - 55.8 \times 0.32^2 = 327 \text{ ft}^4$$

Axis Y-Y:

Part	A	l	M	l^2	I_{CG}	Al^2	ΣI
$ABCG$	22	−1	−22	1	55.5	22	77.5
$GDEF$	33.8	0	0	0	158.2	0	158.2
	55.8		−22				235.7

$$x = \frac{-22}{55.8} = -0.4 \text{ ft}$$

$I_{MN} = I_Y - Ax^2 = 235.7 - 55.8 \times 0.4^2 = 227 \text{ ft}^4$

$P = 236{,}000$ lb (assume it at center of column)

M_X at bottom $= 12{,}000 \times 5.5 + 236{,}000 \times 0.32 = 142{,}000$ ft-lb

M_Y at bottom $= 50{,}000 + 3{,}000 \times 5.5 + 236{,}000 \times 0.4 = 161{,}000$ ft-lb

The greatest pressure is likely to be at B or D of Fig. 9-11(c). Letting the subscripts denote the point under consideration, then

$$p_B = \frac{236{,}000}{55.8} + \frac{161{,}000 \times 2.15}{227} + \frac{142{,}000 \times 4.57}{327} = 7{,}700 \text{ psf}$$

$$p_D = \frac{236{,}000}{55.8} + \frac{161{,}000 \times 4.15}{227} + \frac{142{,}000 \times 0.57}{327} = 7{,}400 \text{ psf}$$

The pressure diagram under the whole footing is shown in Fig. 9-11(*d*). The use of this in designing the footing generally involves broad approximations. A designer might resort to various detailed computations, but it is seldom worth while. His psychological reaction resulting from such tedious work may be beneficial to him, but probably the real value of the answer is not in proportion to the fussiness of the computations. Such footings are generally the exception, having little duplication. Therefore, the designer should think first whether or not his time and that of the checker of his work will cost more than the use of a few more rods in a design that is

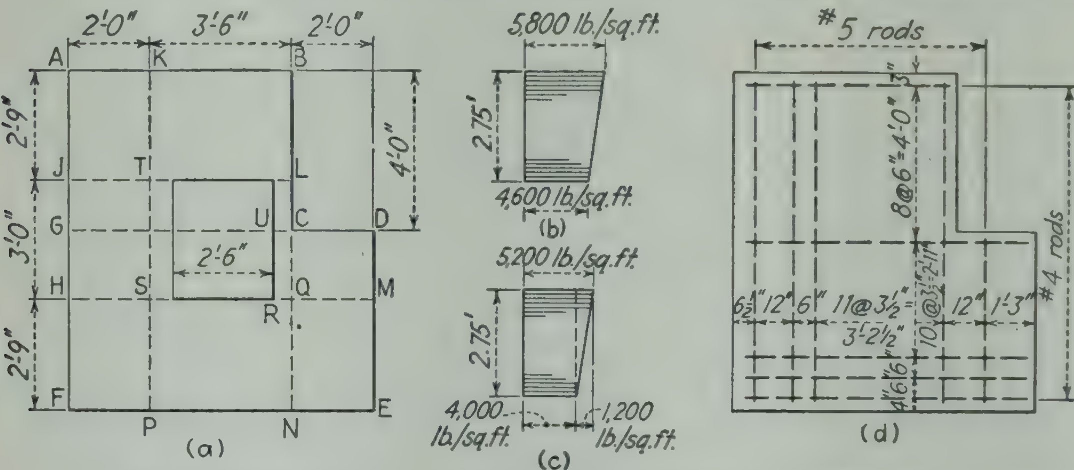


FIG. 9-12. Reinforcement in an unsymmetrical spread footing.

obviously safe. Each special problem requires the use of good judgment in deciding these matters.

The following computations show one way of designing this footing:

1. Looking at Figs. 9-11(*d*) and 9-12(*a*), it seems that a strong band or beam might be made running lengthwise of the footing, such as *KBNP* of Fig. 9-12(*a*). Make it strong enough to carry the shears and moments from the entire portion *JABL* about *TL*, or *HMEF* about *SQ*. Assume it to be 3 ft 6 in., a little wider than the pedestal.

2. Judge whether *TL* or *SQ* is the critical section. Test both if it seems to be necessary. Although the area *JABL* is less than *HMEF*, it is subjected to larger pressures and appears to be the critical side.

3. From Fig. 9-11(*d*) and similar computations, get the pressures at each of the four corners, then average that at *A* and *B*, also at *J* and *L*. The resultant approximate pressure diagram is shown in Fig. 9-12(*b*). The average dead load of the footing and the material over it is 36,000 ÷ 55.8 = about 600 psf. Then the approximate net pressure diagram is that shown in Fig. 9-12(*c*). The bending moment at *TL* is

$$M_{TL} = 5.5\left(4{,}000 \times \frac{2.75^2}{2} + 1{,}200 \times 2.75^2 \times 0.67\right) = 117{,}000 \text{ ft-lb}$$

$$V_{TL} = 5.5\left(4{,}000 \times 2.75 + 1{,}200 \times \frac{2.75}{2}\right) = 70{,}000 \text{ lb}$$

4. Using Eqs. (2-6*a*) and (2-5*a*), with $d = 21$ in.,

$$A_s = \frac{117{,}000 \times 12}{20{,}000 \times 0.88 \times 21} = 3.8 \text{ in.}^2$$

Use 12 No. 5 rods at $3\frac{1}{2}$ in. c.c., hooked.

$$f_c = \frac{6 \times 117{,}000 \times 12}{42 \times 21^2} = 450 \text{ psi}$$

At a distance d from TL

$$v_L = \frac{5{,}800 \times 1 \times 5.5}{42 \times 0.88 \times 21} = 41 \text{ psi}$$

without considering any 45° diagonals, and using only a 42-in. width.

$$u = \frac{70{,}000}{23.5 \times 0.88 \times 21} = 161 \text{ psi} \qquad \text{(near enough)}$$

5. The strip $UDMR$ of Fig. 9-12(a) seems to have the greatest bending of any crosswise to $KBNP$. Arbitrarily assume a piece 12 in. wide with an average net upward pressure of 6,400 psf. For this strip, M_{UR} = 20,000 ft-lb and A_s required = 0.65 in.². Therefore, use No. 4 rods $3\frac{1}{2}$ in. c.c., for which u = 161 psi.

The final reinforcing plan is shown in Fig. 9-12(d). The top hoop and the bars in the pedestal are shown in Figs. 9-11(a) and (b).

9-6. Combined footings. In some cases, where structures are founded upon soils having low bearing capacities, it is advisable to combine the footings of two or more columns. One common example of this occurs when the outside row of columns is close to the building line and it is impossible for the footing to spread over onto adjacent property. The load upon the outer column is also likely to be larger than that on the inner one because of the heavy walls. The footing may be built somewhat as shown in Fig. 9-13, with an enlargement near the outer column so as to produce a uniform intensity of pressure. Sometimes an elongated pedestal may be added in order to make the structure a sort of inverted T beam which is supported by the two columns. Of course, the fundamental objective is to secure uniform settlement and to avoid tipping of the footing.

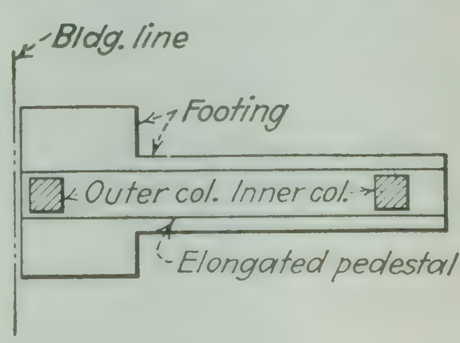


FIG. 9-13.

Many industrial buildings are equipped with heavy cranes that are carried on columns along the exterior walls. The foundations for these may be made with spread footings for the columns and grade beams for the walls, or they may be continuous concrete walls with pilasters and medium-sized footings at the columns. In the latter case, the wall may be designed as a stiffening girder or a long narrow footing to spread the crane column's load. This latter type of foundation may require more excavation and concrete, but it produces a stiff construction.

In other cases, like the invert of the New Jersey shaft of the Lincoln Tunnel (Fig. 9-14), all the footings are combined into a thick continuous

reinforced-concrete slab or mat. This is done here in order to make a boxlike structure which will withstand hydrostatic pressures. In the case of the Merchants Refrigerating Company Building, Varick St., New York City (Fig. 9-15), a solid 5-ft mat was used in order to spread the load to the soil.

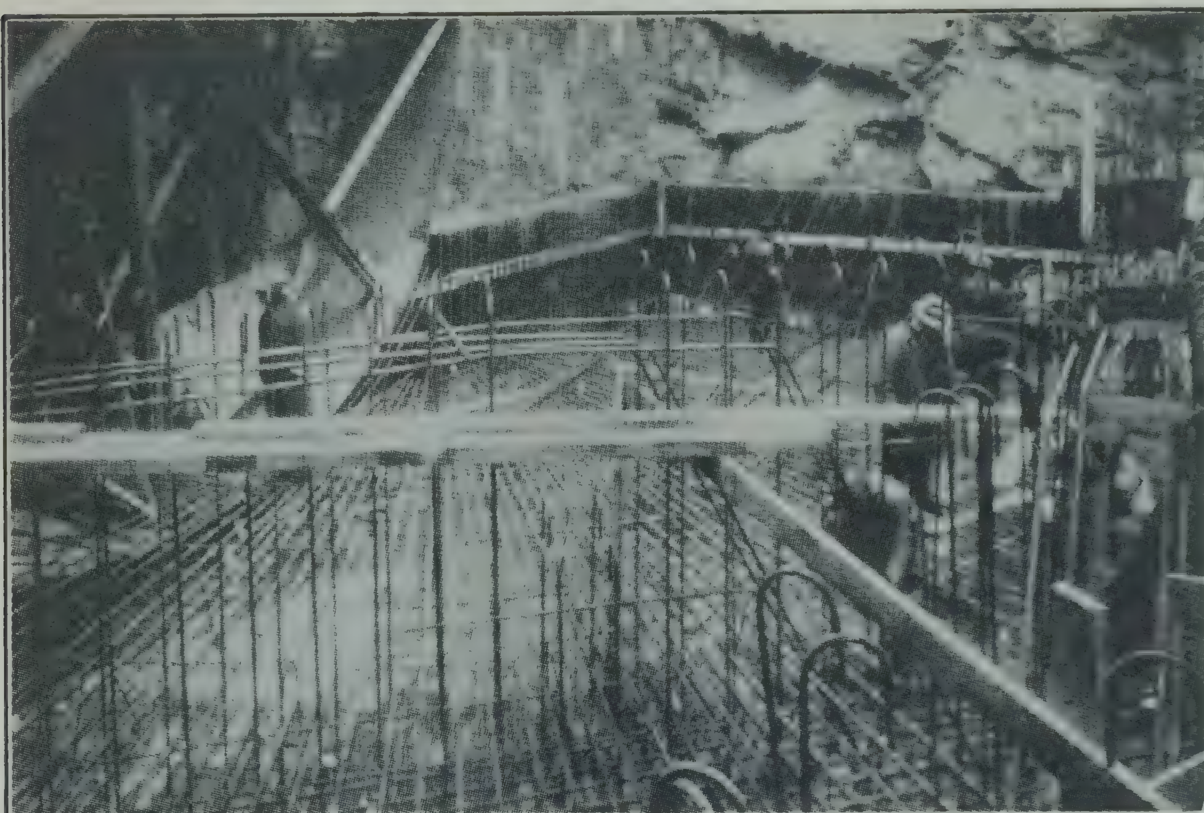


FIG. 9-14. Reinforcement in invert of the New Jersey shaft of the Lincoln Tunnel at New York City.

Example 9-7. Find the stresses in the footing or pier which is pictured in Fig. 9-16. It is part of the viaduct of the New Jersey approach to the Lincoln Tunnel at New York City. Assume $n = 10(f'_c = 3{,}000$ psi) and f_s and $f_c = 18{,}000$ and 1,000 psi, respectively.

This problem will illustrate the use of wooden piles as supporting members for a foundation with overturning forces. It is given here in order to show the nature of many practical cases.

The entire pier is designed to act as a unit, carrying four concentrated loads from the superstructure. Various conditions and combinations of loading must be investigated in the design of such a structure; but in this problem, full live load and dead load will be combined with longitudinal wind and braking forces. The vertical loads are 900 kips at each pedestal; the longitudinal loads are 70 kips acting horizontally at the top of each pedestal, normal to the long axis of the pier. There are 144 piles arranged in 24 rows having 6 piles each.

Two special features must be noticed in this problem: (1) The four vertical loads are equal, meaning that the footing is not like a continuous beam which is on rigid supports; (2) the piles are supported by the frictional resistance of the surrounding material (clay) so that, for dead loads which are constant and continuously applied, the load on the piles is assumed to be distributed equally among them all on account

of the plasticity or minute movement of the clay. It is therefore assumed further that for both live and dead loads, the reactions of all piles are equal because the live loads are relatively small. However, the temporary longitudinal forces are assumed to cause uniformly varying loads upon the piles across the narrow dimension of the footing, and all transverse rows of piles are assumed to act in the same manner.

The total dead load of the footing, neglecting the piles, and assuming the space that is occupied by pile heads to be solid concrete, is about 1,500 kips. This is assumed

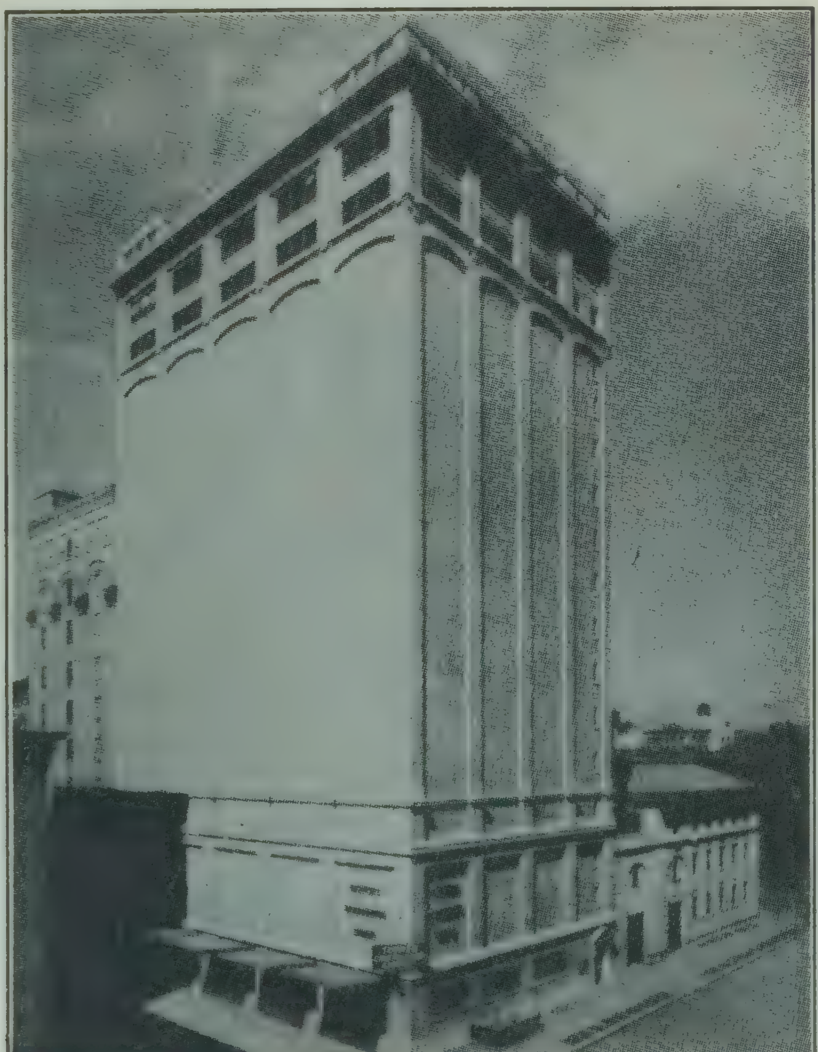


FIG. 9-15. Merchants Refrigerating Company Building, New York City. (Courtesy of Turner Construction Co.)

to be uniformly distributed, and it causes a load of $1{,}500/144 = 10.4$ kips per pile. The superstructure produces a load of $4 \times 900/144 = 25$ kips per pile. These give 35.4 kips vertical load on each one.

On the other hand, the longitudinal forces try to overturn the entire pier. If these forces are applied as shown in Fig. 9-17(*a*), the moment that acts upon one row of piles is

$$M = \frac{4 \times 70}{24} \times 8.17 = 95 \text{ ft-k}$$

The piles are 3.5 ft apart so that the vertical force on the outer piles is

$$P = \frac{Mc}{I} = \frac{95 \times 8.75}{2(1.75^2 + 5.25^2 + 8.75^2)} = \frac{832}{214} = 3.9 \text{ kips}$$

The total vertical reactions of the piles are then as pictured in Fig. 9-17(b). Of course, the horizontal reaction which balances the longitudinal forces comes from the bearing of the clay upon the sides of the footing and of the piles.

It now becomes necessary to make some broad and arbitrary assumptions in order to proceed efficiently with the design, because the loading conditions and the distribution of forces within the footing are too uncertain to justify a long supposedly exact analysis. One must use good judgment and be conservative without undue wastefulness. Therefore, the following will be assumed:

1. The assumed effective cross section of the pier which is counted upon to withstand the lengthwise bending is *ABCDEFGHA* [Fig. 9-18(a)]. The general shape is

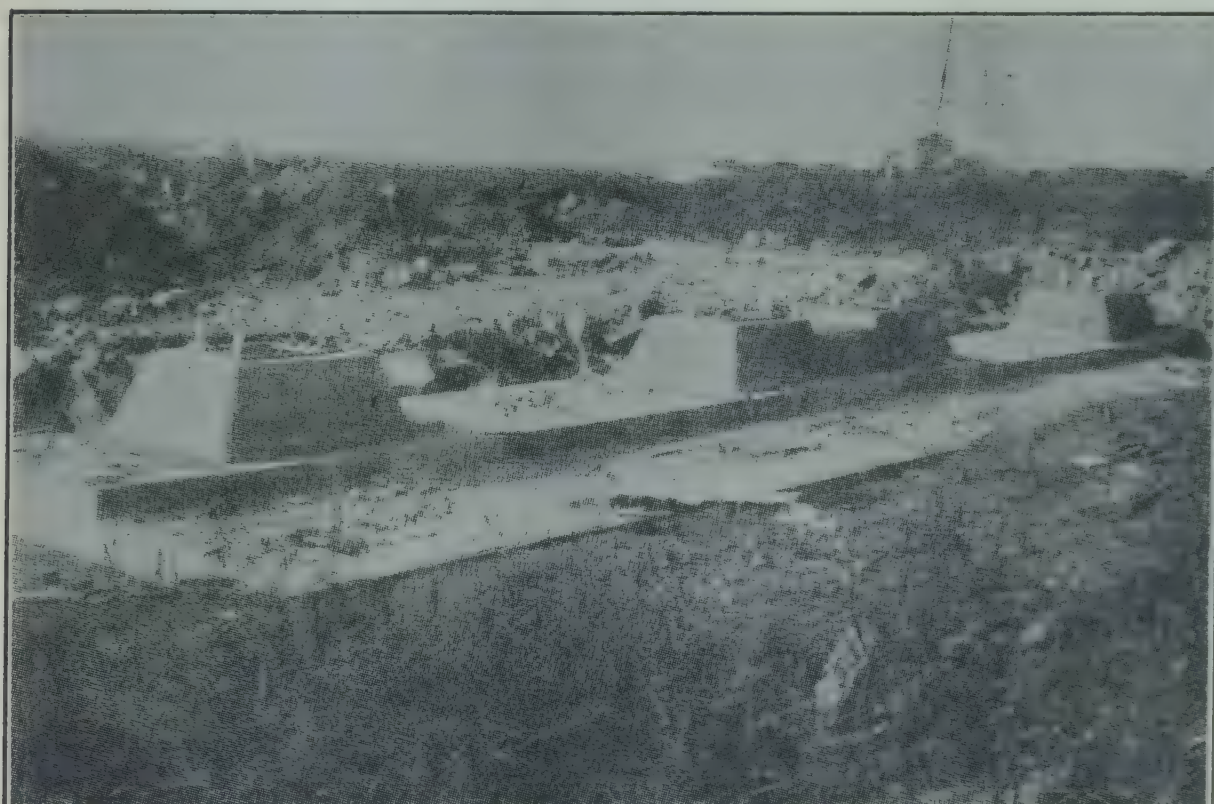


FIG. 9-16. Foundation for six-lane viaduct, New Jersey approach to the Lincoln Tunnel, New York City.

chosen so as to have a narrow strip showing above the soil, to allow pouring a 12-in. sealing slab around the pile heads prior to making the structural base, and to produce a stiff chunky footing. Theoretically the entire width of the footing can be counted in the section resisting longitudinal bending. However, the outer edges beyond the piles will be disregarded, as shown in the illustration. This is done in order to have the required reinforcement within a band close to the 45° lines *HF* and *CE*.

2. The effective depth of the footing is 56 in. (see Fig. 9-21).

3. The effective width of the footing for transverse bending is taken as a width approximately equal to that of the end pedestal plus $2d$. Call it 18 ft. This is to be a "band of resistance." At the inner pedestals, 9 ft can be used at one side, but there is a distance of only 6 ft to the center of the footing. Hence the width at the inner pedestal is assumed to be 15 ft. The average point of greatest transverse bending moment is 2 ft from the center of the pedestals.

4. The stiffening effect of the connecting rib on top of the main footing between the two central pedestals (Fig. 9-20) is not relied upon because it is used primarily as

a filler between these pedestals, and because it is poured after the main footing, causing a plane of weakness at the junction between the base and the top.

5. Each transverse strip is to support the reactions from six rows of piles.

6. The longitudinal bending moments are to be computed at the centers of the pedestals; the diagonal tensions are to be calculated at the edges of the 45° shear cones

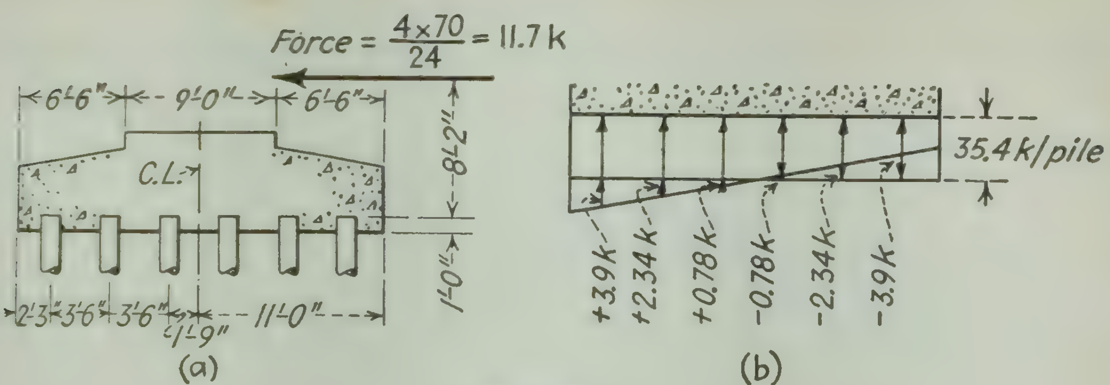


FIG. 9-17.

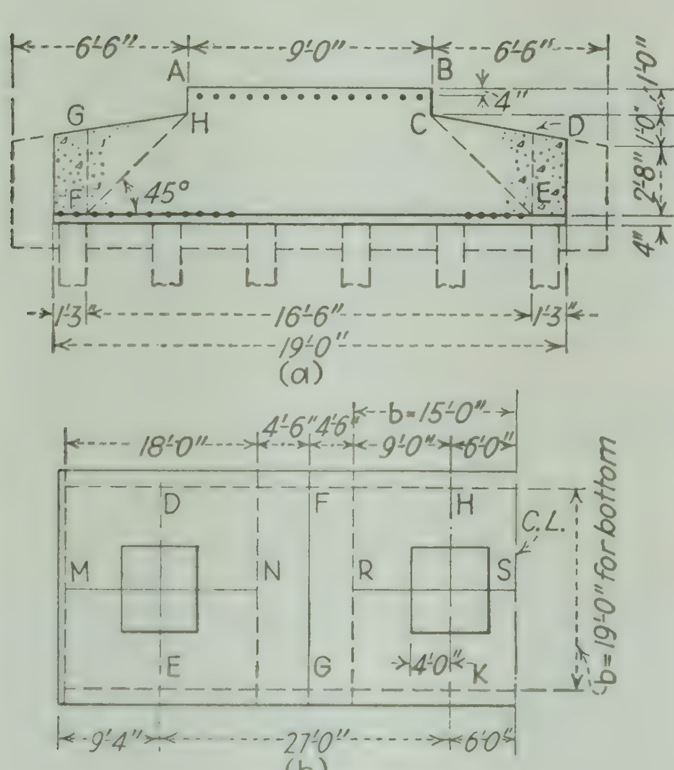


FIG. 9-18.

under the pedestals; and the effective width for punching shear is to be the width of the base of the pedestal.

The bending moments and shears in the longitudinal strip as a whole are caused by the dead and the live loads of the superstructure which are taken as

$$W + L = 4 \times 900 \div 84.67 = 42.5 \text{ kips per ft of pier}$$

The overturning forces do not increase the total vertical loads but merely add to the pile reactions on one side and relieve those on the other. Also, the weight of the footing itself does not cause bending in this strip. The shear and bending-moment dia-

grams are pictured in Fig. 9-19. The values are obtained by working from A to the center E.

From Eq. (2-6a), a trial value of A_s at point D is

$$A_s = \frac{3{,}700{,}000 \times 12}{18{,}000 \times 0.88 \times 56} = 50 \text{ in.}^2$$

If No. 9 rods are placed at 4½ in. c.c., there will be 51 of them across the 19-ft

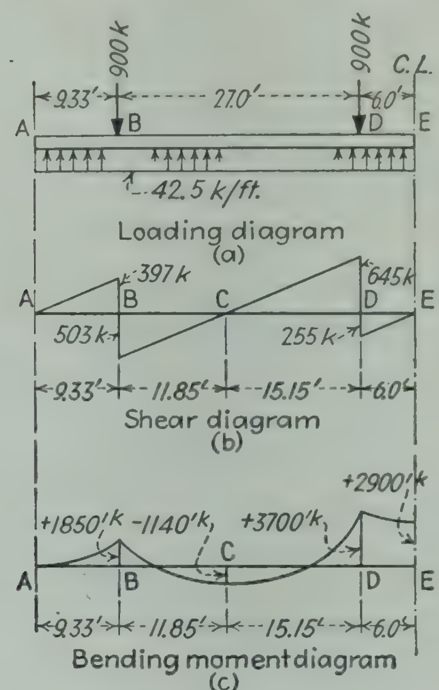


FIG. 9-19.

section shown in Fig. 9-18(a). Similarly, the tensile steel in the top at C, Fig. 9-19(c), should be at least

$$A_s = \frac{1{,}140{,}000 \times 12}{18{,}000 \times 0.88 \times 56} = 15.4 \text{ in.}^2$$

Try 17 No. 9 rods at 6 in. c.c. Since the bending moment at B, Fig. 9-19(c), is half of that at D, use No. 9 rods at 9 in. c.c.

Figure 9-20 pictures the arrangement of the longitudinal reinforcement. The assumed effective beam for bending at D is as shown in Fig. 9-21(a). Solving for the stresses that result from a positive bending moment of 3,700 ft-k on a rectangular section 9 ft wide gives approximately

$$108 \times \frac{(kd)^2}{2} + 9 \times 6(kd - 4) = 10 \times 51(56 - kd)$$

$$kd = 18.5 \text{ in.} \qquad d - kd = 37.5 \text{ in.} \qquad k = 0.33 \qquad j = 0.89$$

$$I_c = 957{,}000 \text{ in.}^4 \qquad S_c = 51{,}800 \text{ in.}^3$$

$$f_c = \frac{3{,}700{,}000 \times 12}{51{,}800} = 860 \text{ psi}$$

$$f_s = \frac{3{,}700{,}000 \times 12 \times 37.5 \times 10}{957{,}000} = 17{,}400 \text{ psi}$$

Similarly, at B of Fig. 9-19(c), with $A_s = 25$ in.^{2}, $f_c = 625$ psi and $f_s = 16{,}200$ psi. The greatest shear is at the left of D [Fig. 9-19(b)]. The bottom of the 45° pressure

cone is $(4 + 4.67)$ ft from D. The total shear at this point is

$$645 - 42.5 \times 8.67 = 277 \text{ kips}$$

Then the diagonal tensile stress at this point is

$$v_L = \frac{V}{bjd} = \frac{277{,}000}{108 \times 0.89 \times 56} = 51 \text{ psi} \qquad \text{(narrow section)}$$

The greatest bond stress, assuming the full shears at B and D, is at B because the

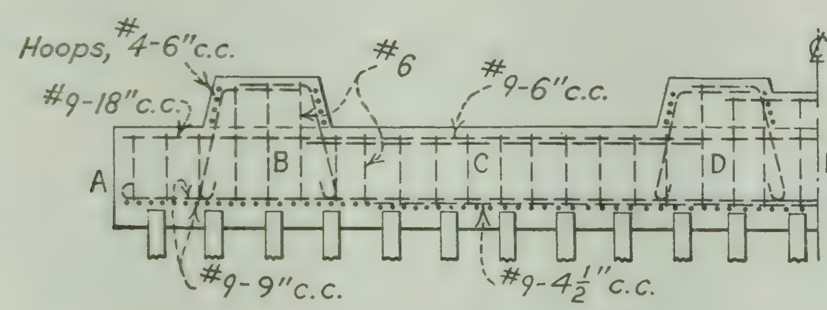


FIG. 9-20.

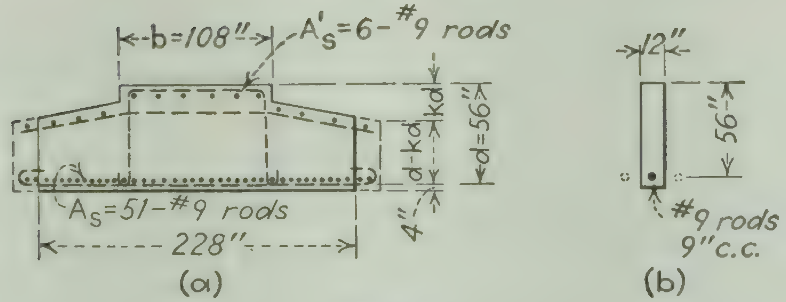


FIG. 9-21.

steel area here is half of that used at D. Then, using the shear at the edge of the pedestal 4 ft to the right of B, and assuming $j = 0.88$,

$$u = \frac{V}{(\Sigma o)jd} = \frac{503{,}000 - (4 \times 42{,}500)}{25 \times 4 \times 0.88 \times 56} = 68 \text{ psi}$$

The transverse shearing stress (punching shear) at the edge of the pedestal 4 ft to the left of D [Fig. 9-19(b)] is

$$v_T = \frac{V}{b(kd)} = \frac{1{,}000(645 - 42.5 \times 4)}{108 \times 18.5} = 238 \text{ psi}$$

The unit stresses at point C of Fig. 9-19(c) are found by using the section of Fig. 9-21(a) as an inverted T beam with tension in the top. Neglect the few rods that serve as temperature reinforcement in the top of the sloping sides of the footing. From Fig. 9-20, $A_s = 17$ in.², and $A_s' = 25$ in.². From Fig. 9-21(a), $b = 228$ in.

$$228\,\frac{(kd)^2}{2} + 9 \times 25(kd - 4) = 10 \times 17(56 - kd)$$

$$kd = 8 \text{ in.} \qquad d - kd = 48 \text{ in.} \qquad I_c = 434{,}500 \text{ in.}^4$$

$$f_c = \frac{1{,}140{,}000 \times 12 \times 8}{434{,}500} = 252 \text{ psi}$$

$$f_s = \frac{1{,}140{,}000 \times 12 \times 48 \times 10}{434{,}500} = 15{,}100 \text{ psi}$$

The maximum bending moment in a transverse strip due to longitudinal forces is found, from Fig. 9-18(b), to occur in the section RS. Applying the pile loads in Fig. 9-17(b), the bending moment in a 1-ft strip at a point 2 ft from the center of the pedestal is (using six rows of piles)

$$M_L = \tfrac{6}{15}(3.9 \times 6.75 + 2.34 \times 3.25) = 13.6 \text{ ft-k}$$

The bending moment in this transverse strip due to the superstructure is

$$M_s = 25(6.75 + 3.25) \div 3.5 = 71.4 \text{ ft-k per ft of pier}$$

where 25 kips is the load per pile (4 × 900/144). This must be combined with the effect of the longitudinal forces so that

$$M = 13.6 + 71.4 = 85 \text{ ft-k}$$

The cross section of a 1-ft strip is shown in Fig. 9-21(b) without considering the No. 6 rods in the top; then

$$kd = 10.1 \text{ in.} \qquad d - kd = 45.9 \text{ in.} \qquad I_c = 32{,}130 \text{ in.}^4$$
$$f_c = 320 \text{ psi} \qquad f_s = 14{,}600 \text{ psi}$$

Since the foregoing calculations show that the stresses at the critical points are safe, then those at other points are automatically known to be safe and therefore need not be computed.

Practice Problems

9-1. Assume a plain concrete pedestal like that of Fig. 9-3. It is 3 ft square and 18 in. deep. It rests upon earth. It supports a column that is 16 in. square and that carries a load of 40,000 lb. What is the tensile stress in the concrete?

9-2. Assume a rectangular plain concrete pedestal like that of Fig. 9-3, supported on earth. It is 3 ft wide, 4 ft long, and 20 in. thick. It carries a 16-in. square column which supports a load of 50,000 lb. What is the maximum tensile stress in the concrete?

9-3. Design a simple square reinforced-concrete footing to support an 18-in. square column which carries a load of 150,000 lb. Let $f'_c = 2{,}500$ psi, $n = 12$, the allowable $f_c = 1{,}000$ psi, $f_s = 20{,}000$ psi, the maximum shearing stress = 75 psi, with hooked rods, $u = 200$ psi, and the allowable soil pressure = 5,000 psf.

Discussion. The footing is like that of Fig. 9-4(a) but without the pedestal. The solution is similar to that of Example 9-2 without the effect of the slope.

9-4. Design a square reinforced-concrete footing with a sloped top for the column of Prob. 9-3 if all the conditions remain unchanged except that the permissible soil pressure = 4,000 psf.

Discussion. Assume that there is a flat top 24 in. square and that the slope is about 1:4 as in Fig. 9-4(b). Design the footing as for Example 9-2.

9-5. Design a square stepped footing like that of Fig. 9-4(a) to support a column load of 300,000 lb. The column is 24 in. in diameter, $c = 3.5$ ft, $n = 12$, the allowable $f_c = 1{,}000$ psi, $f_s = 18{,}000$ psi, the max $v_L = 75$ psi, with hooked rods, $u = 200$ psi, and the allowable pressure on the soil = 5,000 psf. Detail the footing.

Discussion. Follow the same procedure as for Example 9-2 with c instead of a as the dimension that determines the starting points of the 45° sloped shear or distribution lines.

9-6. A stepped footing is to support a column load of 450 kips. Its width is limited to 7 ft. The pedestal is to be 4 ft square and 3 ft high, with its top at ground level.

The allowable soil pressure is 6 ksf. Assume 3,000-lb concrete and f_s = 20,000 psi. Design the footing, using the Code.

9-7. The stepped footing shown in Fig. 9-22(a) is to be designed for the following conditions: P = 250 kips, M = 50 ft-k, S = 10 kips, f'_c = 3,000 psi, and f_s = 20,000 psi. The soil is adequate.

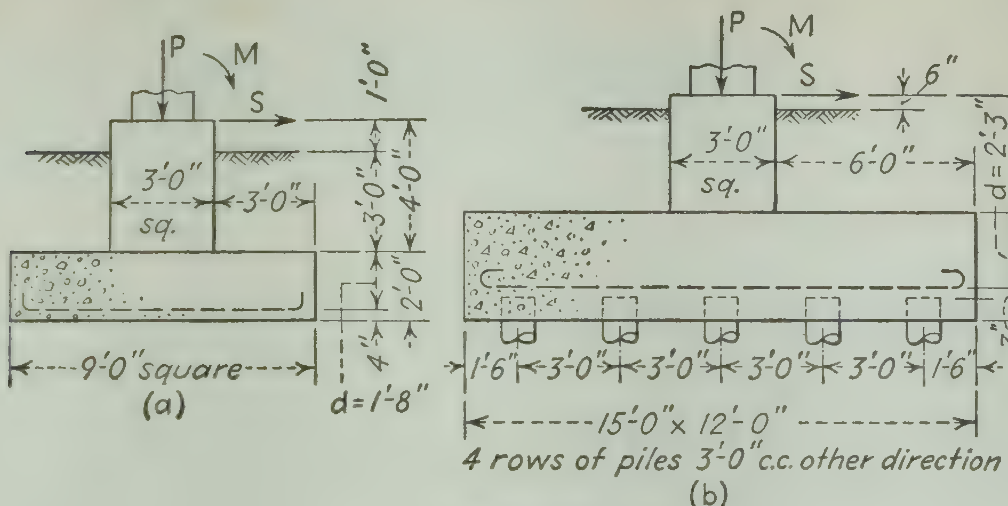


FIG. 9-22.

9-8. The stepped footing shown in Fig. 9-22(a) has the following loads: P = 350 kips, M = 100 ft-k, and S = 20 kips. Compute the maximum bearing pressure.

9-9. The footing of Fig. 9-22(b) has 4 rows of 5 piles each. Design the reinforcement and check footing for the following conditions: P = 800 kips, M = 120 ft-k, S = 20 kips, f'_c = 3,000 psi, and f_s = 20,000 psi.

9-10. Design the reinforcement for the footing of Fig. 9-22(b) and check the footing for the following conditions: P = 900 kips, M = 100 ft-k, S = 15 kips, f'_c = 3,500 psi, and f_s = 20,000 psi.

9-11. Assume the footing shown in Figs. 9-11(a), (b), and (c). Compute the pressure diagram if the column load = 260,000 lb, M = 60,000 ft-lb, and S = 4,000 lb.

10

LARGE SLABS

10-1. Introduction. Large slabs are a very useful type of reinforced-concrete construction. The types to be studied here are two-way ones like Fig. 10-1 and flat slabs as pictured in Fig. 10-5. It is the purpose of this chapter to explain and to illustrate their design as specified in the Code. However, only uniformly distributed loads will be considered here. The analysis of such slabs when subjected to concentrated loads will be discussed in an advanced volume to be published.

Although these slabs are highly indeterminate, they are encountered so frequently in practice that it seems that a method of analysis of them should be included in an elementary text on reinforced concrete. The methods specified by the Code are exceedingly empirical, and they seem at first to be very complicated. However, they really are not too difficult to follow, and they seem to yield safe results.

These codes are the result of much study, experience, and discussion. They serve a very useful purpose in establishing procedures that are recognized as producing structures that are acceptable. The specified methods of analysis are intended to provide a simplified procedure, whereas anything approaching an "exact" analysis of such indeterminate structures would be very complicated.

10-2. Two-way slabs. Large rectangular slabs that are supported at all four edges and that have reinforcement in two perpendicular directions, as shown in Fig. 10-1(a), are frequently used, especially in large continuous monolithic floor systems. When the short side is less than one-half of the long side, the slabs are generally analyzed as though they were one-way members spanning the short direction. Otherwise, they are to be analyzed upon the basis that the loads are supported by bending in both directions.

First of all, it is well for one to try to visualize how a two-way slab behaves under load. Assume that the slab pictured in Fig. 10-1(a) is covered with a 2-ft layer of sand so as to cause a uniform load of 200 psf. The slab sags under this weight. The central portion can deflect but the beams shown dotted are very strong and stiff so that their deflec-

tion is very small, and they are supported by columns at their intersections. Thus the beams form a stiff supporting grid.

If the slab were made of two sets of 2- by 12-in. planks laid at right angles over the beams as shown by the two strips in Fig. 10-1(a), the shorter plank would obviously be stiffer. If a man were to stand on their intersection, more of his weight would be supported by the short one than by the other. In the case of the concrete slab, any equally reinforced imaginary strips must act similarly because the deflection of their intersection must be the same.

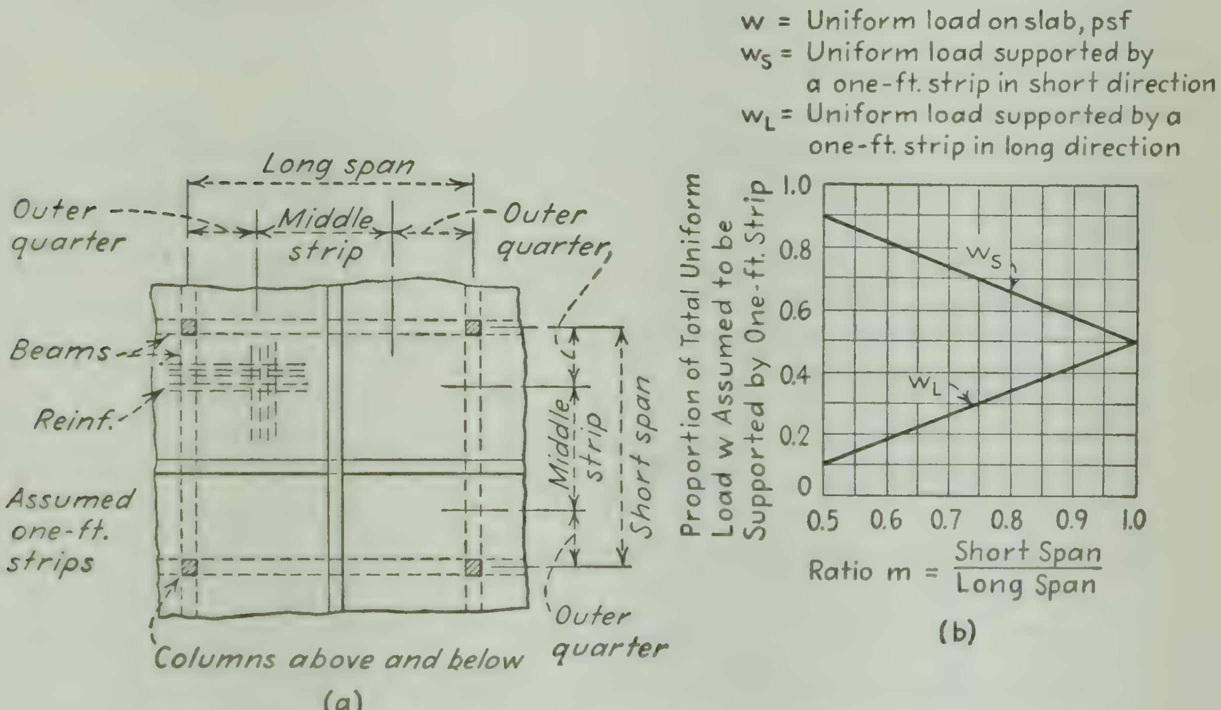


FIG. 10-1. Assumed distribution of uniform load on two-way slabs.

Naturally, the central portions of the slab will deflect the most. Therefore, the middle strips shown on the drawing are the critical ones whether the edges of the slabs are simply supported or continuous over the beams. The portions near the beams are affected by the T beam action of the beams themselves.

Especially in the case of heavy mats, thick side walls of structures subjected to large earth pressures, and even heavy floors or roofs, it is convenient to have some means of approximating the bending moments in two-way slabs. In Fig. 10-1(b) is shown a diagram for estimating the proportion of a uniform load that may be assumed to be supported by the bending resistance in the short and in the long directions. A typical 1-ft strip may then be assumed to carry its proportion of the loads as though the slab were a one-way slab in that particular direction. If the central one-half of the slab is designed in this manner for each direction, it will serve to give a "scale" to the size needed. The outer quarters

near the beams may be reinforced as though the moments varied from that of the middle strip to 25 or 50 per cent thereof near the beams that are parallel to the strips being considered.

The Code gives two methods for the analysis of two-way slabs. One method will be given here. It is selected upon the assumption that the reader has not made a study of indeterminate structures. The following outline is an attempt to state briefly the rules given in the Code:

1. Slabs must be monolithic with supports, solid or ribbed both ways, and supported on all four sides with edges continuous or discontinuous.

2. Span = center to center of supports, or clear span + twice the slab thickness, whichever is the smaller.

3. S denotes short span.

4. m denotes ratio of short span to long span. Min m (for computations) = 0.5.

5. Middle strip is a strip across panel, one-half panel in width, symmetrical about center line of panel. If m is less than 0.5, middle strip width = long span minus short span.

6. Column strip = remainder outside of adjacent middle strips and centered along support, as pictured in Fig. 10-2(a).

7. w = uniformly distributed load (panel fully loaded).

8. Principal design sections:

   a. For negative moments: along edges of panel at faces of supports.

   b. For positive moments: along center lines of panel.

9. Bending moment coefficients C—as shown in Table 10-1. Interpolate for intermediate values of m. $M = CwS^2$ for a 1-ft strip. For column strip moment coefficients, use two-thirds of value used for middle strip as average, varying from maximum at edge of middle strip, where value equals that of middle strip, to one-third that value at edge of panel.

10. Reinforce top and bottom of exterior corners of panels for same moment as positive moment in middle strip. Critical section in top of slab is perpendicular to diagonal; in bottom, parallel to diagonal. Effective area of diagonal rod = $A \times$ sine of angle between rod and critical section.

11. When varying spans or loads on opposite sides of supports cause unequal negative moments, especially when the smaller is less than 80 per cent of the larger, distribute two-thirds of unbalanced moment to the two spans in proportion to their I/L (stiffness factors). When the difference between the unadjusted negative moments is large (perhaps 20 per cent of the larger), investigate mid-span positive moments as for continuous beams, or assume such moment = 1.5 $\times$ the original end moment in that panel minus the average of the end moments in the adjacent panels.

12. Distribution of load for shear in slabs and for design of supporting beams assumed to be uniform load on area within 45° lines from corners and median line of panel parallel to long side (Fig. 10-2), giving the following total loads for design of beams, considering panel on one side of beam:

   a. Shears:

   (1) For short spans:

$$W_S = \frac{wS^2}{4} \tag{10-1}$$

(2) For long spans:

$$W_L = \frac{wS^2}{4}\left(\frac{2-m}{m}\right) \tag{10-2}$$

NOTE: Adjust reactions to take care of effect of different end moments.

TABLE 10-1. Bending Moment Coefficients C for Two-way Slabs
For moments in middle strips in ft-lb per ft width of slab

Bending moments	Short span S						Long span, all values of m
	Values of m						
	1.0	0.9	0.8	0.7	0.6	0.5 and less	
Case 1. Interior panels:							
Negative moment at continuous edge	0.033	0.040	0.048	0.055	0.063	0.083	0.033
Positive moment at mid-span	0.025	0.030	0.036	0.041	0.047	0.062	0.025
Case 2. One edge discontinuous:							
Negative moment at continuous edge	0.041	0.048	0.055	0.062	0.069	0.085	0.041
At discontinuous edge	0.021	0.024	0.027	0.031	0.035	0.042	0.021
Positive moment at mid-span	0.031	0.036	0.041	0.047	0.052	0.064	0.031
Case 3. Two edges discontinuous:							
Negative moment at continuous edge	0.049	0.057	0.064	0.071	0.078	0.090	0.049
At discontinuous edge	0.025	0.028	0.032	0.036	0.039	0.045	0.025
Positive moment at mid-span	0.037	0.043	0.048	0.054	0.059	0.068	0.037
Case 4. Three edges discontinuous:							
Negative moment at continuous edge	0.058	0.066	0.074	0.082	0.090	0.098	0.058
At discontinuous edge	0.029	0.033	0.037	0.041	0.045	0.049	0.029
Positive moment at mid-span	0.044	0.050	0.056	0.062	0.068	0.074	0.044
Case 5. Four edges discontinuous:							
Negative moment at discontinuous edge	0.033	0.038	0.043	0.047	0.053	0.055	0.033
Positive moment at mid-span	0.050	0.057	0.064	0.072	0.080	0.083	0.050

Moment $= CwS^2$ for both long and short spans. w = load in psf. Slabs interposed between heavy masonry walls are considered as discontinuous.

b. Approximate uniform load per foot of beam for computing bending moments:

(1) For short span:

$$w_S = \frac{wS}{3} \tag{10-3}$$

(2) For long span:

$$w_L = \frac{wS}{3}\left(\frac{3-m^2}{2}\right) \tag{10-4}$$

13. Minimum slab thickness $t = 4$ in. but not less than the perimeter of the slab divided by 180.

Example 10-1. Assume a building floor like that in Fig. 10-2 with several bays 22 ft long. The live load = 150 psf, and $f'_c = 3{,}000$ psi. Find the maximum bending moments and shear in the interior bay shown, using the procedure given in the preceding outline.

Assume $S = 18$ ft, $m = 18/22 = 0.82$.

$$\min t = (2 \times 18 + 2 \times 22)12/180 = 5.3 \text{ in.}$$

For the interior panel, $t = 5.6$ in.

TABLE 10-2. Bending Moments in Middle Strips of Slabs
For Example 10-1

Panel	Edge	Center of span	m	Coef. C	S, ft	S^2	$M = CwS^2$, ft-lb per ft	Min A_s, in.^{2}
	AA'	....	0.82	0.026	18	324	−1,900	0.29
	BB'	....	0.82	0.054	18	324	−3,940	0.57*
$AA'B'B$	AB	....	0.82	0.041	18	324	−2,980	0.45
	$A'B'$	....	0.82	0.041	18	324	−2,980	0.45
		Short	0.82	0.040	18	324	+2,910	0.44
		Long	0.82	0.031	18	324	+2,260	0.34
	BB'	....	0.91	0.039	20	400	−3,520	0.57*
	CC'	....	0.91	0.039	20	400	−3,520	0.53
$BB'C'C$	BC	....	0.91	0.033	20	400	−2,970	0.45
	$B'C'$	....	0.91	0.033	20	400	−2,970	0.45
		Short	0.91	0.030	20	400	+2,700	0.41
		Long	0.91	0.025	20	400	+2,250	0.34

* Adjusted by distribution of unbalanced M with $j = 0.88$; $d = 4.5$ in.

Assume $t = 6$ in. throughout, and $w = 150 + 75 = 225$ psf. Then interpolate in Table 10-1 to find the moment coefficients and to compute the data given in Table 10-2

The negative moments along BB' for the middle strip appear to differ by

$$3{,}940 - 3{,}520 = 420 \text{ ft-lb}$$

This should be adjusted as follows:

Outer panel:

$$\frac{I}{L} = \left(\frac{12 \times 6^3}{12}\right) \div 216 = \frac{216}{216} = 1.0$$

Interior panel:

$$\frac{I}{L} = \left(\frac{12 \times 6^3}{12}\right) \div 240 = \frac{216}{240} = 0.9$$

M for outer panel:

$$3{,}940 - \frac{1}{1.9} \times 420 \times 0.67 = 3{,}790 \text{ ft-lb}$$

M for interior panel:

$$3{,}520 + \frac{0.9}{1.9} \times 420 \times 0.67 = 3{,}650 \text{ ft-lb}$$

Having the bending moments, the reinforcement can be determined. Furnishing the required steel areas with a practical bar arrangement is not as simple as it may appear. Generally, part of the bottom steel should extend throughout the slab in both directions; some may be bent up to resist negative moments, although the slab is too thin for this to be very advantageous. The bars should not be farther apart than three times the slab thickness, and A_s should not be less than that required for shrinkage.

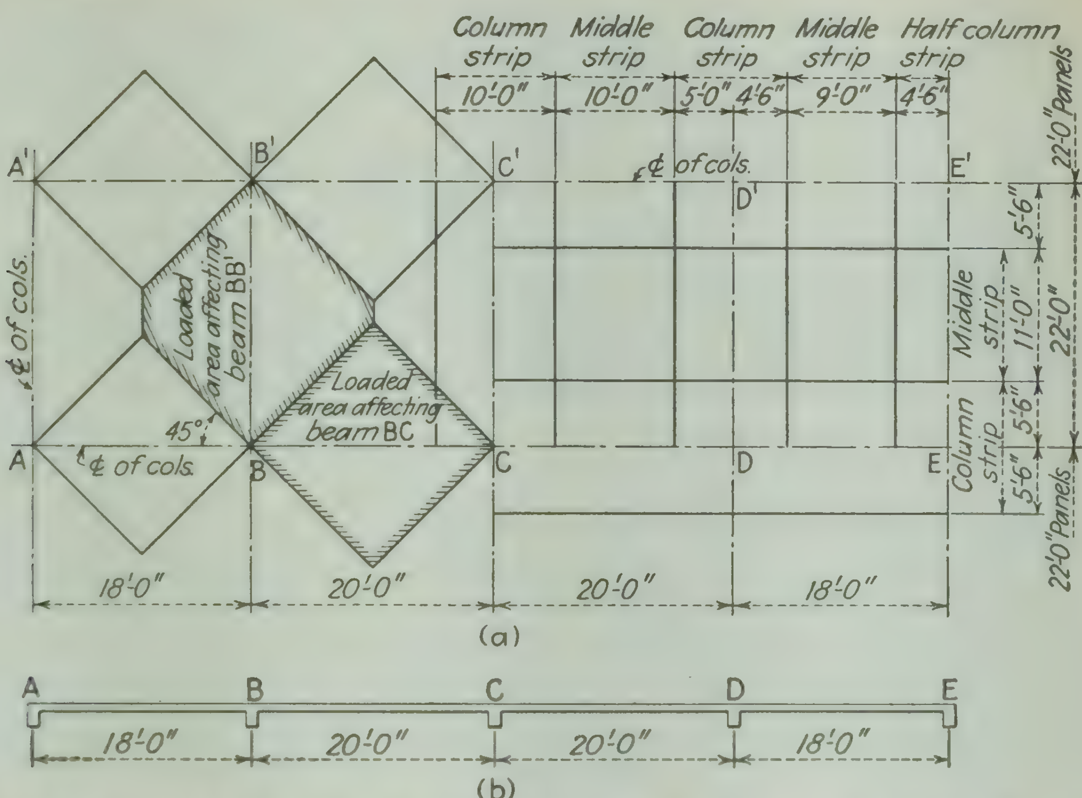


FIG. 10-2. Assumed distribution of loads to beams supporting two-way slabs with uniform loads.

The greatest shear will be in the middle strip of the short span in panel $AA'B'B$. It is $225 \times 0.65 \times 9 = 1{,}316$ lb for a 12-in. strip. [See Fig. 10-1(*b*).]

In some cases the stiffness of such large thin slabs may need to be investigated.

In building construction for which the live loads are not large, it is often economical to use large slabs with hollow-tile filler blocks as pictured in Fig. 10-3(*a*) or with hollow spaces which are produced by the use of thin steel forms, or "pans," as shown in Sketch (*b*). Both these types may be built with the one-way or two-way system of reinforcement, but the latter is generally more rigid. The two-way system is made by using square or rectangular tiles or pans which are separated on all four sides so as to form a series of small T beams at right angles to each other.

The design of these special floors is based upon the same principles as those which have been explained for solid slabs.

10-3. Flat slabs. The term "flat slabs" denotes large rectangular slabs of approximately uniform thickness which are supported on columns but which have no beams or girders to carry these slabs—except

possibly at the outside of the structure or at openings. Decreased height of each story, excellent lighting and ventilation, better fire resistance because of the absence of projecting corners, easier formwork, and better economy for heavy uniform loading—these are some of the advantages of flat-slab construction. Figure 10-4 shows its use for both an elevated railway and a warehouse.

The essential parts of this type of construction are pictured in Fig. 10-5. This shows that, for design purposes, the slab is divided each way into column strips, which serve the purpose of beams between the columns; and middle strips, which may be regarded as suspended spans

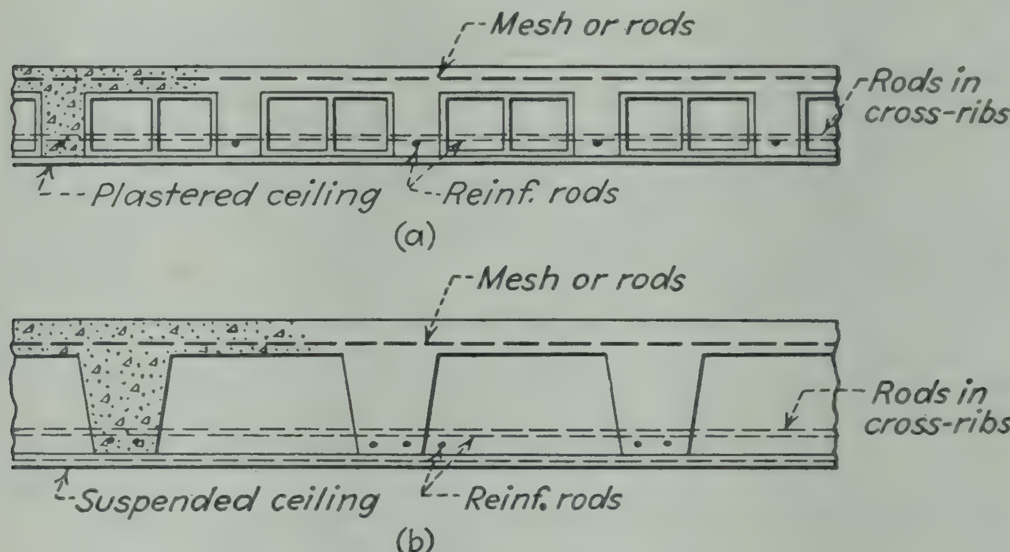


FIG. 10-3.

that are carried by the column strips for two-way systems—reinforcement parallel to the column rows in both directions. In a four-way system, as indicated in Fig. 10-5, the middle strips are crossed by two sets of diagonal reinforcement which carry the loads more directly to the columns. However, the four-way system is more complicated, and the "packing up" of the four layers of rods is objectionable if the slab is relatively light. Figure 10-6 should be studied very carefully because it shows a two-way system that is under construction.

When a flat slab is loaded, it deflects on all sides of the columns which tend to punch through the floor. In fact, an exaggerated example can be seen when one props up a large canvas with a system of poles. Negative moments exist around the edges and across the tops of the column capitals which are flared out in order to reduce these moments and the shears—the same being the function of the drop panels. The determination of the magnitudes and distributions of the stresses is exceedingly complex if one attempts to compute them theoretically.

The construction to be illustrated is that used for floors and roofs. Drop panels, and even capitals, may be omitted in some cases. If so,

structural-steel crossed members or special rings of bent shear reinforcement may be used to improve the strength of the slab in the vicinity of the tops of the columns. The heavy flat slabs that are used as foundation mats[1] may be from 1 to 3 or 4 ft thick, and their analysis may be made by other means with sufficient accuracy.

FIG. 10-4. Construction of Lackawanna Terminal Warehouse, Jersey City, N.J. (*Courtesy of Turner Construction Co.*)

The system of moment coefficients recommended by the Code will be used here. The following is an attempt to condense the specifications into outline form, and a few extra ideas are added:

A. Limitations for applicability of specifications.
   1. Slabs must be rectangular and monolithic with columns.
   2. There should be three or more rows of panels in each direction.
   3. Maximum ratio of length to width of panel = 1.33.
   4. Dimensions of adjacent panels should not vary by more than 20 per cent of the shorter span.
   5. Slabs may be solid or ribbed as in Fig. 10-3.

B. Principal design sections (see Fig. 10-5).
   1. Negative moments—along edges of panel. Notice that these sections for column strips cross the columns. For convenience $-M_c$ will be used to denote negative moment in column strips, and $-M_m$ will denote negative moment in middle strips.

[1] See C. W. Dunham, "Foundations of Structures," Chap. 7, McGraw-Hill Book Company, Inc., New York, 1950.

2. Positive moments—along center lines of panel. For convenience, $+M_c$ will denote positive moment in a column strip, and $+M_m$ will denote positive moment in a middle strip.
3. For computing compression due to bending, use three-fourths of width of strip as effective. If section passes through drop panel, use three-fourths of width of drop. The latter seems to be too severe a restriction when the drop panels are relatively small and are considerably less than the width of the column strips. However, it is to allow for nonuniformity of resistance across the section. Make reductions for recesses that weaken section.

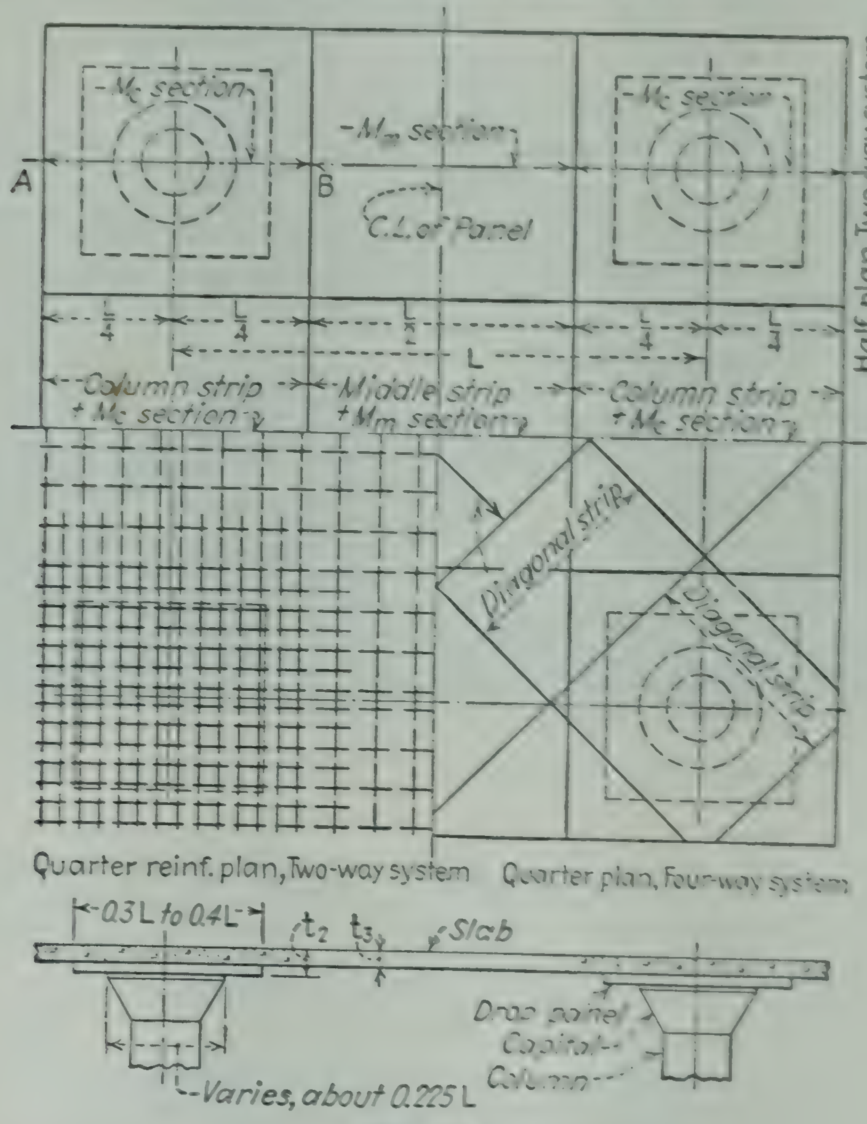


FIG. 10-5.

4. In two-way system, reinforcement to resist computed bending in a strip must lie within strip itself, and all tensile bars in the strip may be included.
5. In four-way system, reinforcement to resist computed bending in a strip = sum of normal areas of all bars in the strip times cosine of angle between each bar and the longitudinal axis of the span.

C. Bending moments.

1. For interior panels, the total bending moment M_o in the direction of either side of the panel for a total uniform load W on the panel is

$$M_o = 0.09WL\left(1 - \frac{2c}{3L}\right)^2 \tag{10-5}$$

M_o is the sum of positive and average negative bending moments at critical design sections of the flat-slab panel. L is span length center to center of columns in direction being considered. The symbol c = effective diameter in feet of a round capital at its junction with slab or drop panel except that one must disregard any part of capital outside the largest 90° cone that can be included within the outline of the capital. If the capital is rectangular, c is the diameter of a circle having an area equal to the base of a similarly inscribed 90° pyramid.

TABLE 10-3. Bending Moments in Interior Flat-slab Panel

With drop panel:		
Column strip..........	Negative moment	$0.50M_o$
	Positive moment	$0.20M_o$
Middle strip..........	Negative moment	$0.15M_o$
	Positive moment	$0.15M_o$
Without drop panel:		
Column strip..........	Negative moment	$0.46M_o$
	Positive moment	$0.22M_o$
Middle strip..........	Negative moment	$0.16M_o$
	Positive moment	$0.16M_o$

TABLE 10-4. Bending Moments in Exterior Flat-slab Panel

With drop panel:		
Column strip..........	Exterior negative	$0.45M_o$
	Positive moment	$0.25M_o$
	Interior negative	$0.55M_o$
Middle strip..........	Exterior negative	$0.10M_o$
	Positive moment	$0.19M_o$
	Interior negative	$0.165M_o$
Without drop panel:		
Column strip..........	Exterior negative	$0.41M_o$
	Positive moment	$0.28M_o$
	Interior negative	$0.50M_o$
Middle strip..........	Exterior negative	$0.10M_o$
	Positive moment	$0.20M_o$
	Interior negative	$0.176M_o$

2. Distribution of total moment M_o at principal design sections of interior panel is to be as shown in Table 10-3.
3. Minimum reinforcement at any section of any strip is $A_s = 0.0025bd$.
4. Discontinuous panels.
   a. Bending moments at critical sections are to be as shown in Table 10-4 if exterior supports are concrete walls or beams that are integral with slab, and ratio of stiffness of support to that of slab at least equals ratio of live load to dead load and is not less than 1.
   b. Determine M_o as for interior panel.
   c. When discontinuous edge is restrained, assume negative moment more nearly equal in both column and middle strips, with that of column strip not less than $0.30M_o$, and the sum remaining as in Table 10-4.

d. Positive bending in middle strip parallel to discontinuous edge (except at corners) = same as for interior panel. At corners with unrestrained edges it may be advisable to increase $+M_m$ by 10 or 20 per cent but not to decrease $-M_m$.

e. With masonry walls and other unrestraining edge supports, use Table 10-4 for bending at critical sections of strips normal to edge except as follows:

$-M$ at inner face of edge support $= 0.05M_o$
Increase $+M$ by 40 per cent
Increase $-M$ at first interior columns by 30 per cent

TABLE 10-5. Bending Moments in Panels with Marginal Beams or Walls

		Marginal beams with depth greater than $1\frac{1}{2}$ times the slab thickness; or bearing wall		Marginal beams with depth $1\frac{1}{2}$ times the slab thickness or less	
a. Load to be carried by marginal beam or wall		Loads directly superimposed upon it plus a uniform load equal to one-quarter of the total live and dead panel load		Loads directly superimposed upon it exclusive of any panel load	
		With drop	Without drop	With drop	Without drop
b. Moment to be used in the design of half column strip adjacent and parallel to marginal beam or wall	Neg.	$0.125M_o$	$0.115M_o$	$0.25M_o$	$0.23M_o$
	Pos.	$0.05M_o$	$0.055M_o$	$0.10M_o$	$0.11M_o$
c. Negative moment to be used in design of middle strip continuous across a beam or wall	Neg.	$0.195M_o$	$0.208M_o$	$0.15M_o$	$0.16M_o$

f. Use Table 10-5 for moments in strips parallel and close to edge supports—also for edge beams and $-M_m$.

g. Panels with marginal beams on opposite edges are to be designed as one-way or two-way slabs.

h. Coefficients in Tables 10-3, 10-4, and 10-5 may be varied by not over 6 per cent, but sum of $+Ms$ and $-Ms$ in a panel is not to be less than specified there.

D. Thickness of slab and drop panel:

1. Slab thickness shown in Fig. 10-5 shall not be less than

$$t_3 = \frac{L}{40} \qquad \text{with drop panels}$$

or

$$t_3 = \frac{L}{36} \qquad \text{without drop panels}$$

2. Thickness of drop panel below flat slab shall not exceed one-fourth the distance from edge of capital to edge of drop. No data are given for the widths of drop panels, but they may be from $0.3L$ to $0.4L$.
3. Thicknesses shall be such that compressive stress caused by bending at critical sections of strips, and shearing stress about capital and edge of drop, shall not exceed allowable unit stresses for concrete of quality used.

E. Shearing stresses.

1. Referring to Fig. 10-5, compute shearing stress on the following vertical sections.
   a. $t_3 - 1\frac{1}{2}$ in. outside edge of column capital and parallel to or concentric with it when no drop panels are used.
   b. $t_2 - 1\frac{1}{2}$ in. outside capital with drop panels.
   c. $t_3 - 1\frac{1}{2}$ in. beyond and parallel to edge of drop panel.

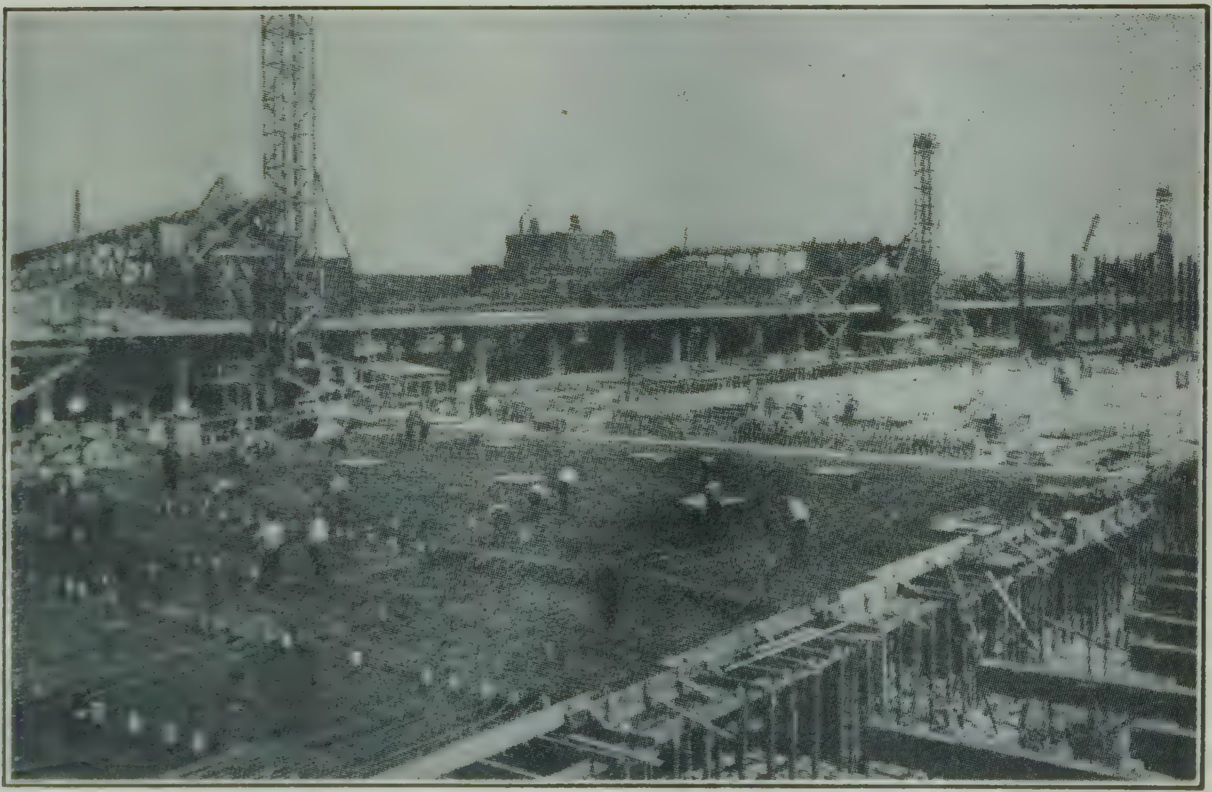


FIG. 10-6. Construction of Lackawanna Terminal Warehouse, Jersey City, N.J. (Courtesy of Turner Construction Co.)

2. Use $v_L = V/bjd$, where d = depth of slab or drop $-1\frac{1}{2}$ in.
3. Shearing stress near capital should not exceed
   a. $0.03f'_c$ when at least 50 per cent of bars for $-M_c$ pass directly over capital.
   b. $0.025f'_c$ when 25 per cent or less of bars for $-M_c$ pass directly over capital.
   c. For percentages between 50 and 25, interpolate accordingly.
4. Shearing stress in slab near edge of drop shall not exceed $0.03f'_c$, and at least 50 per cent of negative reinforcement in column strip must pass directly over the drop panel.

F. Capitals and brackets.

1. With no capital, c of Eq. (10-5) = diameter of column. Allow for effective structural steel or other detail embedded in slab or drop panel if it serves somewhat as a capital.

2. When beam frames into column with no capital or supporting bracket, c = width of column plus twice the depth of the stem of the beam (for strips parallel to beam only).
3. Brackets may be substituted for capitals at exterior columns if effective. Then c = twice the distance from center of column to where bracket is $1\frac{1}{2}$ in. thick, but not to exceed thickness of column plus twice the depth of the bracket.
4. Where c varies, use average of four corners to find bending in middle strips, but use average of c for the two columns at the ends of a column strip to determine bending in it.

G. Arrangement of reinforcement.

1. Arrange rods for intermediate as well as for critical sections.
2. Space bars evenly across strips. Spacing not to exceed $3\times$ slab thickness.
3. At exterior supports, extend bars perpendicular to edge for $+M$ so as to secure at least 6-in. embedment in supports and columns. Anchor negative reinforcement into edge beams, walls, and columns.

H. Beams.

1. Beams (or equivalent) should be used at all discontinuous edges unless cantilevered.
2. Edge beams are to support one-fourth of uniform load on adjacent panel plus all direct loads. See Table 10-5.
3. Interior beams are to support one-fourth of uniform load on both adjacent panels—plus all direct loads—as T beams.
4. If interior beams or walls off column center lines interfere with flat-slab action, it may be desirable to frame entire panel as beam-and-slab construction.

I. Openings.

Openings may be used if moments and shears can be resisted safely.

Example 10-2. Assume that an industrial building is 6 panels wide and 15 panels long. It is to be two-way flat-slab construction with drop panels. All spans are 20 ft both ways; the live load = 250 psf; the drop panels are 8 ft square; and c for the capitals is 4.5 ft. Design the slabs of the exterior and the first interior panels of a typical intermediate bay as shown in the key plan in Fig. 10-7. Assume $f_c' = 3{,}750$ psi, $n = 8$, and the allowable f_s and f_c = 20,000 and 1,700 psi, respectively. Use the Code's procedure. Use a concrete edge beam at the exterior wall, but assume that the slab is not strongly restrained.

Discussion. The numbers and letters given along the right-hand margin of the calculations refer to the applicable item in the preceding outline.

1. *Interior Panel.* Using $t_3 = L/40$, $t_3 = 20 \times 12/40 = 6$ in. However, this seems to be rather thin. Try 7 in. Then $w = 250 + 88 = 338$ psf for the slab and perhaps 400 psf at the drop panel. However, the latter's weight is not important and the average load will be called 350 psf for design purposes. D1

Next, determine the drop. Its maximum is $(4 - 2.25)\frac{12}{4} = 5.25$ in. Call it 4 in. Then $t_2 = 7 + 4 = 11$ in., and d at the edge of the capital $= 11 - 1.5 = 9.5$ in. D2

$$M_0 = 0.09(350 \times 20^2)20\left(1 - \frac{2 \times 4.5}{3 \times 20}\right)^2 = 182{,}000 \text{ ft-lb} \qquad \text{C1}$$

Column strip:

$$-M_c = -0.50 \times 182{,}000 = -91{,}000 \text{ ft-lb} \qquad \text{C2}$$
$$+M_c = +0.20 \times 182{,}000 = +36{,}400 \text{ ft-lb} \qquad \text{C2}$$

Middle strip:

$$-M_m = -0.15 \times 182{,}000 = -27{,}300 \text{ ft-lb} \qquad \text{C2}$$
$$+M_m = +0.15 \times 182{,}000 = +27{,}300 \text{ ft-lb} \qquad \text{C2}$$

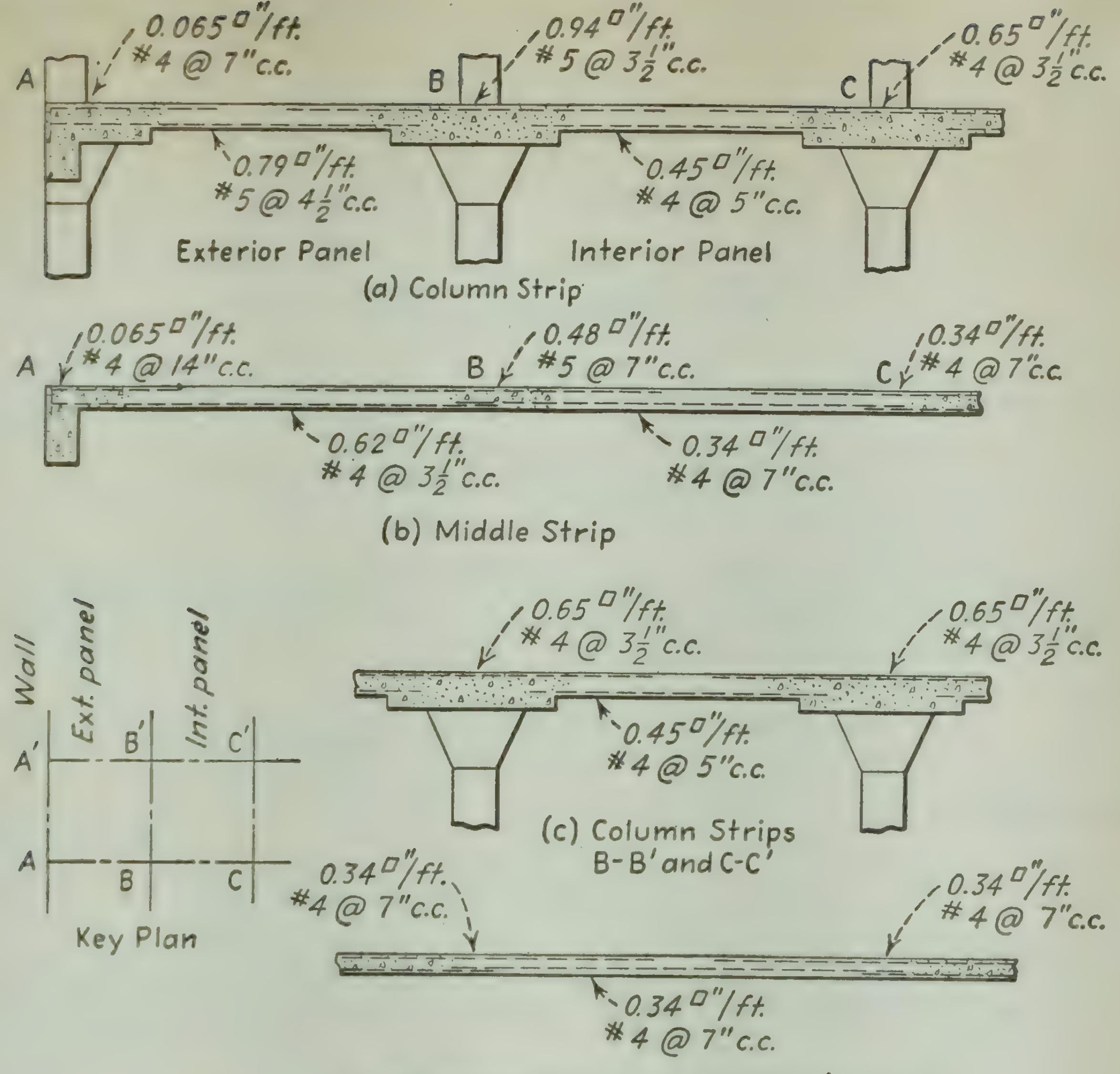


FIG. 10-7. Schematic diagrams of flat-slab reinforcement.

The critical section will be at the capital. The assumed b through the drop is

$$b = \tfrac{3}{4} \times 8 \times 12 = 72 \text{ in.} \qquad \text{B3}$$

From Table 6 in the Appendix, $K = 298$ for a balanced design with $f_c = 1{,}700$ psi. For $-M_c$, this would give the required effective depth

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{91{,}000 \times 12}{298 \times 72}} = 7.14 \text{ in.}$$

This is less than the 9.5 in. available, but the latter will be retained. Similarly, with

$$b = \tfrac{3}{4} \times 120 = 90 \text{ in.} \qquad \text{B3}$$

a balanced design for the center of the column strip requires

$$d = \sqrt{\frac{36{,}400 \times 12}{298 \times 90}} = 4.04 \text{ in.}$$

This is less than the 5.5 in. available.

Since both these depths used are well above the minimum, f_c in the concrete will be under the allowable value.

Now determine the trial reinforcement, using $d = 9.5$ for $-M_c$, $d = 5.5$ elsewhere, and $j = 0.88$.

Column strip:

$$\text{For } -M_c\text{: } A_s = \frac{91{,}000 \times 12}{20{,}000 \times 0.88 \times 9.5} = 6.54 \text{ in.}^2$$

Use No. 4 bars at 3½ in. c.c.

$$\text{For } +M_c\text{: } A_s = \frac{36{,}400 \times 12}{20{,}000 \times 0.88 \times 5.5} = 4.51 \text{ in.}^2$$

Use No. 4 bars at 5 in. c.c.

Middle strip:

$$\text{For } -M_m \text{ and } +M_m\text{: } A_s = \frac{27{,}300 \times 12}{20{,}000 \times 0.88 \times 5.5} = 3.38 \text{ in.}^2$$

Use No. 4 bars at 7 in. c.c.

Next, check the slab for shear. At a section $d = 9.5$ in. beyond the capital (c), the load on the two adjacent quarters beyond this section must be withstood by a half circle of radius $2.25 \times 12 + 9.5 = 36.5$ in.

$$V = \left[10 \times 20 - \frac{\pi}{2}\left(\frac{36.5}{12}\right)^2\right] 350 = 65{,}000 \text{ lb} \qquad \text{E1b}$$

Then, using $j = 0.88$,

$$v_L = \frac{V}{bjd} = \frac{65{,}000}{(\pi \times 36.5)0.88 \times 9.5} = 68 \text{ psi} \qquad \text{E2}$$

This is satisfactory. More than 50 per cent of the bars will be over the drop. E3a
At a section $d = 7 - 1.5 = 5.5$ in. beyond the drop panel, the load on two adjacent quarters beyond this section must be withstood by one-half the total perimeter of the section. Hence

$$b = (96 + 5.5 \times 2) + 2(48 + 5.5) = 214 \text{ in.}$$

and

$$V = \left(10 \times 20 - \frac{107 \times 53.5}{144}\right) 350 = 56{,}000 \text{ lb} \qquad \text{E1c}$$

Again assuming $j = 0.88$,

$$v_L = \frac{V}{bjd} = \frac{56{,}000}{214 \times 0.88 \times 5.5} = 54 \text{ psi} \qquad \text{E2}$$

The shearing stress is satisfactory. E4

Check the bond stresses as follows:

At edge of capital:

$$V = \left(10 \times 20 - \frac{\pi}{2} \times 2.25^2\right) 350 = 67{,}200 \text{ lb}$$

$$\text{Perimeter} = \pi \times 2.25 = 7.06 \text{ ft}$$

$$\text{Average } V \text{ per ft} = \frac{67{,}200}{7.06} = 9{,}500 \text{ lb}$$

At 3½ in. c.c., $\Sigma o = 5.39$ in.^{2} per ft

Then

$$u = \frac{V}{(\Sigma o)jd} = \frac{9{,}500}{5.39 \times 0.88 \times 9.5} = 211 \text{ psi} \qquad \text{(safe)}$$

At edge of drop panel:

$$V = (10 \times 20 - 8 \times 4)350 = 58{,}800 \text{ lb}$$
$$\text{Perimeter} = 8 + 2 \times 4 = 16 \text{ ft}$$
$$\text{Average } V \text{ per ft} = \frac{58{,}800}{16} = 3{,}700 \text{ lb}$$
$$\text{At } 3\tfrac{1}{2} \text{ in. c.c., } \Sigma o = 5.39 \text{ in.}^2 \text{ per ft}$$

Then

$$u = \frac{3{,}700}{5.39 \times 0.88 \times 5.5} = 142 \text{ psi}$$

If the point of inflection should occur before the edge of the drop is reached, u for 5-in. spacing would be

$$u = \frac{142 \times 5}{3.5} = 206 \text{ psi, which is still safe}$$

2. *Exterior Panel.* From Table 10-4, the bending moments at critical sections, with M_o = 182,000 ft-lb, are C4a

Column strip:

$$\text{Exterior } -M_c = 0.05 \times 182{,}000 = 9{,}100 \text{ ft-lb} \qquad \text{C4e}$$
$$\text{Interior } -M_c = 0.55 \times 182{,}000 \times 1.30 = 130{,}000 \text{ ft-lb} \qquad \text{C4e}$$
$$+M_c = 0.25 \times 182{,}000 \times 1.40 = 63{,}700 \text{ ft-lb} \qquad \text{C4e}$$

Middle strip:

$$\text{Exterior } -M_m = 0.05 \times 182{,}000 = 9{,}100 \text{ ft-lb} \qquad \text{C4e}$$
$$\text{Interior } -M_m = 0.165 \times 182{,}000 \times 1.30 = 39{,}000 \text{ ft-lb} \qquad \text{C4e}$$
$$+M_m = 0.195 \times 182{,}000 \times 1.40 = 49{,}700 \text{ ft-lb} \qquad \text{C4f}$$

The required reinforcement, using No. 4 bars where practicable, is the following:

Column strip:

$$\text{For exterior } -M_c\text{: } A_s = \frac{9{,}100 \times 12}{20{,}000 \times 0.88 \times 9.5} = 0.65 \text{ in.}^2$$

Use No. 4 at 7 in. c.c., although this is excessive. The restraint caused by the column may be appreciable. This steel will be available if needed. G2

$$\text{For interior } -M_c\text{: } A_s = \frac{130{,}000 \times 12}{20{,}000 \times 0.88 \times 9.5} = 9.35 \text{ in.}^2$$

No. 4 bars would have to be about 2½ in. c.c. It will be better to use No. 5 rods at the 3½-in. spacing selected for interior panels and to extend these clear across the first interior column.

$$\text{For } +M_c\text{: } A_s = \frac{63{,}700 \times 12}{20{,}000 \times 0.88 \times 5.5} = 7.9 \text{ in.}^2$$

Use No. 5 bars at 4½ in. c.c.

Middle strip:

For exterior $-M_m$: A_s = same as for column strip but use No. 4 at 14 in. c.c. C3 and G2

$$\text{For interior } -M_m\text{: } A_s = \frac{39{,}000 \times 12}{20{,}000 \times 0.88 \times 5.5} = 4.84 \text{ in.}^2$$

Use No. 5 bars at 7 in. c.c. so as to keep typical spacing.

$$\text{For } +M_m\text{: } A_s = \frac{49{,}700 \times 12}{20{,}000 \times 0.88 \times 5.5} = 6.16 \text{ in.}^2$$

Use No. 4 bars at 3½ in. c.c.

For the one-half column strip along the beam, use

$$-M_c = 0.125M_o = 0.125 \times 182{,}000 = 22{,}800 \text{ ft-lb} \qquad \text{C4f}$$

and

$$+M_c = 0.05M_o = 0.05 \times 182{,}000 = 9{,}100 \text{ ft-lb}$$

Use No. 4 bars at 7 in. c.c. across the columns and 14 in. c.c. through the middle.

The preceding reinforcement is shown in Fig. 10-7. None of it is bent from areas of negative to positive reinforcement because this is unnecessary and would complicate the job on account of different spacing. Not all the bars have to extend the full lengths shown. For example, in Sketch (*b*), the bottom steel could be arranged with alternate bars full length and intermediate ones a little shorter.

Obviously, the shearing and bond stresses will be satisfactory since those found for the interior panel are so.

However, a rough check of the compressive stresses produced by $-M_c$ at the interior column line and by $+M_c$ should be made. Use Eq. (2-5*a*).

$$f_c = \frac{6M}{bd^2} = \frac{6 \times 130{,}000 \times 12}{72 \times 9.5^2} = 1{,}440 \text{ psi}$$

at the interior column and

$$f_c = \frac{6 \times 63{,}700 \times 12}{90 \times 5.5^2} = 1{,}680 \text{ psi}$$

at the middle of the column strip. These will be accepted. However, the results show that, with simply supported or weakly restrained exterior edges, the first panel might well have a slightly shorter span than the interior ones in order to ease the critical bending moments.

Practice Problems

For these problems assume $f'_c = 3{,}000$ psi, $f_s = 20{,}000$ psi, and other stresses as allowed by the Code.

10-1. A two-way slab is 20 ft square, continuous at all four edges, and 10 in. thick. It is to support a uniform live load of 150 psf. Determine the reinforcement for it.

10-2. A two-way slab is 18 by 20 ft; one 20-ft edge is simply supported; the other three are continuous. Design the slab for a live load of 200 psf.

10-3. A floor of flat-slab construction has the columns spaced 20 ft c.c. both ways. Design the floor for a live load of 200 psf. Assume strong restraining edge beams. Use drop panels.

10-4. A flat-slab building is to have outer panels 18 ft wide and interior ones 20 ft wide. In the other direction, all columns are 20 ft c.c. Assume unrestrained exterior edges. Design a typical exterior and interior panel of the floor for a live load of 300 psf.

11

FORMS

11-1. Introduction. The purpose of this chapter is to point out some of the important principles to bear in mind when planning the forms for cast-in-place concrete construction. It is especially important for the designer to consider them when making his plans so that he may attain the optimum economy and practicability. A thorough knowledge of how his structure is to be built, and provisions in the design for efficiency in the field work, are attributes of the expert in contrast to the novice. Furthermore, it is not conducive to good relations between the contractor and the design engineer when the former has to come to the latter with various explanations as to why certain features of the design should be changed in order to make the field work feasible.

Of course, the designer should not dictate to the contractor as to exactly how the latter is to conduct his operations. If he does so then, in the eyes of the law, the designer, or the owner whom he represents, becomes an employer and thereby assumes the responsibility for the operations. The design should show the results desired, but it should be such that these results can be secured efficiently and economically.

Details of forms will vary widely. The character of the structure, the availability of form materials, the amount of duplication, the equipment and supplies owned by or available to the contractor, and the methods formerly employed by the contractor—all these will affect what is to be done in constructing a particular job.

It is obvious that the forms are to support the plastic concrete until it has hardened and attained sufficient strength to support itself. They are also to give to each member the desired shape, dimensions, and surface finish.

11-2. Strength of forms holding vertical loads. Forms that support the weight of plastic or "green" concrete can be designed to hold up these fairly definitely determinable loads. However, a very important feature is that of stiffness. Lack of this quality may cause unexpected and unfortunate results.

The forms in some cases can be made with enough camber so that,

after the loads are applied and the forms have deflected, the members will have the right shape and position. In other cases, deflection is to be minimized so that no damage will occur to the member itself or to other portions of the structure.

The forms should be designed for all the weights that are likely to affect them. Among these are the following:

1. The dead load of the forms themselves, including their supports.
2. All the plastic concrete that is above them, using a unit weight of 150 pcf to allow for the weight of some reinforcement.
3. The weight of any additional pours placed over the first one if the latter cannot safely support this weight itself.
4. The weight of a 200-lb man standing in any probable position. This may affect light forms for thin floors.
5. The weight of any equipment that may be placed upon the forms; *e.g.*, buggies full of concrete, especially the motorized ones that have been used so efficiently on some modern jobs; a concreting bucket that is likely to be set down temporarily; and piles of materials placed on green concrete that merely transfers the load to the forms. Of course, abuses like some of these should be guarded against.
6. The effect of impact due to vibrators. This is difficult to determine but should be minimized by proper supervision.

The design of the forms to hold vertical loads is a matter of planning more for efficient fabrication, erection, and removal than for strength, at least in many cases. Experience in such work is extremely valuable. It is not practicable to attempt to show in this chapter the vast variety of details used for such work. Some are pictured merely to suggest ideas and arrangements.

Forms have failed. Probably this trouble has been due to attempts to economize a little too much, because forms are often very costly. On the other hand, some inadequate detail that was overlooked can cause the failure of otherwise excellent work.

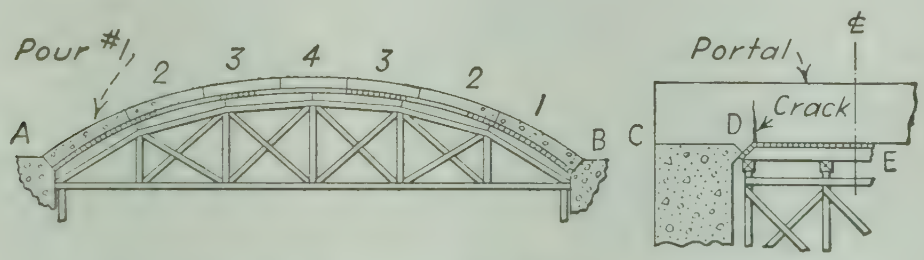
One illustration of such a failure[1] is that of a railroad overpass in Tennessee which failed just after the concrete was poured. The accident killed one man and injured three others.

As an illustration of the need for adequate stiffness, refer to Fig. 11-1(a). This arch is to be poured in seven sections in the sequence indicated in order to minimize the effect of shrinkage. The main support for the forms is steel trussing. It is used in order to maintain traffic below the forms. Pours No. 1 will adjust themselves to any deflection of the trusses and their supports if they are completed fast enough. When the next sections are poured, each successive pour will cause more deflection of the trusses and the end posts. These deformations tend to

[1] *Eng. News-Record,* May 22, 1952, p. 24.

cause shear in the first pours near A and B and rotation about these ends. As the work progresses, this effect at the ends, and distortion of the weak concrete of other pours, increases. The effect may be very harmful. One remedy is that of leaving a foot or two open between the abutments and pours No. 1, and between each successive section. When the trussing has deflected under the weight of these portions, the closing pieces may be poured without serious harm.

Another illustration is pictured in Fig. 11-1(b). These were low heavy piers with large portals. The shafts were poured first up to the construction joints shown. The portal forms were made of wood and were theoretically strong enough but they were highly stressed. However, no consideration was given to the deflection of the posts or to the crushing of wooden wedges on top of the posts. Each portal required several hours for pouring. Shortly after the forms were removed, cracks appeared somewhat as shown. Although shrinkage may have aggravated the cracking, it seems that, when the portion CD was rigidly supported but DE was on yielding supports (with probable increase of deformation with time), there may have been a shearing failure of the weak concrete near D which made a plane of weakness that later opened up.



(a) Truss Centering for an Arch (b) Forms for a Heavy Portal

FIG. 11-1. Examples showing need for consideration of deflection of forms.

It is obvious, therefore, that the portal of Fig. 11-1(b) should be poured quickly so that all the concrete is placed, compacted, and adjusted to any settlement of the forms before the initial set takes place. Another and better scheme is to use very big and stiff members as shores under the forms so that settlement will be negligible. Remember that it is easy to plan rapid work in the office, but it may not be easy or even possible to conduct the field operations so expeditiously. Therefore, a plan that is safe for adverse conditions will probably be advisable, and it certainly will not cause trouble if everything goes smoothly.

On the other hand, if one tried to pour the shafts and portals in one big rapid operation, wet shrinkage of the shafts might cause settlements there, whereas the portal would be "hung up" on the central forms. This might also be harmful, and it certainly would be more expensive to

erect all the reinforcement, to build all the forms, and to place concrete way down in the bottom of such a "mess."

To reiterate, remember that, unless they are oak or other suitable hardwood, timber caps, sills, and wooden wedges may crush because of high pressures perpendicular to the grain. In fact, their crushing may proceed slowly, and it may continue for several days. In the meantime, the concrete has set, the settlement causes shearing deformations in the weak

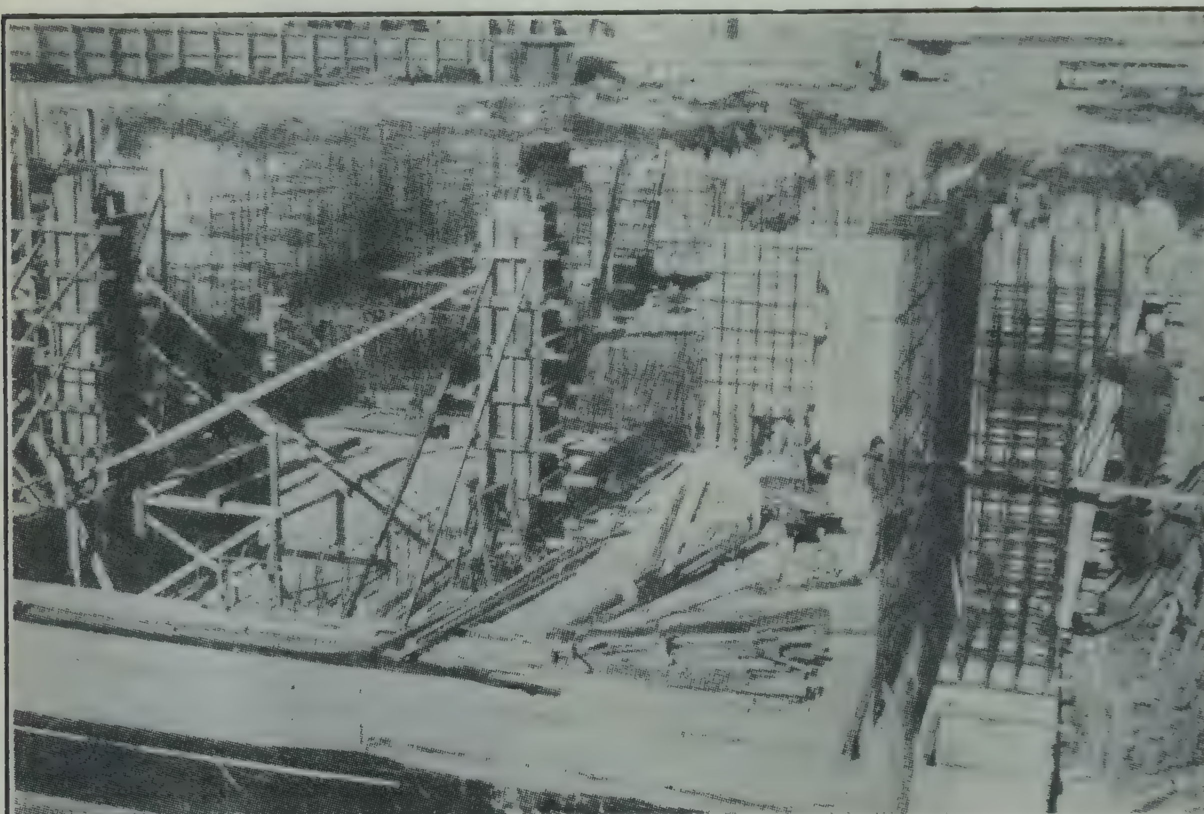


FIG. 11-2. Some heavy construction for an industrial plant. *(Courtesy of the International Smelting and Refining Co., Perth Amboy, N.J.)*

concrete, and these may constitute planes of weakness. Thus the forms may be very economical but they may ruin the structure.

Not only must forms be strong vertically, they must be well braced, as indicated for the columns in Fig. 11-2. A long retaining wall was being built without the projecting portions shown at the base of the stem in Fig. 8-18 to hold the bottoms of the forms in line. The braces at one side of the bottom of one portion slipped about 1 in. The entire bottom bowed outward. This was discovered after the concrete had been placed, but it was impossible to push it back again. After the forms were removed, the bulge looked so bad that the contractor was compelled to replace that portion of the wall.

In another case, a long steel bin and some building columns were to be supported upon a very heavy concrete floor resting upon a series of walls and concrete columns. The formwork was not adequately braced laterally. Somehow one end swayed sideways about 2 in. at the top. This

was not noticed until the forms were removed. The cost of replacement would be so great that the owner finally accepted the defective structure. However, the contractor had to pay for revising the steelwork to fit the displaced anchor bolts.

11-3. Lateral pressure on forms. The intensity of the lateral pressure produced by newly poured concrete is uncertain. It is generally assumed to be hydrostatic in character, but to a limited extent. The unit pressure seems to depend upon the rate of pouring, the temperature of the concrete, the nature of the coarse aggregate, the richness of the concrete in cement, the relative amount of fine aggregate, the slump, and the use or absence of vibrators.

The setting of the cement soon causes plastic concrete to stiffen so that it becomes a weak solid. It no longer acts as a fluid when more concrete is deposited on it. Probably the temperature of the concrete is influential only when it is high enough to cause more rapid setting of the cement or so low that it retards this action. Furthermore, it seems that the coarse aggregate, especially if it is angular, tends to form a somewhat rigid mass near the bottom of a pour so that it supports itself to a considerable extent. This probably applies in the case of sections under a foot or two in width more than it does in massive construction.

It also seems that a lot of cement with enough water to produce a slump of 2 to 4 in., and perhaps a large amount of sand, may tend to increase the fluidity of the newly poured concrete, thereby causing more pressure on the forms. On the other hand, too wet a mix will cause the grout or mortar to cease to lubricate the mixture and to permit the coarse aggregate to settle out, with a resultant decrease in the lateral pressure.

The effect of vibrators in augmenting lateral pressures is likely to be important only in plastic concrete that has been poured for less than an hour or two. They probably will not have power enough to move concrete in which the formation of the cement gel has stiffened the mass.

An illustration of the utilization of this stiffening effect is the practice of pouring the bottom of a stepped footing, letting it stand for 30 to 40 min, then pouring the pedestal with side forms only, and doing this without having the second pour squeeze out the concrete under the edge of the forms.

The pressure caused on forms for tremie concrete, or concrete otherwise deposited under water, will probably be similar to that of concrete deposited in air, but the intensity may be about two-thirds of that for the latter.[1]

[1] Halloran and Talbot, The Properties and Behavior Underwater of Plastic Concrete, *J. ACI*, June, 1943; also Angas, Shanley, and Erickson, Concrete Problems in the Construction of Graving Docks by the Tremie Method, *J. ACI*, February, 1944.

Of course, any predictions of pressures indicated here are made upon the basis of proper deposition of the concrete. If the concrete is dropped as a large gob falling through a height of several feet, the lateral pressure may be increased considerably.

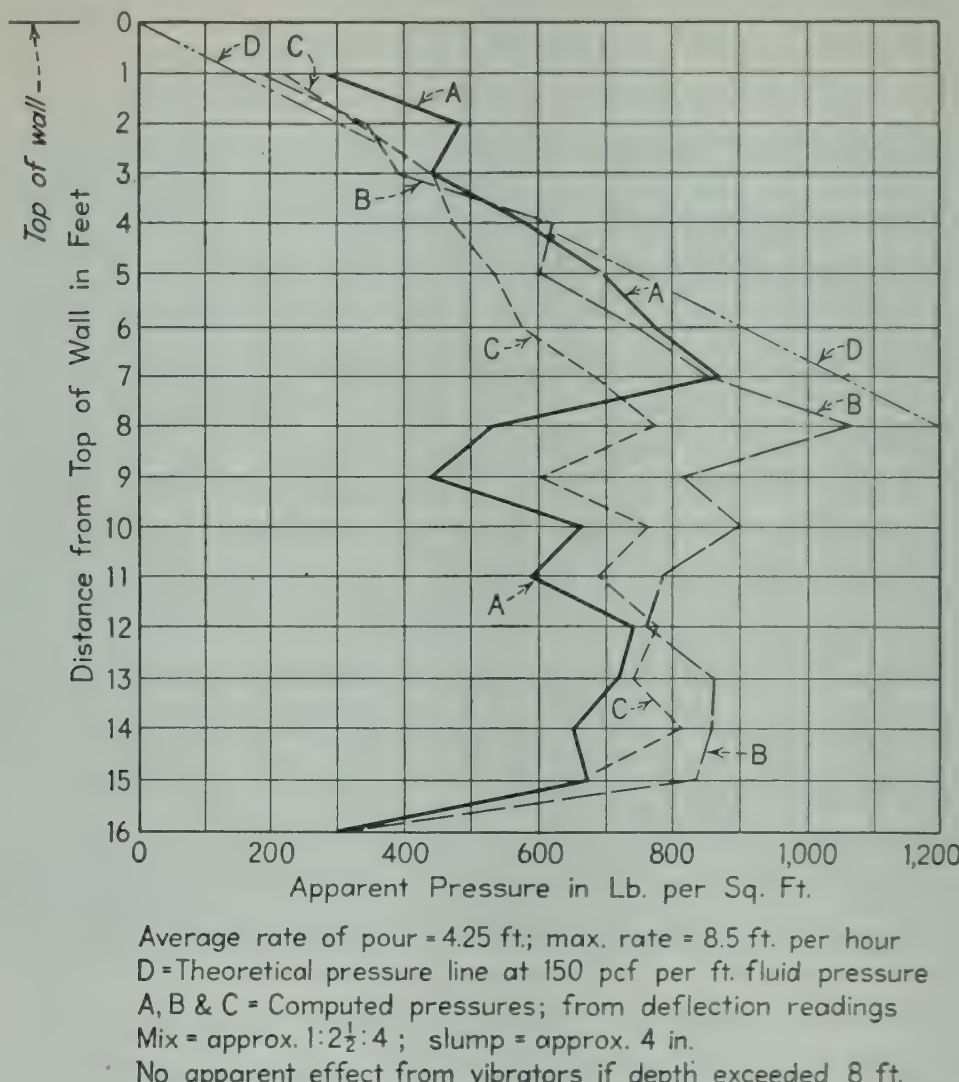


FIG. 11-3. Pressures on wooden forms with plywood lining. ***(Courtesy of Charles Macklin, Architect and Structural Engineer, Springfield, Ill.)***

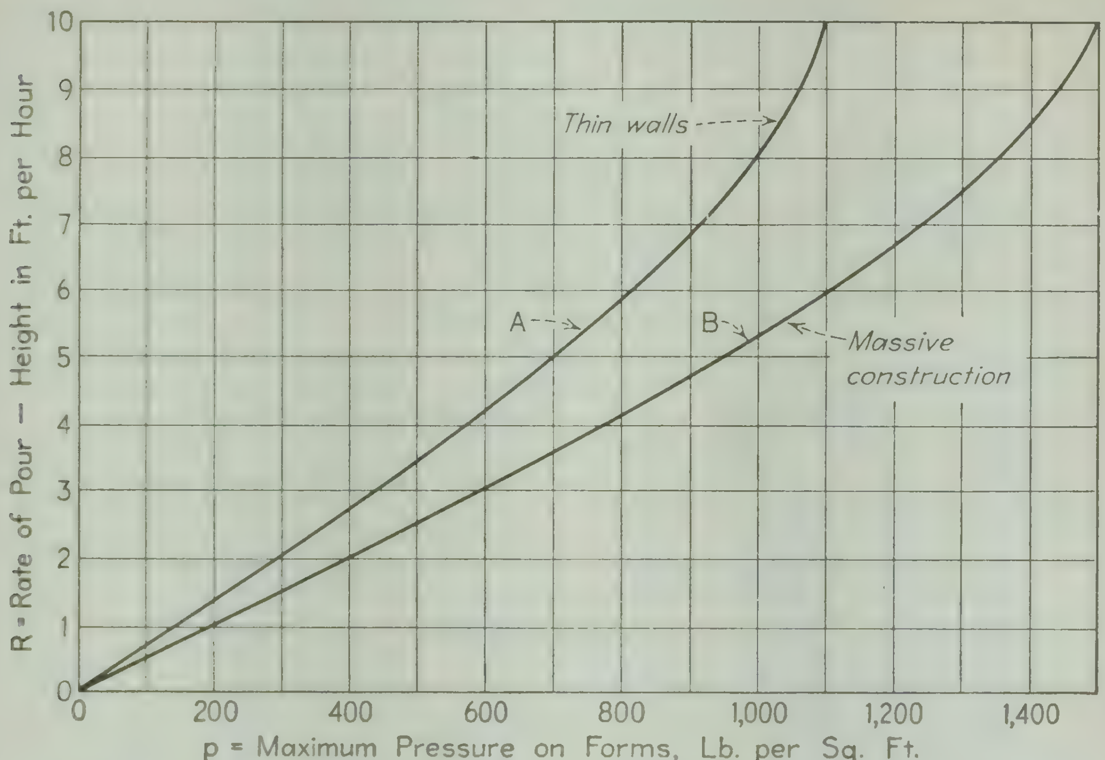
The height of pours or lifts and the time required to elapse between successive pours are questions about which there is considerable debate. The following considerations may well be borne in mind in this connection:

1. Pressure on forms will not increase as the hydrostatic head if the lower portion has been deposited and undisturbed for 2 or 3 hr. See Fig. 11-3 as an illustration of this. The lower portion of the forms need not be unduly strong if the rate of pouring is not too rapid.

2. Bleeding may be troublesome if deep pours are made too quickly and vibrated excessively. Slower progress of the pour will permit the setting of the concrete so as to avoid serious bleeding.

3. If one lift is allowed to set for several days, considerable heating and expansion will occur initially; cooling and shrinking will then follow. When another pour is

placed on top of the first one, the former is in its warmer and expanded condition as it sets. Then, when it tries to shrink, the lower pour tends to prevent this shrinkage and to cause cracks in the upper lift. It would seem, therefore, that a massive tall structure should be poured full height by increments that are added at intervals of a few hours in order to have each lift placed before the one under it has set too long and shrunk too much.



Ordinary columns are poured rapidly. Use p = 150 lb. per sq. ft. per ft. of height
A = Walls 12 in. or less in thickness
B = Walls 24 in. or more in thickness
Assumptions: ordinary mixes, slump approx. 2 to 4 in., moderate vibration, 60° to 70° F, friction decreases pressure in case of thin walls and columns, and vibration negligible below 6 or 7 ft.

FIG. 11-4. Curves for estimating pressure of concrete on forms (for preliminary use when lacking better data).

4. The cleaning of horizontal joints—removal of laitance—is slow and costly. Continuous but slow pouring of deep sections will often minimize or eliminate such cleaning.

5. Vibration or other methods of compaction should be conducted so as to avoid harm to set or partially set concrete. Good judgment used in this connection will generally permit slow but rather continuous pouring.

Figures 11-3, 11-4, and 11-5 are given to show the results of some tests that have been made to ascertain the pressures on particular forms and to make some recommendations. These do not agree with the data given in Art. 1-16 but they are nevertheless very instructive. It seems that experimental data vary tremendously and that this whole question of lateral pressures on forms is not subject to exact theoretical determination. Conditions for any particular job will differ from those of another

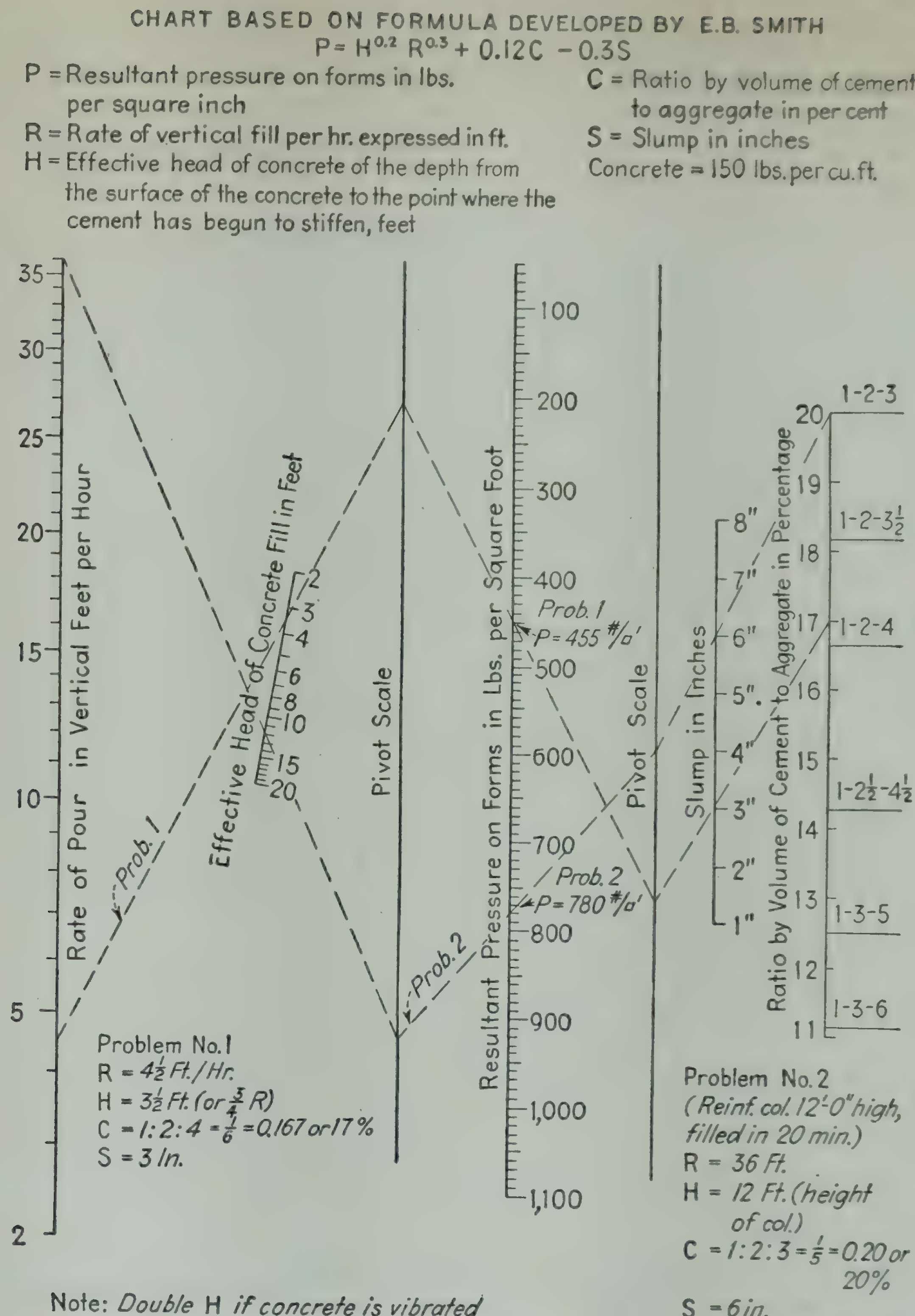


FIG. 11-5. Pressure on forms. *(A chart devised by C. E. Thunman, Springfield, Ill.)*

job. The design of forms is therefore primarily an art that is developed through experience, but it still should be developed as good engineering.

Form ties are customarily used to resist the lateral pressure of the concrete when there are forms on opposite sides of the section, as for a wall. If a form is to be used on one side only, as when concreting against a side of rock, earth, or previously set concrete, a strong set of lateral bracing will be needed to resist the thrust on the forms.

Some varieties of form ties are shown in Fig. 11-6. It is generally desirable to avoid having any steel at or very close to the surface of the concrete. That is why the ties are made so that the outer ends can be removed or broken off and the holes pointed up after removal of the forms. If the embedment of the weak section or the connection point is too great, it may be difficult to remove the outer portions and to fill the deep void completely with mortar when the end is taken out.

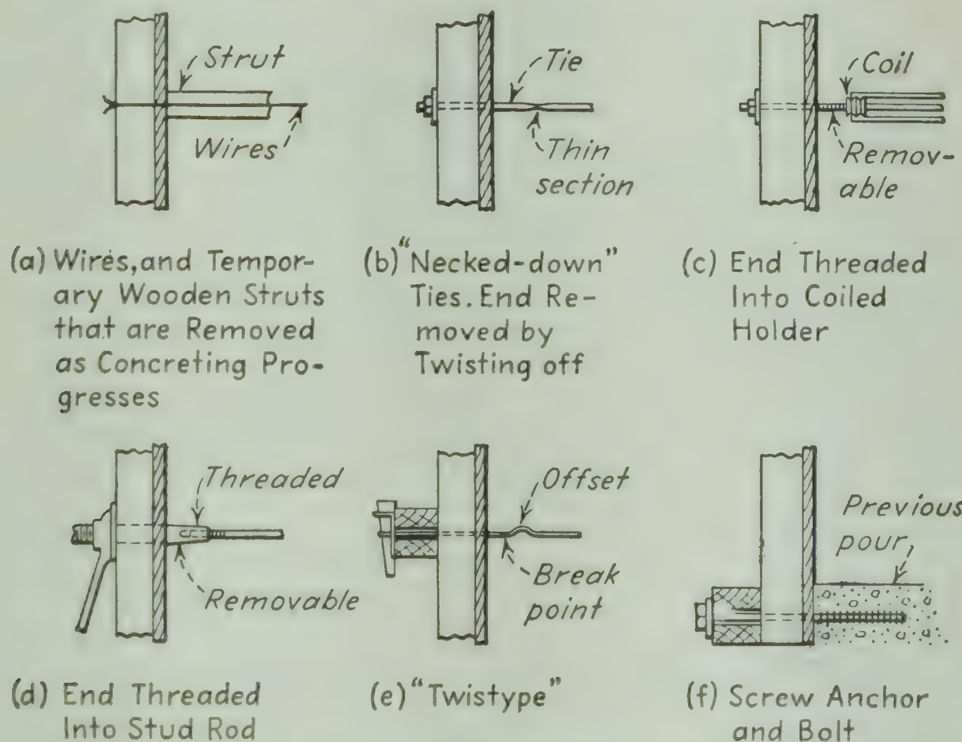


FIG. 11-6. Some principles of form ties. (Wires are not to be used when work will be exposed. Point up holes with mortar after ends are removed.)

Another feature to avoid is "hydrostatic" uplift on the forms. This results when the forms are sloped considerably. When such shapes are necessary, it is often desirable to embed bolts in the previous pours to which the forms can be attached, to brace the forms against something heavy and strong, or to make the pour slowly and in small lifts so that the initial set or some stiffening can occur in a lower portion before more concrete is placed, thus minimizing the uplift.

Where possible, it is advantageous to have ledges to support the forms, as in the case of footings under walls. It is difficult to build wall forms on irregular ground, also to hold them high up in the air.

A special case that deserves some consideration is that of the pouring of concrete walls directly against the steel sheet piling of a cofferdam that is subjected to unbalanced water pressure from the outside. It is probable that there will be some leakage at the interlocks. Therefore, when concrete is poured, the water may force its way into the concrete and thereby cause weakness and bleeding.

To illustrate this, refer to Fig. 11-7. Sketches (*a*) and (*b*) show a heavy concrete wall that is to be poured between piling *A* and an inner form *B*. Assume that there is a leak at *D* and that the water pressure outside the cofferdam exceeds the hydrostatic pressure of the plastic concrete. Water may then force its way into the neighboring concrete, causing a porous pocket; it may force a channel through the concrete to

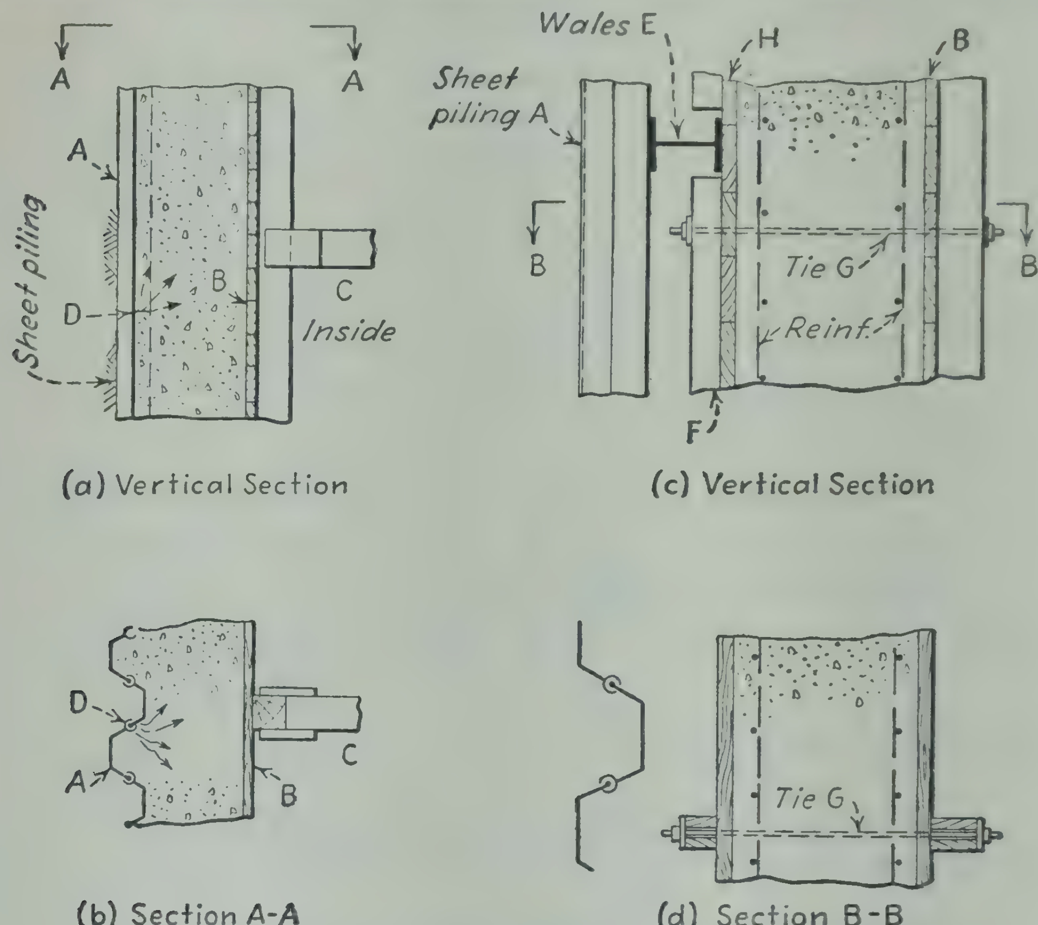


FIG. 11-7. Concrete walls inside cofferdams.

some outlet through form *B*, thus washing out the cement and fine aggregate through this channel; or it may work its way up alongside the piling, thereby washing out the cement along some upward channel.

Furthermore, Figs. 11-7(*a*) and (*b*) indicate that the form *B* must be braced in some manner to resist the hydrostatic pressure of the plastic concrete. This bracing may be costly and troublesome. As an alternate, form ties might be welded to the sheet piling. Any wales and bracing to support the cofferdam itself are likely to interfere with the reinforcement and with the placing of the concrete.

Another arrangement is pictured in Figs. 11-7(*c*) and (*d*). Here the inner form *B* is set farther from the sheet piling *A* so that an outer form *H* can be built past wales *E*. It may also be desirable to extend form braces *F* past the wales. The form ties *G* then resist the pressure of the plastic concrete and permit the use of a minimum of bracing to steady

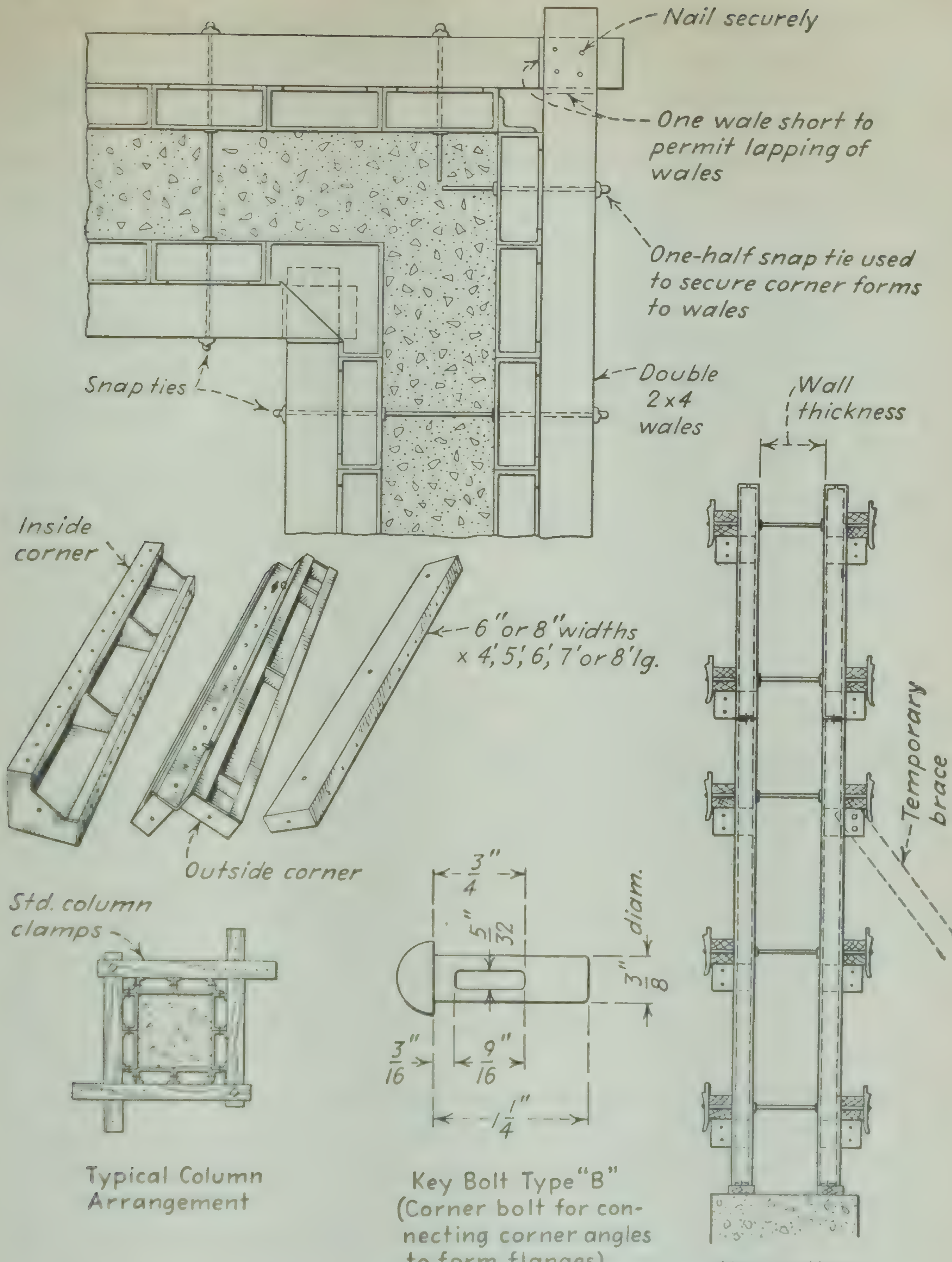
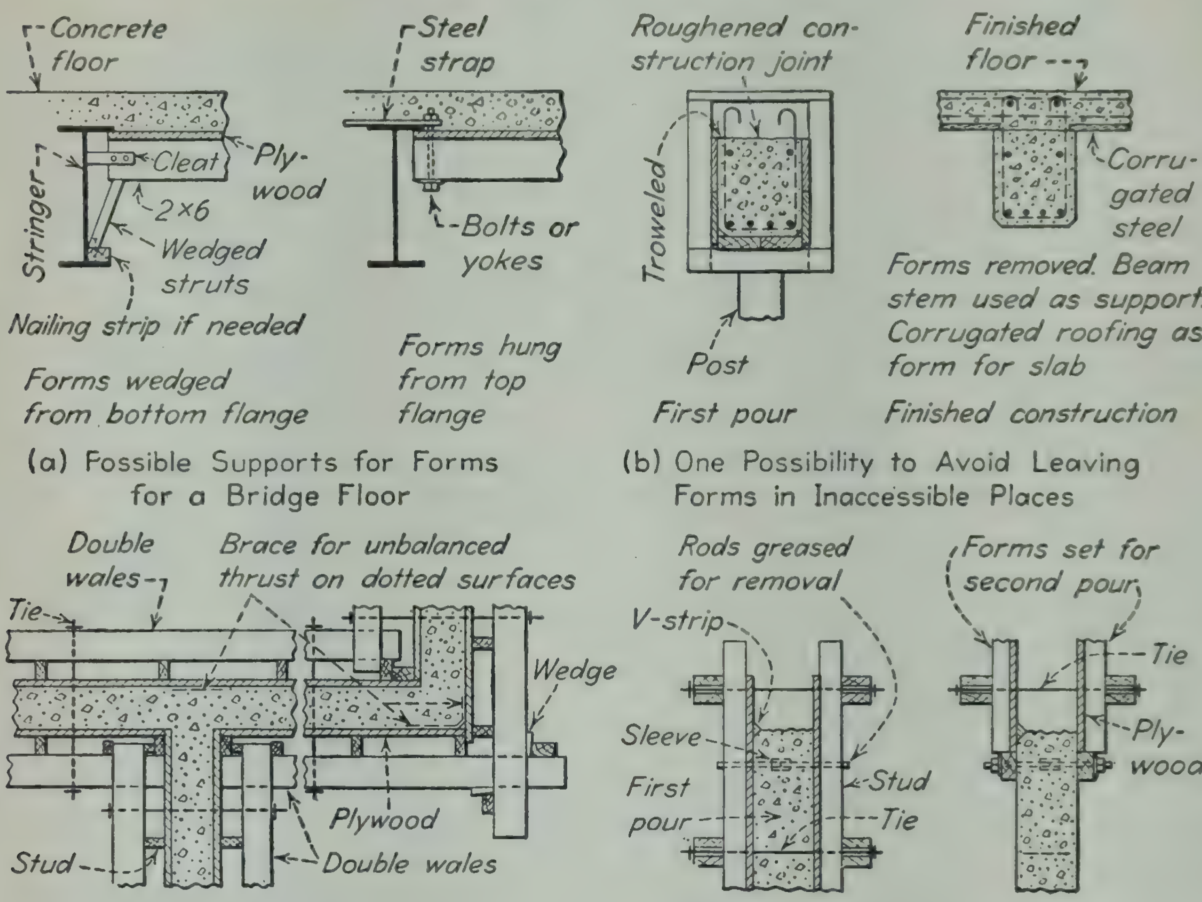


FIG. 11-8. Standard construction details, Atlas Speed Forms. (Courtesy of Irvington Form and Tank Corporation, New York, N.Y.)

the forms. Any leakage through the piling can then be collected at the bottom of the space between the piling and the back form, it can be pumped out, or provisions can be made to drain it away temporarily. The forms H are generally left in place.

If the cofferdam is in impervious material, the leakage may be small so that the method of Fig. 11-7(a) will be satisfactory. The wall may



(a) Possible Supports for Forms for a Bridge Floor

(b) One Possibility to Avoid Leaving Forms in Inaccessible Places

(c) Plan View of Some Forms for Walls

(d) Vertical Sections of Forms for a Wall

FIG. 11-9. Some details of formwork.

then be made thick enough to extend the outer reinforcement past the wales if these are to be embedded permanently. If the soil is porous and the external water pressures are high, the scheme of Fig. 11-7(c) may be preferable.

In some cases, if the sheet piling leaks it may be possible to close the leaky places by calking, to lower the water table by means of well points, or even to box out locally and conduct the leakage to a temporary sump and pumps. If the sheet piling is to be removed later, bond of the cement to it should be prevented by painting the piling with oil or some suitable coating, or by covering it with plywood, tar paper, old boards, or even cardboard. In one case about a 10-ft depth of concrete was poured against 40-ft sheet piling that was supposed to be pulled. How-

ever, the difficulty and cost of breaking it away from the concrete were so great that most of the piling was abandoned.

11-4. Details and miscellaneous data. Forms are generally made of lumber and plywood or of steel. The first two are very common, although, if there is to be enough duplication to permit extensive reuse, steel forms may be really economical. Figure 11-8 shows steel forms that are made in standard units that can be assembled as needed to produce a vast variety of sizes and shapes. It is sometimes advisable to plan a concrete structure with dimensions chosen to fit these forms.

When beams are planned, for example, it is advisable to consider how the forms are to be made, especially when they are to built of wood.[1] It is important for economy that the carpentry work be minimized by the use of standard lumber without a lot of cutting. As an illustration, the width of a beam might be made 9½ instead of 10 in. so as to use a standard 1 by 10 to form the bottom, or the width might be 15 instead of 16 in. so as to use two 1 by 8 pieces. Correspondingly, the depth of haunch or stem might well be made in multiples of standard sizes. Some illustrative sketches are pictured in Figs. 11-9 and 11-10.

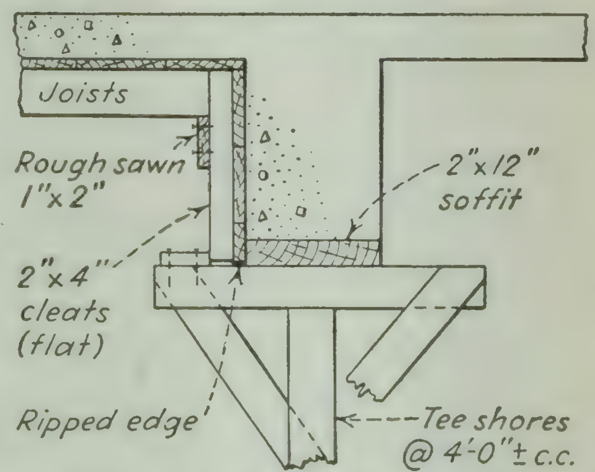


FIG. 11-10. Partial section of beam-and-slab form.

Furthermore, it is economical to plan a job so as to use a series of various sizes that differ by 2 or 4 in., not by fractions or odd dimensions. This simplifies the formwork and permits a maximum of reuse. The reinforcement can be varied easily as the loads require

It is sometimes worth while to make elaborate forms that are mounted on tracks or rollers so that they can be moved quickly from point to point. Forms for side walls and roofs of long tunnels are examples of this.

For tall structures, such as bridge piers, "slip forms" are used so that they can be raised slowly by jacks or screws as the concrete is poured. If the shafts are tapered, this causes complications in adjustments. Stepped piers may be used instead of those with battered sides.[2] Of course, various details will have to be invented to suit each particular job.

The special forms developed by the Vacuum Concrete Corp. have great

[1] Formwork Simplified for Apartment House, *Eng. News-Record*, Aug. 16, 1951.

[2] Ralph Holt, Slip Forms Reduce Cost of Tall Bridge Piers, *Civil Eng.*, December, 1950.

possibilities for substantial economies,[1] especially if a structure is designed in advance for their use. The drying and stiffening effect produced by the vacuum enables side forms to be removed in from a few hours to a day instead of a few days after pouring of the concrete.

Removal of forms may be troublesome unless this operation is planned in their design. Oiling the forms or coating them with some other suitable material will help to avoid bond between the forms and the concrete. Adequate cleaning of a used form is essential prior to its reuse.

Heavy centering is usually supported upon wedges, shims, or jacks so that it can be "struck" or lowered away from the underside of the structure. Sand jacks are boxes filled with sand that will support the posts when desired but will lower them when the boxes are broken or the sand removed.

When one pour is made on top of a lower one from which the forms have been removed, special care is needed to prevent leakage of mortar where the upper forms rest upon or lap over the completed concrete. Pinching of the forms against the concrete, calking of joints, or mortaring the junction may remedy the difficulties.

After forms are set, or before the last side is closed in, all debris, dirt, and laitance should be removed. Reinforcement often has to be erected before the forms are completed unless the member is so large that a man can work inside the forms. This is shown in Fig. 11-2.

In pouring columns or other members with considerable height, panels of one side of the form may be left off until the top of the concrete approaches their location. Then they can be attached and the pouring continued.

In ordinary construction it is customary to build the concrete work of the main structure prior to completion of many of the minor parts, erection of equipment, installation of piping and wiring, finishing of floors, etc. Steelwork, anchor bolts, and various materials to be embedded often cause delays because they are not on hand or the details for them cannot be determined soon enough. As an example, curb angles that are to be held by welded anchors buried in the concrete of floors might well be redesigned to provide attachment by means of anchor bolts that can be set beforehand. Similarly, holes can be left for piping as shown in Fig. 11-11. Plans that are made carefully to avoid delays from such troubles may result in substantial economies.

Obviously the position and details of forms should be carefully checked before any concrete is poured in them. In one case, some 20 ft of the bottom of a bridge pier about 100 ft high had been poured and set before it was noticed that the forms for the pier had been placed off center by 1 ft. The steel superstructure was fabricated and shipped to the job.

[1] K. P. Billner, Applications of Vacuum Concrete, *J. ACI*, March, 1952.

What could be done about it? Should the bearings be set close to one edge, should the batters of the sides be changed so as to bring the top in proper position (causing an appearance of leaning), or should the completed work be demolished and the base rebuilt? Such questions cause many a headache.

Forms may be lined with various materials to produce a desired texture for the finished surface of the concrete. Cloth, masonite, steel

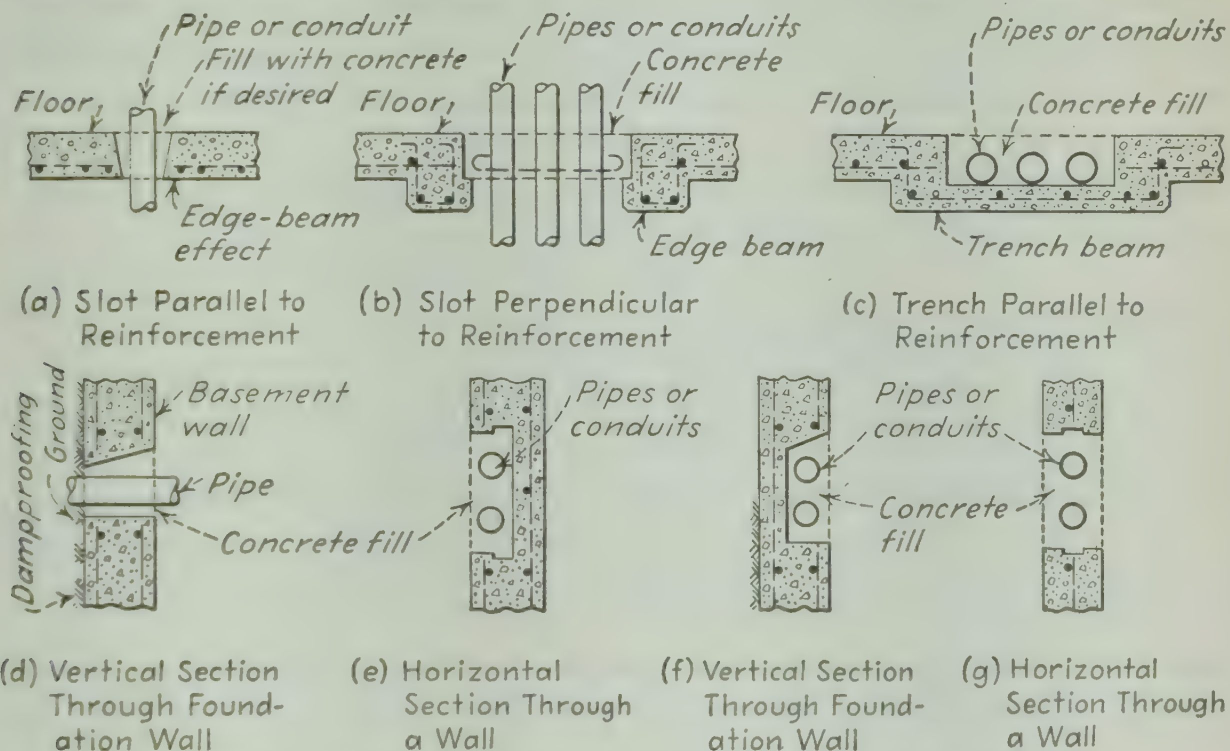


FIG. 11-11. Studies of provisions for installation of utilities.

sheets, plywood, and even clapboards may be used to get a surface varying from glazed for steel forms to make-believe clapboards. The forms for the Harlem River Speedway Connection of the New York Approach to the George Washington Bridge in New York were made with matched boards that had one half of the sides of the groove removed. This was done in order to make ridges on the surface that would collect dust and make the structure appear rough and aged. Even plastic-faced plywood is being offered as a material for forms.

There seems to be some question regarding the best material to use for forms in order to obtain the optimum durability of the surface of the finished concrete. Naturally, steel forms do not absorb the water in the concrete, whereas wooden forms do so to a certain extent, even when they are oiled somewhat for easy removal. A moderate amount of such absorption automatically produces a lower water-cement ratio near the surface, and this may be beneficial if it cannot proceed so far that it interferes with proper curing. One job reportedly constructed in 1905 had the forms lined with duck for the purpose of absorbing surface water.

This structure is said to be in such good condition that the marks of the threads of the duck are still clearly visible.

When adequate fire protection will be provided, steel beams under concrete slab or ribbed floors may be housed in lath and plaster instead of being encased in concrete. This is especially advantageous when hung ceilings are used under ribbed floors that rest on top of the beams. The savings in weight, concrete, and forms may be considerable.

Steel pans placed on planks are used for one type of floor that is good and that minimizes the formwork.[1] It produces a one-way or two-way system of small T beams that provides a light but stiff floor.

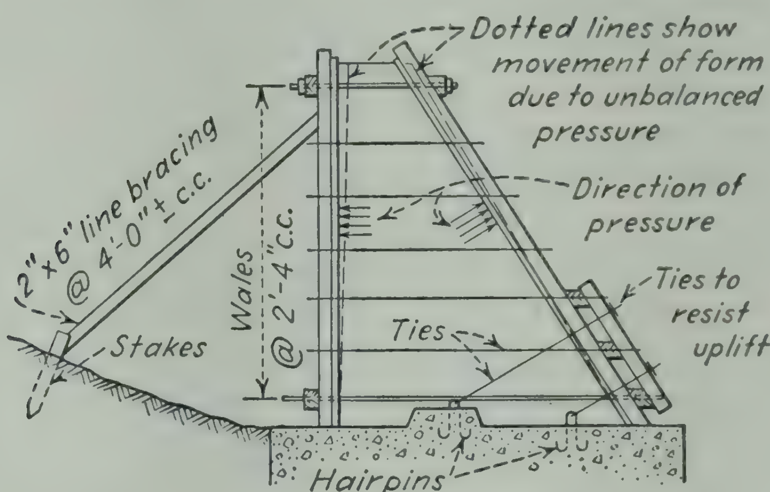


FIG. 11-12. Illustration of movement of forms for a retaining wall because of unbalanced pressure. *(Courtesy of C. E. Thunman, Springfield, Ill.)*

The Dox floor construction illustrated in Chap. 12 has been developed to eliminate forms. The precast members automatically serve this purpose. They are useful for moderate spans and light loads when covered with an adequate mesh-reinforced topping.

Recesses in the faces of concrete construction are easy to make. Pieces of the right shape can be attached easily to the insides of the forms. They should be beveled or rounded at the edges so as to facilitate removal of the forms without spalling the corners of the offsets in the finished concrete. The details should be made so that air pockets and honeycombing of the concrete are guarded against. Remember that it is difficult to make concrete completely fill the spaces at undercut horizontal offsets. The edges are likely to be irregular and rounded off.

Projecting panels, brackets, corbels, and similar details are the cause of trouble and expense in building forms. If they are necessary, the details should be designed so as to simplify the formwork as much as possible. Sections tapered on four sides, conical shapes, and warped

[1] R. L. Reid, Steel-pan Forms Provide Economical Long-span Roof, *Civil Eng.*, September, 1951.

surfaces require costly forms, but slopes and circular or other simple curves in one direction are not too difficult to build.

It is desirable in many cases to have the forms prefabricated at some convenient shop where all the necessary equipment is available. They are made in panels that can be assembled on the job.[1] As this is written, a subcontract for all the forms for an industrial plant in the Southwest has been let to a shop approximately 40 miles from the site. The forms for about 50,000 yd^3 of concrete will be shipped from the shop to the job.

In some cases, the forms, or a large portion of them, can be assembled as big units that are placed by cranes. This is especially helpful in the case of underwater construction.

When concrete floors are to be placed upon steel framing, the steelwork itself can often be used to support the forms, thus avoiding costly shoring. Precast members may be used similarly.

The forms for thin concrete members may cost more than the materials for the concrete itself. In such cases, and in most concrete structures, excellent planning in the design can yield big savings in cost. To a large extent, designers might well plan their structures directly in terms of the economies in formwork as well as for architectural and engineering requirements.

[1] Prefab Forms Erected and Stripped Fast, *Eng. News-Record,* Feb. 2, 1950.

12

PRECAST CONCRETE

12-1. Introduction. Precast concrete is not a special kind of concrete but a method of fabrication. The term means any reinforced-concrete member, or even one of plain concrete, that is cast in forms somewhere other than its final position, then erected in place. Usually the casting is done on the ground near the site or in a shop that makes a specialty of such work.

The use of precast members is increasing considerably. As engineers become more familiar with the possibilities of such construction and with the great economies that can be made by proper use of it, they will undoubtedly resort to it more and more. However, it is seldom practicable to substitute precast members piece by piece for those which have been designed for pouring in place. Basically, a proper use of precasting involves an adequate knowledge of what can be done in this line, then the planning of a structure to use these products.

Prestressed concrete, which will be explained in the next chapter, is especially well adapted to combination with precasting. The two are so closely interrelated—or probably soon will be—that it is a question as to which one should be explained first. This chapter will deal with precasting in general, and it will leave most of the adaptation of precasting with prestressing to the next chapter.

12-2. Economics. Precasting may be a means of making considerable savings when compared to cast-in-place construction. This is primarily because of economies in the cost of forms. Working on the ground with all the necessary equipment handy, and with multiple use of forms, is far easier than building heavy forms, pouring concrete, finishing surfaces, sprinkling during the curing period, and removing forms when all these operations must be done above ground. This is obvious.

On the other hand, there are costs that may tend to offset these savings. Among these are the following:

1. Large area and accommodations required for production in volume.
2. Large storage space and facilities for curing.

3. Handling in the casting yard from casting position to the curing room or yard, and perhaps moving again to storage.

4. Large equipment for the transportation of long heavy members that must be handled carefully.

5. Large equipment for raising such heavy members into position.

6. Special details, particularly at junctions of members.

The designer should have intimate knowledge of what can and cannot be done practicably in precasting, or he should obtain expert and reliable advice regarding such matters. Then he should design his structure for the use of precast members in the first place. In fact, he should work out all important details and prepare the contract and specifications on that basis. He should let the contractor decide how and where he will make the members, and what means he will use to erect them, but the plans should be very specific and complete in showing what the final result is to be. The contractor should not have to try to adapt the design to precasting.

The use of the vacuum can be very helpful in shortening the time that forms must be left in place. It, with the special equipment that has or may be developed, can also cut down the cost and difficulty of handling precast members, especially when they are thin panels or long slender pieces. Steam curing can also be used to shorten the time between casting and erection or shipping of members.

Precasting in large volume is an assembly-line job. Space, equipment, and operations should be planned in great detail just as they would be when laying out the production line for a factory. This is properly the function of the contractor or of the manufacturer of concrete products.

12-3. Design features. In general, precast members are designed as structurally determinate ones; *i.e.*, they are not continuous beams or frames. However, in some cases, they may be incorporated in structures that appear to be indeterminate; and continuity may even be secured to a certain extent and in certain cases. The variety of construction is so great that the best one can do is to illustrate different instances where precasting has been or can be used. From these, the reader can get an idea of the great possibilities for it.

The design of a precast member itself to withstand bending will ordinarily be similar to that of a corresponding part that is cast in place. Provisions to support the end reaction may be considerably different, and special reinforcement may be required because of stresses that will exist during the handling and erection of the piece. This is obvious in the case of a long precast pile in which the critical bending moments will probably occur when the pile is lifted from a horizontal to a vertical position.

As stated previously, the design of a precast-concrete structure requires

different thinking and planning than does a conventional concrete one. It is almost as though one were using a different material. In some ways, it approaches the character of the planning involved in heavy timber construction. It is the planning of a structure to use standard premade articles, plus the planning of those prefabricated articles themselves in many cases.

The end details, provision for supporting the end reaction, shearing stresses in the ends, provisions for proper fit, provisions for seating or insertion of members, means for transferring end or sideward shears and moments when they are desirable, and provisions for tying the entire

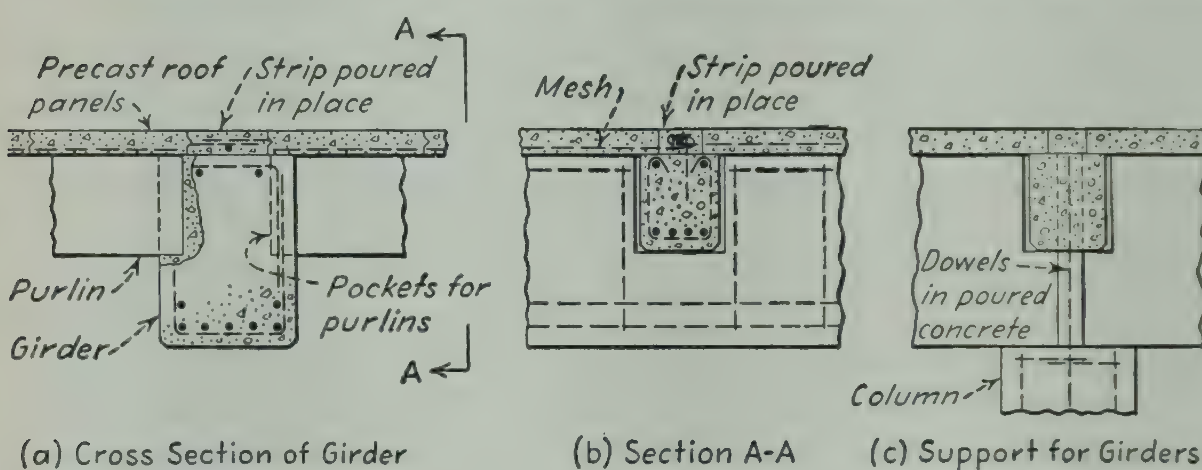


FIG. 12-1. One type of precast-concrete construction for a factory roof.

structure together—these are some of the special features that call for ingenuity and engineering skill. They are special problems of precast-concrete construction.

Provisions for conduits, piping, fixtures of various sorts, windows and doorways may be troublesome when walls, roofs, and floors are precast. All must be planned in advance, and all details incorporated in them.

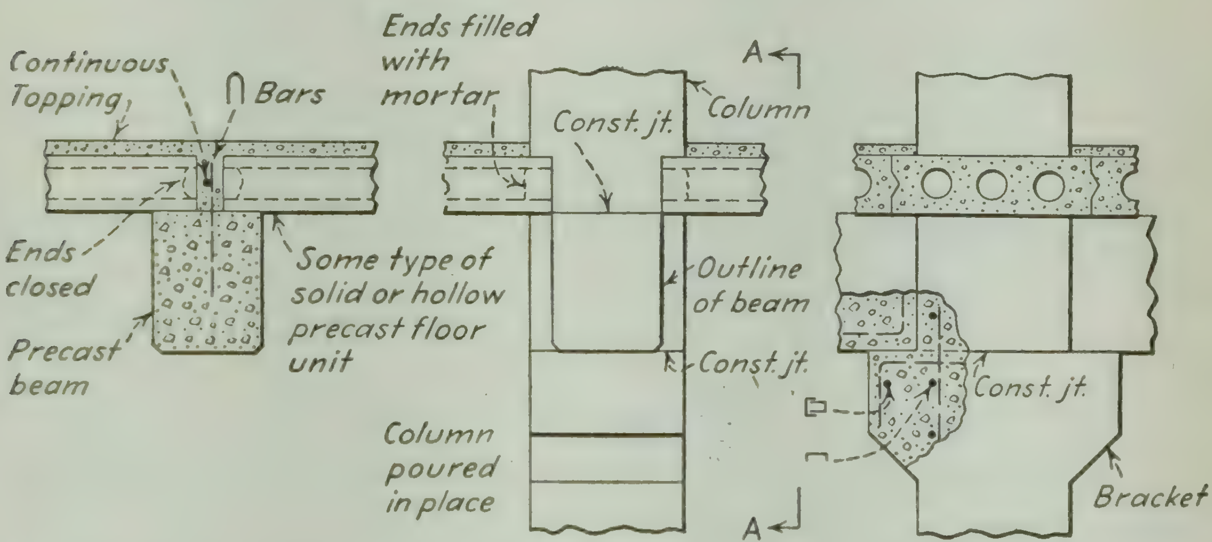
Bearings of simply supported members may be narrow. Therefore, special care should be used to be sure that bond, end anchorage of reinforcement, and bearing pressures are adequately provided for. One case of this is shown in Fig. 12-1(a) where precast rectangular purlins are supported upon shelves on the sides of a main precast girder. The bearing of the girder on a column is pictured in Sketch (c). Both drawings show the utilization of embedded rods on top of the purlins and the girder to knit the structure together both ways. This may not be necessary, but some kind of positive tie is desirable. Bolts might be used, as shown in Fig. 12-9.

Precast members are to be designed in such a way as to facilitate the formwork and its reuse. Members may be poured in multiples or individually. In any case, recesses, holes, ribs, and other details should be made with bevels in order to make it easy to remove the members from

the forms, or to remove the forms from them. The members should be shaped so as to facilitate the placing of the concrete and complete filling of the forms. They may be cast in normal position (right side up), on their side, or even bottom side up.

For precast members it is often advantageous to make up the reinforcement as prefabricated cages. The bars may be wired together or even tack-welded at their junctions.

In order to expedite the work it is often desirable to use quick-setting cement. The concrete should be designed for at least $f'_c = 3{,}000$ to



(a) Section Through Beam and Floor (b) View Along Face of Column (c) Section A-A

FIG. 12-2. Building construction using precast members except for columns and topping.

4,000 psi. It should have a small slump, and it should be compacted well by vibration or other means. As indicated in Fig. 12-1, the compressive stress in the concrete is likely to be higher than in ordinary members because of the narrow width of the top. Diagonal tension and shears due to handling generally make it desirable to use an effective system of web reinforcement when the members are long and heavy.

It is also desirable to use small-sized aggregate in order to have the rods closely spaced because a relatively high percentage of steel and close spacing of bars are customary.

12-4. Small bridges. Precast construction can be very useful for the superstructures of small highway bridges. The parts can be handled from the casting plant to the site by trucks, then lifted into place by one or two truck cranes, or erected by any other hoisting equipment that is suitable. Since the members are already cured, interference with traffic is often minimized. Where old deck spans are to be replaced, it is also practicable in many cases to maintain traffic on one longitudinal half of

the old structure while the new half is being erected, then to transfer traffic to the latter during the completion of the job.

Figure 12-3 pictures one type of construction that might be used for spans of 15 to 25 ft, or possibly longer, if the depth, width of ribs, reinforcement, and other features are made adequate. The longitudinal channel-shaped members are made so that their bottoms are close together at the joints *A* and *B* but their tops are about 1 to 1½ in. apart. The keyway can thus be grouted from the top, and it will lock the members together vertically. The sheet asphalt is used as a finished pavement and to protect the structure from water to a considerable extent.

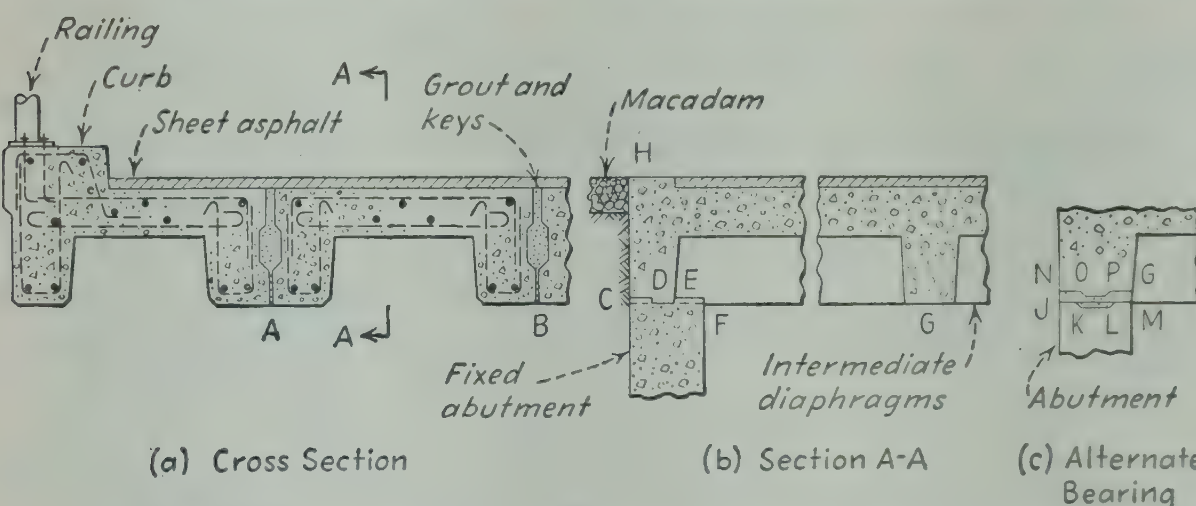


FIG. 12-3. Some details for a short precast-concrete highway bridge.

It is possible to use these sections of Fig. 12-3(*a*) without the poured-in-place keyways if the spans are short and if leakage through the joints is not objectionable. However, even ordinary keys will help to distribute wheel loads to adjacent members.

The bearings of precast bridges are likely to be troublesome details. In Fig. 12-3(*b*) there is a keyway *DE* and projections *CD* and *EF* between the ribs. The ribs at *CF* extend straight across. These two give a two-way keying effect that locks the members in position longitudinally and transversely. However, the formwork and accuracy of finishing of the abutment are difficult. At the expansion bearing, keyway *DE* is omitted and the top of the abutment is coated with asphalt, oil, or some other compound to prevent bond.

The diaphragms *G* are used to distribute loads between the two ribs of each member. The bulkhead strip *H* is used to terminate the sheet asphalt.

Another detail of a bearing is pictured in Fig. 12-3(*c*). The abutment is finished level except for intermittent depressed keyways *KL* under the end diaphragms. The diaphragms are shallower than the ribs, and they have a projecting key *OP*. After erection the space *NOPGMLKJ* is dry-

packed with a stiff 1:2 mortar which serves to lock the deck and the abutment together horizontally.

Figure 12-4 shows another arrangement that was used to replace a series of old timber bridges in Florida. Here the precast members are separated and the transverse bars extended into the open spaces where they are connected by means of cable clamps as well as by the bond produced when the open strips are concreted. Dowels projecting from the

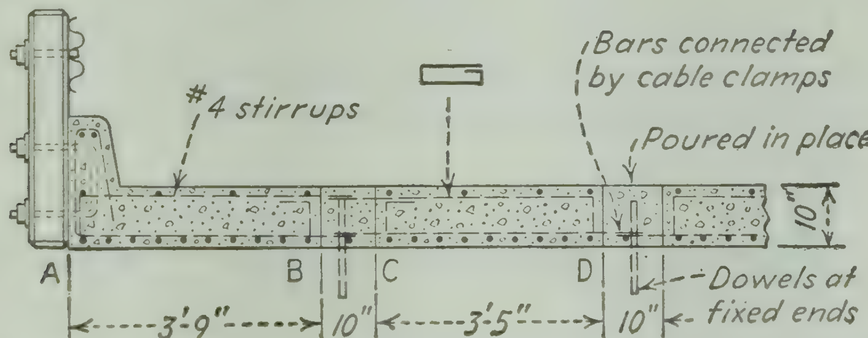


FIG. 12-4. Precast units 15 ft long used for some bridges built by Florida State Road Department.

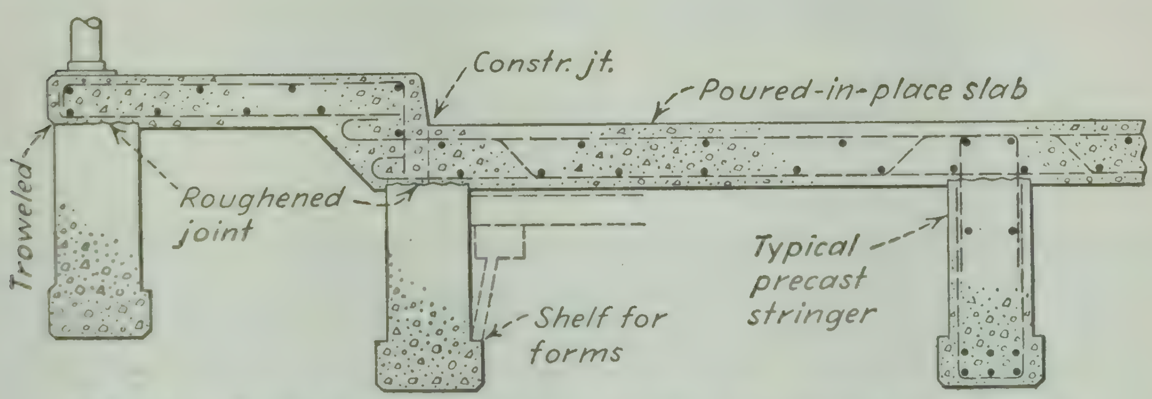


FIG. 12-5. Girder bridge with precast stringers and poured-in-place deck.

abutments and piers into these open strips are concreted in. They then serve to hold the superstructure in place. The transfer of vertical shear between adjacent members depends upon the bond between them and the 10-in. strips, but keys might be used. Generally, it is necessary to provide some means of attaching hoisting devices to these precast members, especially when they are to be placed close together. One method is to provide two holes near each end so that eyebolts can be inserted. The holes are later grouted.

Another arrangement for bridges of moderate spans—from 25 to 50 ft—is shown in Fig. 12-5. The precast longitudinal stringers are designed as rectangular simply supported beams that are strong enough to support the weight of the forms, the concrete of the slab, and any concreting equipment that is to be used. They may be handled by means of padded slings, embedded eyebolts, or transverse bolts placed through

holes in the webs. The last is the best because of its cheapness and because it is not necessary to get under the members to lift them off the bottom forms. After erection the forms are supported upon the shelves at the bottoms of the beams. The slab is then poured and finished. The completed structure acts as a series of T beams. The tops of the precast members are roughened in order to develop the necessary resistance to longitudinal shear. They are also gripped laterally by the slab.

12-5. Building construction. Buildings, if properly designed, constitute a field in which precast concrete can be very useful. They are often composed of slender members and thin panels that are high above ground so that forms and shoring are relatively costly when the concrete is poured in place.

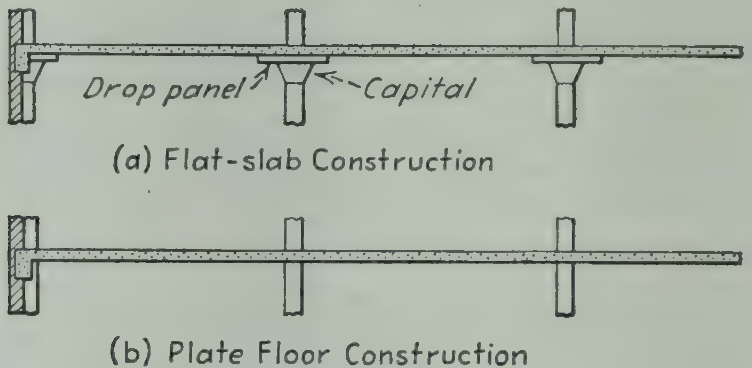


FIG. 12-6.

On the other hand, some building types are suitable primarily for cast-in-place construction. Among these are the following:

1. Flat-slab construction with drop panels and capitals of the type shown in Fig. 12-6(a). Here continuity is vital, and it is difficult to secure otherwise when large floor areas are involved. Shearing stresses around the drop panels and capitals may also be critical so that a monolithic floor is preferable.

2. Flat "plate" floors that are a modification of the flat-slab type but without drop panels and capitals, as in Sketch (b). Continuity and shearing resistance again may eliminate precasting when the floor area must be large.

3. Very large multistory buildings of beam-and-slab construction where continuity, frame action, shallow depths, and relatively small-sized columns are desired. A study of Fig. 12-1 will show how these could be accomplished better if the floor were monolithic. Furthermore, with precast construction, the column loads from above cause difficulties with the details of end bearings and column sizes. However, certain precast parts may be incorporated advantageously in such buildings. When the loads are light enough, the ends of the beams may be supported upon brackets on the columns, as in Fig. 12-2.

4. Any floor construction that must be self-supporting but must carry heavy machines or large live loads, and where a continuous well-integrated structure is necessary.

5. Large arches, domes, barrel-type construction, and other special cases. These generally require that the structures be poured in place, although it is sometimes possible to incorporate some precast parts into them.

Besides Figs. 12-1 and 12-2, which show a sort of imitation of heavy timber construction, there are many ways of using precast concrete in whole or in part for buildings. One is the combination of precast beams with poured-in-place floors as pictured for the bridge in Fig. 12-5. If a

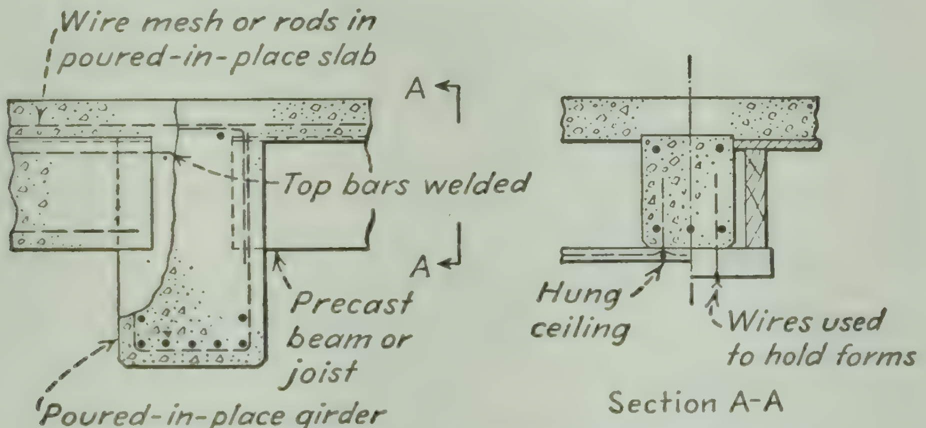


FIG. 12-7. Combination construction for a floor.

hung ceiling is to be used, it may be preferable to hang the forms from the beams rather than to use shelves for their support. Any wires or bolts that are visible when the forms are removed will be concealed by the ceiling. The girders that hold the beams, and the columns also, might have to be poured in place, somewhat as pictured in Fig. 12-7. This construction is not particularly economical for buildings.

A better way is to use large precast panels for floors, roofs, and side walls with the supporting beams poured in place after the slabs are erected on a few posts or bents. Figure 12-8 shows one application of this idea. The rigid frames are cast in place and serve as the "backbone" of the structure. The hollow panels were made on wooden forms on a concrete casting slab as follows:

1. Place reinforcement for one thin side.
2. Pour concrete of first thin side, screed it, and let it develop its initial set.
3. Place strong waterproofed corrugated-paper cartons about ⅛ in. thick to form hollow spaces.
4. Pour concrete around and over cartons.
5. Place reinforcement for other thin side.
6. Pour last concrete and finish the surface.

After erection, the joints between roof and wall panels are mortared. Some sort of tie between the two sides is desirable. Wires through the ribs between cartons may be sufficient if attached to the reinforcement of both faces.

Two large single-story warehouses[1] at the Naval Supply Depot, Mechanicsburg, Pa., were made with bolted channel-shaped sections as shown in Fig. 12-9. Even the columns were precast and hollow. The welding of reinforcement and the concreting of the joints shown in the

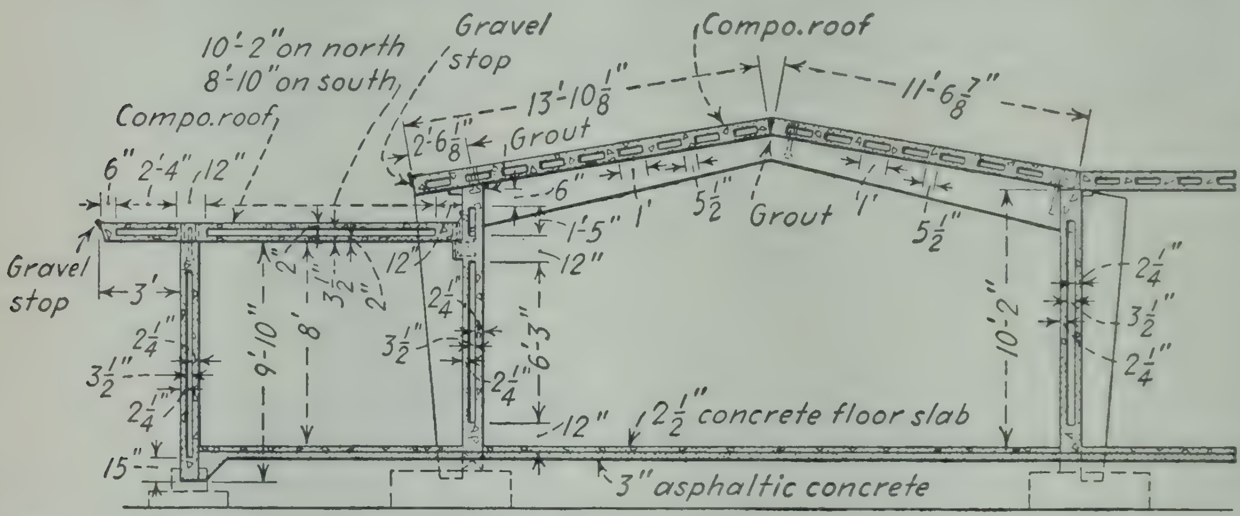


FIG. 12-8. Hollow precast-concrete slabs used for walls and roofs of one-story infirmary buildings at Los Angeles County Hospital. (*Eng. News-Record, Mar.* 8, 1951.)

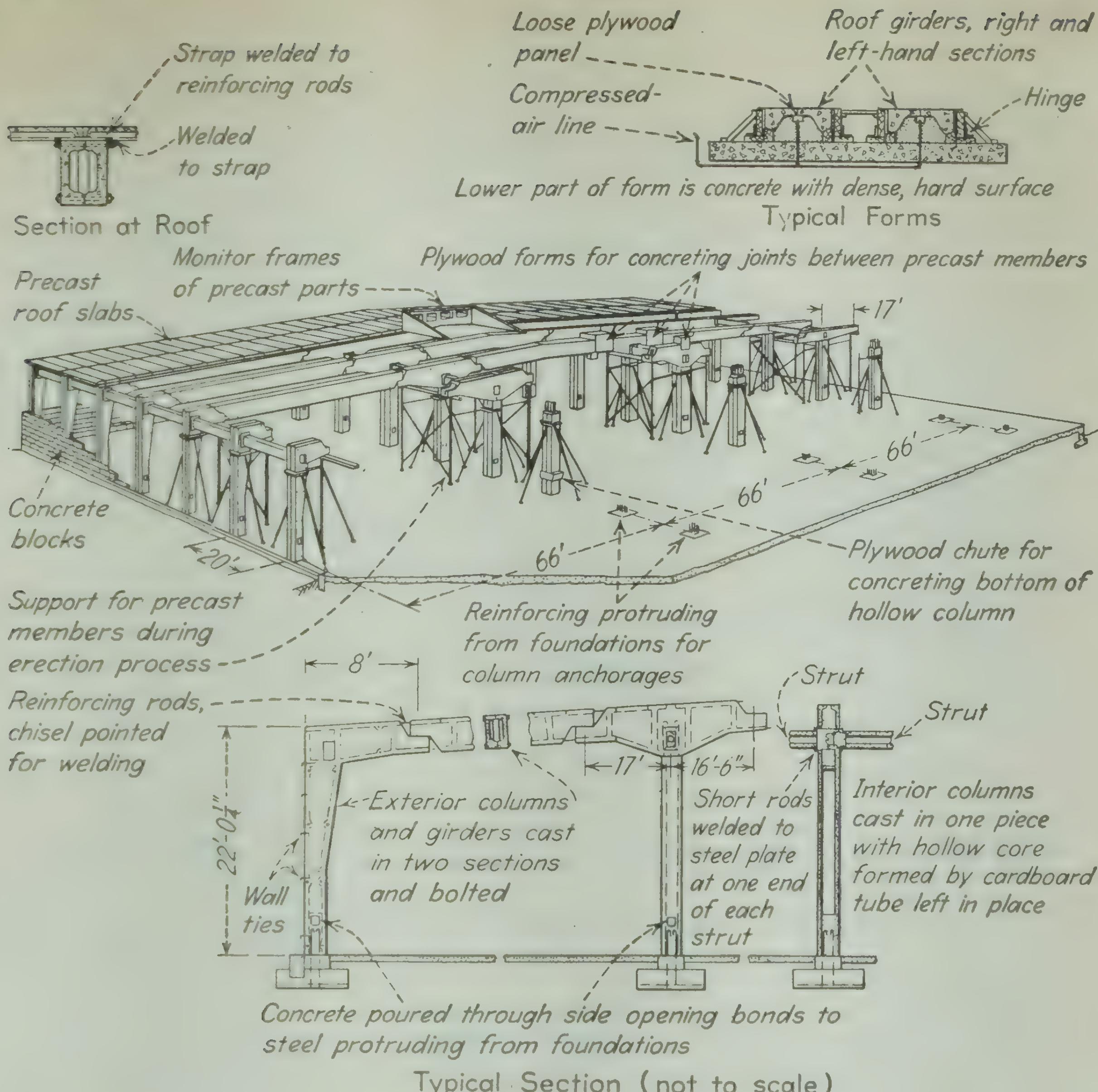
figure transformed the structure so that it could act as a rigid frame. The roof panels are 5 ft wide and 20 ft long, of channel type, and with intermediate cross ribs.

The forms were hard-surfaced smooth-troweled concrete with hinged wooden sides. A vacuum process was used with high-early-strength cement. The sections were removed from the forms within 24 hr after pouring, using vacuum lifters.

Notice that the roof panels are held by welded straps. The struts that connect the various rigid frames are also attached by welding at one end.

It is possible to cast rigid frames in pieces as solid units of the same general shape as those parts shown in Fig. 12-9. However, the columns will have to be supported laterally until the structure is framed together. The columns can have square ends that rest in pockets in the foundations. Struts to provide longitudinal stiffness of the structure may have their main portion precast, but the junctions with the frames may have to be concreted in place so as to engage proper keys and dowels, or some other positive attachment may have to be invented.

[1] W. Mack Angas, Precast Structural Members Facilitate Speedy Erection of Rigid-frame Buildings, *Eng. News-Record*, Apr. 18, 1946.



Major Dimensions

	Length	Width	Height	Thickness
Warehouse	600'	200'	24' av.	
Ext. girders	42½'	1'-8"	3'	2" or 2½" web
Int. girders	32'	1'-8"	2'-9"	3½" flge.
Ext. columns	22'	1'-8"	2'	2", 2½" or 3" web; 3½" flge.
Int. columns	17'	1'-8"	1'-8"	3" min.
Roof panels	20'	5'		1¼"
End panel beams (2)	5'		6"	2" to 3"
Int. panel beams (3)	5'		6"	1½" to 2½"
Panel girder (2)	20'		8"	2½" to 3"
Struts	20'	1'	1'-4"	1¾" min. web; 3¾" min. flge.

FIG. 12-9. Schematic drawing of a precast warehouse. (*Eng. News-Record*, Apr. 18, 1946.)

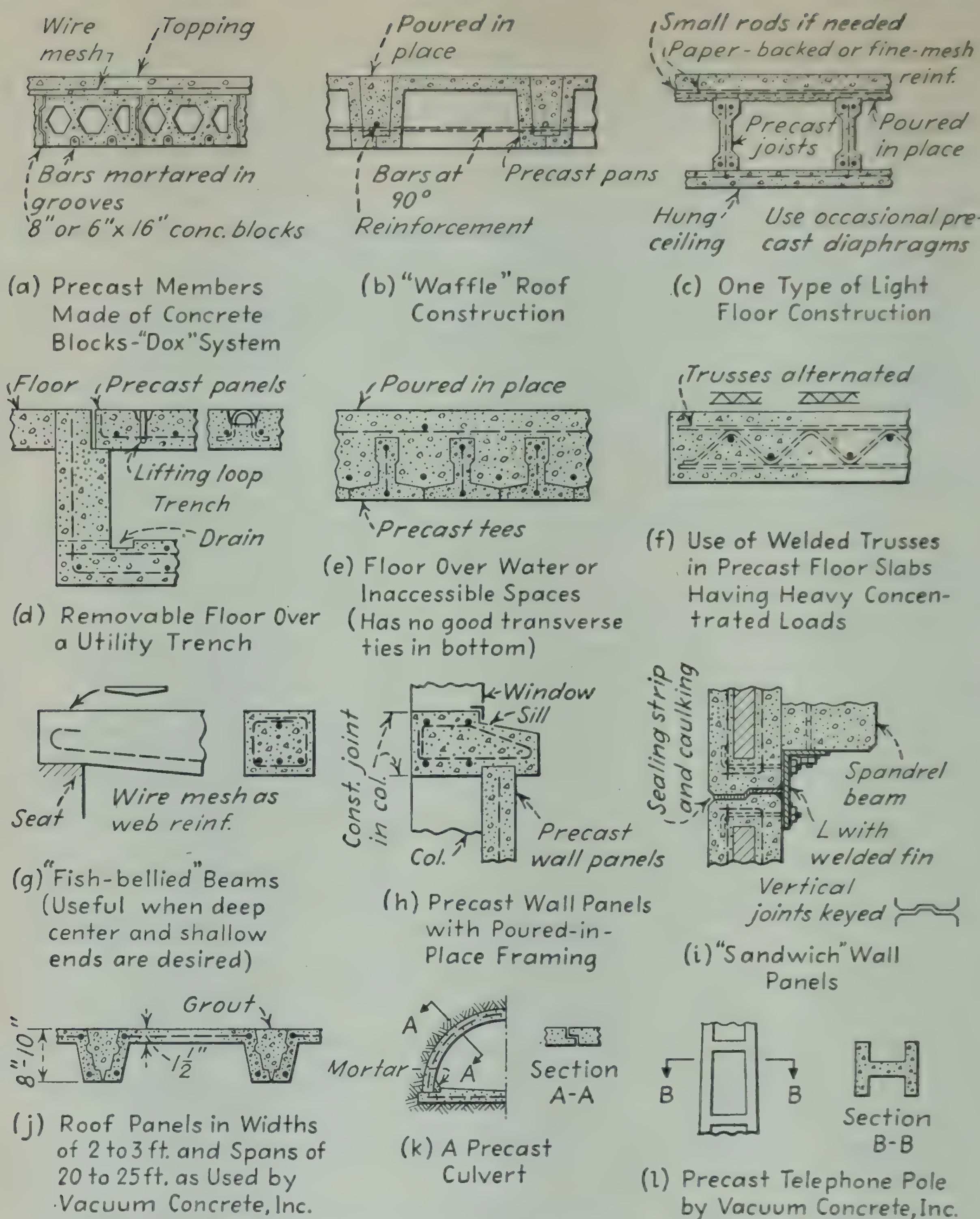


FIG. 12-10. Some miscellaneous details of precast-concrete construction.

Precast wall construction has been used considerably. Tilt-up walls are one adaptation. These walls are cast on the ground in one or more sections, then lifted or tilted upright and attached to each other and to whatever other parts may be necessary. Sometimes these panels are made with foam glass or other insulating material inside. They are built similarly to those explained in connection with Fig. 12-8. In the structure, the panels may be connected to the floor, to partitions, to col-

umns, and to each other by welding steel details together, by bolting, by mortaring, or by any other suitable and positive means.

Precast wall panels can also be used in large multistory structures. When made with an insulating material in the middle, they are often called "sandwich" walls.[1] Much experimental and development work in this line is under way. Condensation, durability, watertightness, manufacturing procedures, and structural details are among the problems that are being tackled. As stated previously, if a structure is to utilize such precast parts, it must be designed accordingly, and in great detail.

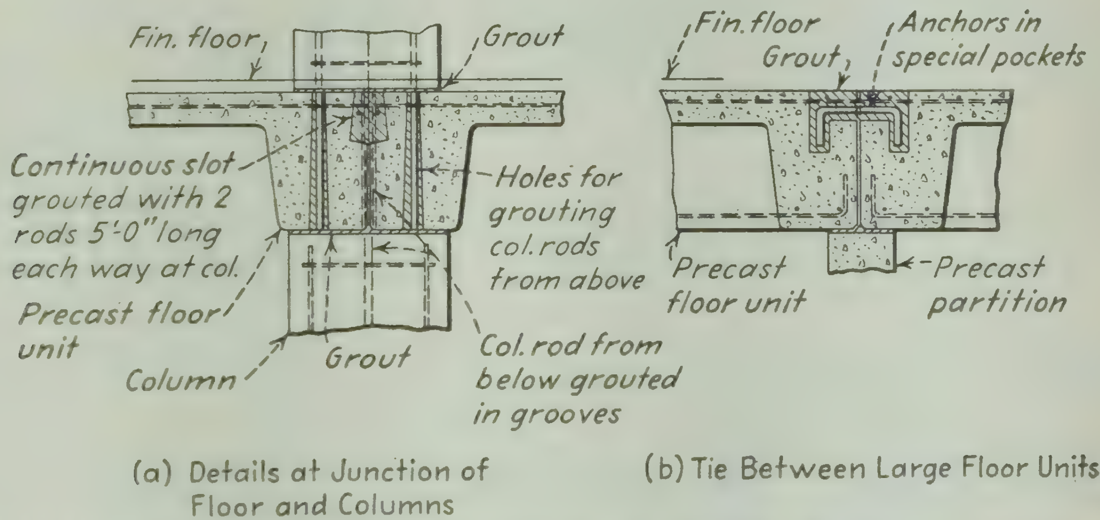


FIG. 12-11. Some details of precast construction for multistory buildings. (*Patterned after work of Vacuum Concrete, Inc., Philadelphia, Pa.*)

Devices have been invented for casting roofs or floors on the ground or on another floor in very large panels, then raising them into place by "jacking" them up the columns or by using other special hoisting devices.

Precast concrete blocks are used in making precast members of the general type shown in Fig. 12-10(*a*), one of which is the Dox system. These are useful in some cases for moderately light loads and short spans when simply supported and when not cut up by pipes and utilities. The blocks are machined, assembled in a row, bottom side up, and pulled tightly together. The reinforcement is mortared in the grooves, and the member is cured. It is shipped to the site and erected right side up. These units serve as forms to support the topping when it is poured. Together with the topping, they make a sort of T beam construction.

Precast concrete boxes, such as shown in Fig. 12-10(*b*),[2] have been

[1] S. B. Roberts, Sandwich Walls Precast for Pulp Mill, *Eng. News-Record*, Feb. 22, 1951; also, P. M. Grennan, Precast Sandwich Wall Makes Costs Tumble, *Eng. News-Record*, Jan. 24, 1952.

[2] Concrete Block Laid on Floor Slab Jacked Up on Columns to Form Roof, *Eng. News-Record*, Feb. 21, 1952, p. 30.

used to form large roof slabs of "waffle" type, or two-way pan construction. They serve as forms for the poured-in-place concrete ribs that encase the bars, and their tops assist in resisting compressive stresses. They could also be used if erected directly in place by supporting them on a series of temporary bents.

Designers of precast-concrete structures should bear in mind the problem of stiffness. This may involve the knitting of a structure together

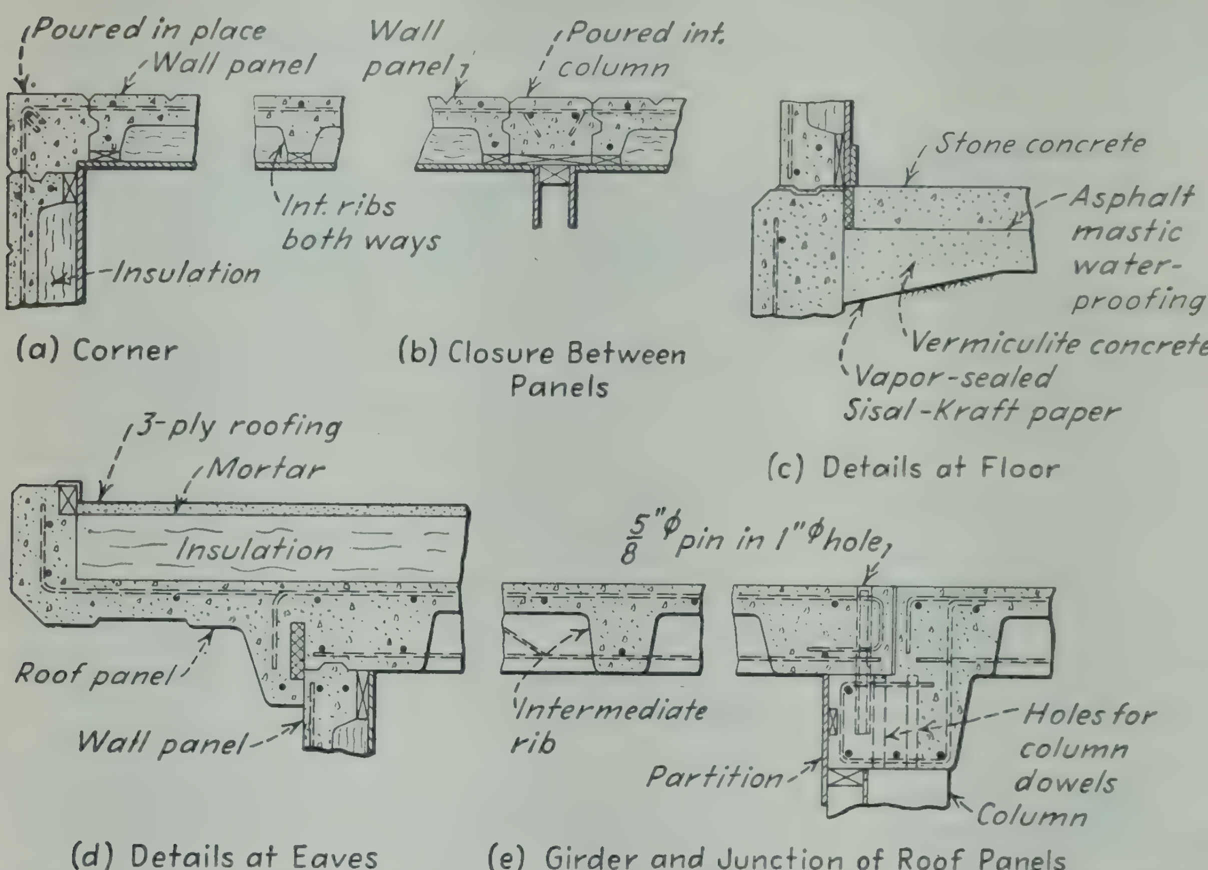


FIG. 12-12. Some details for one-story dwellings. (*Patterned after work done by Vacuum Concrete, Inc., Philadelphia, Pa.*)

by poured-in-place junction pieces as already shown. In other cases, it may be a question of the stiffness of individual members. For example, precast joists of relatively long span were used successfully when a poured-in-place floor slab was built on top of them, causing combined T beam action. In another structure the same members were used except for the fact that the floor slab was made of precast-concrete panels laid on the joists but not rigidly connected to them. Because of the much smaller effective moment of inertia, the joists deflected sufficiently to cause serious cracking of the partitions erected above them. Each such problem is one to be studied in the light of the particular conditions that apply to it.

These are just a few of the applications of precasting. The illustrations here may give the reader some additional useful suggestions. Var-

ious persons will have various ideas about how best to do things. By no means are these illustrations complete or even expected to be the best examples of what ought to be done in precast-concrete construction.

By working together, engineers, architects, and the manufacturers of concrete products will undoubtedly develop precast-concrete construction to a point where much will be standardized. Also, engineering and architectural data regarding what has been and can be done will become available for all those who are in the design and construction business.

13

PRESTRESSED CONCRETE

13-1. Introduction. Although prestressed-concrete construction has come into prominence in this country only within the last few years, the basic idea is not very new. Its adaptation to reinforced concrete, however, is a relatively recent development that has grown considerably in Europe under the leadership of such men as Gustave Magnel and M. Freyssinet. Interest in this field is now growing rapidly in the United States.

European leadership in prestressed-concrete construction seems to be due in large part to the relatively high cost and scarcity of materials, especially steel, and the low wage rates for and abundance of labor. In the United States, our mass-production methods and vast steel production have provided steel and many other materials in large volume at low cost, whereas labor has been both scarce and expensive. Design and construction methods have been developed so as to minimize the labor required and particularly to increase the use of mechanization. As time goes on, this situation will probably change, materials like steel will become scarcer, and more thought will be given to economies in their use. It is therefore probable that the use of prestressed concrete will grow in popularity for construction to which it is adaptable. This does not mean that standard types and methods will be abandoned, but prestressing will probably become one of the alternatives to consider when concrete structures are planned.

Prestressed-concrete structures generally require less concrete than standard designs—perhaps 20 to 25 per cent less. The saving in steel is likely to be even greater—maybe 50 to 60 per cent—but it is high-strength bars or wires. When long spans, shallow depths, and light dead loads are desired, prestressing may be very advantageous. For example, the depth-span ratio may be $\frac{1}{20}$ or $\frac{1}{30}$. In general, such members are also relatively stiff until the prestressing is overcome. After that, because of shallowness and small steel areas, they are likely to be quite flexible.

By no means is prestressed concrete to be considered as a substitute

for all other reinforced-concrete construction. It has disadvantages as well as advantages. Among the former are the difficulty of securing continuity, the lack of adaptability for reversal of stress, and uncertainties of action under the effect of large moving live loads. These will be discussed later.

Space is not available for more than the basic principles of the analysis of prestressed-concrete construction.[1] Much remains to be done in codifying the subject and in adapting it to engineering practice in the United States. Because developments in this direction will probably be many and extensive the author will try to stick to fundamentals. A student of structural engineering should have a fair knowledge of them on which

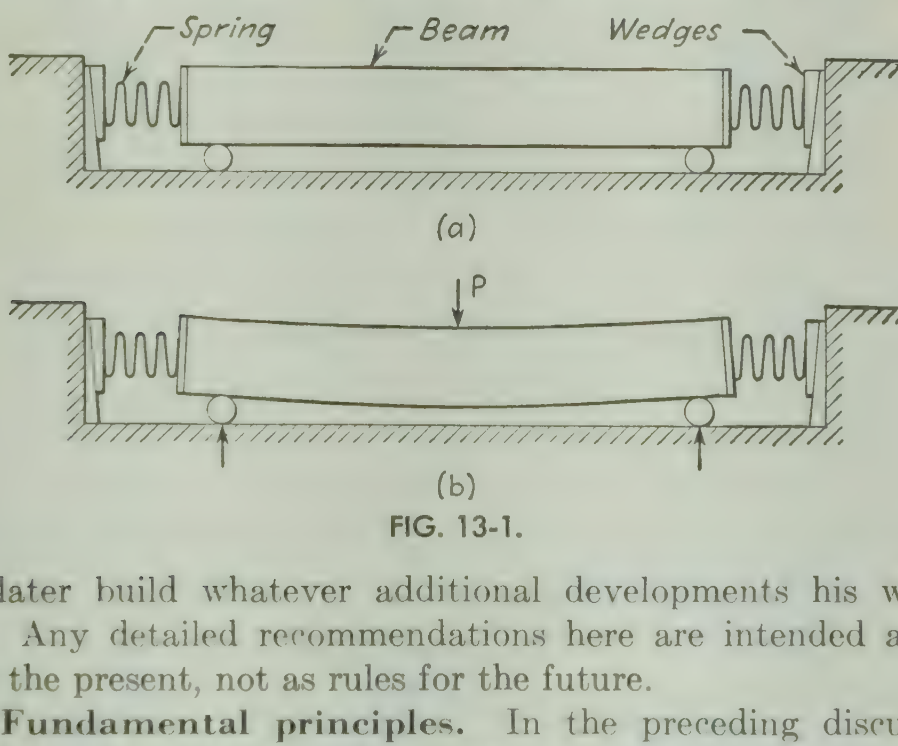


FIG. 13-1.

he can later build whatever additional developments his work may require. Any detailed recommendations here are intended as suggestions for the present, not as rules for the future.

13-2. Fundamental principles. In the preceding discussions of standard construction of reinforced concrete considerable attention has been given to consideration of the effect of cracking due to the elongation of the reinforcement. The big difference in prestressed members is the use of a means of preventing these cracks. Thus the members more nearly approach solid monolithic ones.

What prestressing does is first to put the member under compression. Figure 13-1*(a)* shows an analogy of the condition. Here the springs squeeze the beam. Then, when a load is added, the beam deflects as shown to exaggerated scale in Sketch *(b)*, but it is not bent so much that tension in the bottom will overcome the compression that the springs caused initially. Therefore, the beam will not crack because there is

[1] See Gustave Magnel, "Prestressed Concrete," Concrete Publications, Ltd., London; P. W. Abeles, "Principles and Practice of Prestressed Concrete," Frederick Ungar Publishing Co., New York; August E. Komendant, "Prestressed Concrete Structures," McGraw-Hill Book Company, Inc., New York, 1952.

no tension to make it open up. Of course, the top gets compressed more severely than it was in the first place.

Let Fig. 13-2(*a*) represent a piece of a solid rectangular uncracked beam of elastic material that is in equilibrium as a free body. The external bending moment M will be counteracted by the resisting moment of the compressive stresses and the tensile stresses pictured by the triangles ACE and DCB, respectively. These act on face AB. In Sketch

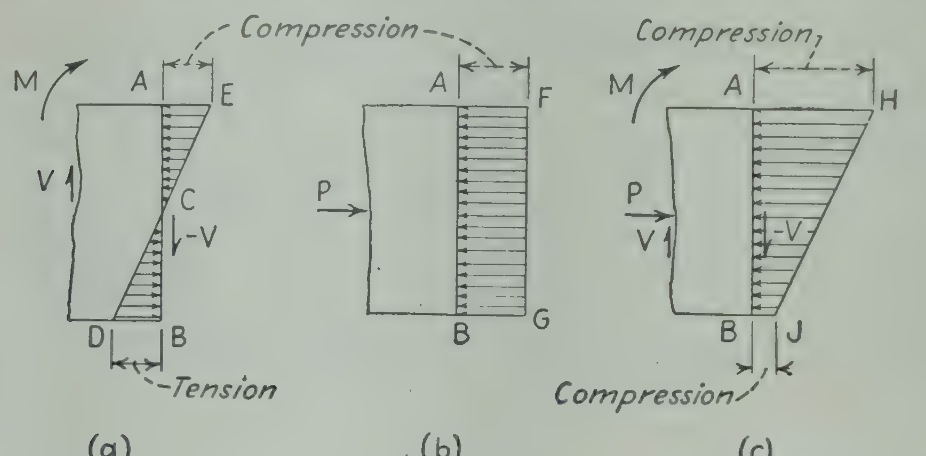


FIG. 13-2. Assumed stress conditions.

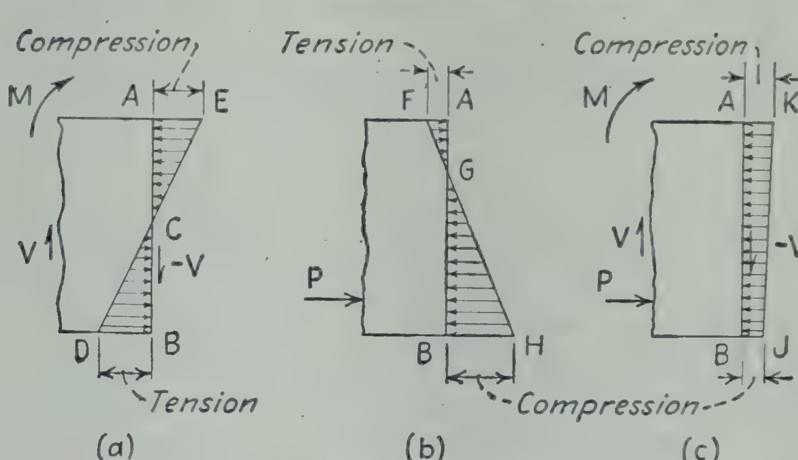


FIG. 13-3. Direct load applied eccentrically.

(*b*) the same piece is shown with a centrally applied direct load P which causes the compressive stresses illustrated in the diagram $AFGB$. Now, if P is applied first and then M is applied later, the condition may be as represented by the stress diagram $AHJB$ in Sketch (*c*). This means that $BG - DB$ results in a remaining compressive stress BJ, whereas $AF + AE$ produces a large compressive stress AH. This in a broad sense is what prestressing tries to accomplish.

Compare Fig. 13-2 with Fig. 13-3. In the latter, the bending effect of M is pictured in Sketch (*a*) as before, but the load P in (*b*) is applied eccentrically so as to produce a combination of direct stress and bending. The large compression BH is at the bottom. There may be little or no

compressive stress at A, or even a small tension as pictured by FA if the material can withstand it. Now, when P is applied and M follows, the combined condition may be somewhat as pictured in Sketch (c). This is obviously much less severe in the resultant compressive stress AK at the top of the beam, and it is therefore more desirable.

Figure 13-4 shows some general arrangements of reinforcement to accomplish this eccentric pressure at the critical section near the middle. For the present, assume that the reinforcement has been tightened up in

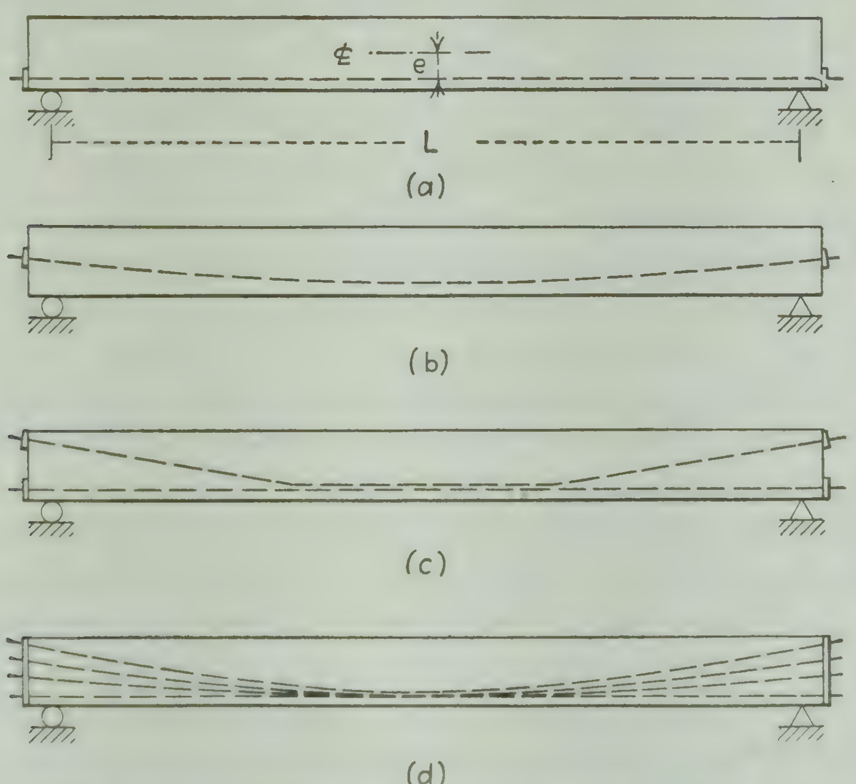


FIG. 13-4. Various arrangements for prestressing systems in simply supported beams.

some way. Also, assume that the transverse load on each beam is represented by its own weight w so that the bending-moment diagram due to these weights is a parabola and equal to $wL^2/8$ at the center.

In Sketch (a), this tension in the steel will cause a direct compression P and a moment Pe throughout the entire length of the beam. The compressive stress at the bottom of the beam may be largely offset by the moment $wL^2/8$ at the center of the span, but the dead-load moment will decrease toward the ends and will be unable to counteract the stresses caused there by the steel.

If the steel is curved as in Fig. 13-4(b) in the form of a parabola, the eccentricity at the ends will be negligible and the remaining stress condition near the ends will be the compressive stresses due to P if the moments produced by P and w largely counteract each other. Furthermore, the steel now acts somewhat like the cable of a self-anchored suspension bridge. It tries to straighten out and therefore tends to produce an

uplift on the central portion of the beam. Sometimes the tensioning cables are made flat in the central portion, then sloped upward at the ends, or the cables may be straight and the beam built with a "hump" in the middle. All these are for the same general purpose of producing a large eccentricity where it is most useful. However, these systems alone are not suitable when large moving live loads may be applied, especially near the ends of the beam.

In Fig. 13-4(c), part of the reinforcement is shown straight in the bottom and part is bent up above the center at the end so as to obtain part of each of the effects described in connection with (a) and (b). Sketch (d) is the same idea carried still further.

There are two general methods of prestressing. The word *prestressed* means that the steel is placed in tension before the main loads are applied to the structure. The words *pretensioned* and *pretensioning* will be used to denote members and processes in which the reinforcement is placed under tension before the concrete is cast around it. When the concrete has cured and the tensioning devices have been released, the bond on the surface of the steel prevents slippage so that the attempt of the steel to shorten under tensile stress produces a compression in the concrete that opposes such shortening. This pressure in the concrete makes it shorten more or less elastically, and this shortening relieves a corresponding amount of strain and stress in the steel. When equilibrium is attained it is obvious that the steel is not stressed as highly as it was initially.

The terms *posttensioned* and *posttensioning* will be used to denote members and processes in which the concrete is first poured and cured and then the tension is applied to the reinforcement, which is not bonded to the concrete. As the tension increases, the reaction of the tensioning devices against the concrete causes the concrete to shorten accordingly. When the bars are fully stressed, their ends are locked in place. Thus compressive deformation of the concrete does not annul part of the effective stress carrying capacity of the steel. This is an obvious advantage of posttensioning.

In pretensioning, the shrinkage of the concrete causes, or tries to cause, some shortening of the member. If the bond holds, this effects a corresponding shortening of the reinforcement or else it produces tension or hair cracks in the concrete that will be relieved or closed, respectively, when the tensioning devices are released from the steel.

Supposedly, most of the shrinkage has taken place before posttensioning is applied. This is another advantage of posttensioning.

In both systems, plastic flow of the concrete may occur under the compressive stresses produced by the attempt of the steel to shorten under tensile stress. This again causes a tendency to lose part of the

tensile stress in the steel. Furthermore, if the steel tends to yield (creep) under the effect of high stress, this also causes a loss of some of the tension in the reinforcement.

These ideas are illustrated to some extent in Fig. 13-5. In (*a*) the pressure due to tension in the steel is centrally applied on a rectangular or symmetrical section. It is supposed to cause a stress diagram *ACDB* in the concrete. However, the actual pressure diagram is *AEFB* because of the shortening of the concrete due to shrinkage, plastic flow, compressive deformation, or even creep, or any combination of them. Sketches

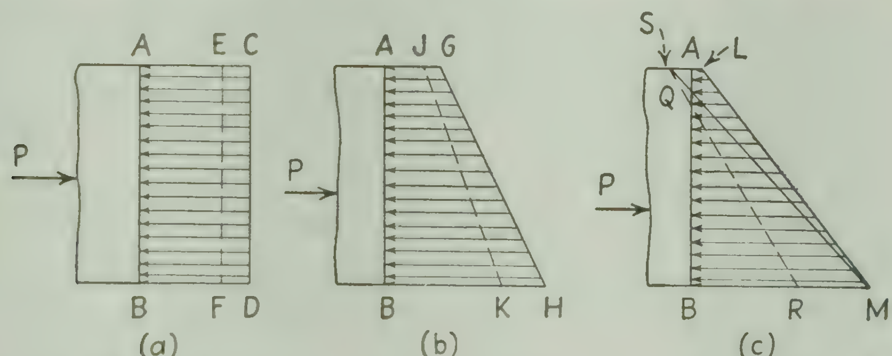


FIG. 13-5. Effect of shrinkage and plastic flow of concrete or creep of steel.

(*b*) and (*c*) are to illustrate the idea when the pressure and pretensioning are eccentric. Of course, the tensile deformation of the steel must be large or the deformation of the concrete will be sufficient to counteract it and cause complete loss of the prestress. An approximation of the loss of prestress due to shrinkage, etc., is 20 to 25 per cent. Magnel recommends the former figure.

Examine the situation for pretensioning particularly. Assume the following data (plastic flow and creep neglected for the present):

$$E_c \text{ for concrete} = 4{,}000{,}000 \text{ psi}$$
$$E_s \text{ for steel} = 30{,}000{,}000 \text{ psi}$$
$$\text{Shrinkage} = \tfrac{3}{8} \text{ in. in 100 ft} = 0.00031 \text{ in. per in.}$$

Average resultant compressive stress in concrete = 1,000 psi
The deformation per inch due to compression is

$$\delta_c = \frac{f_c}{E_c} = \frac{1{,}000}{4{,}000{,}000} = 0.00025 \text{ in. per in.}$$

Add to this the shrinkage. Then the total unit deformation of the concrete is 0.00025 + 0.00031 = 0.00056 in. per in. For the steel, this amount of deformation of the concrete corresponds to a tensile stress of

$$f_s = E_s\delta_s = 30{,}000{,}000 \times 0.00056 = 16{,}800 \text{ psi}$$

If the maximum pretensioning in the steel is 150,000 psi, this lost stress alone is approximately 11 per cent of the prestress. This shows that, if

creep and plastic flow are included, ordinary reinforcing steel is not the thing to use as a prestressing medium because most of the pretensioning is likely to be lost when even the yield point cannot be more that 40,000 to 50,000 psi. Therefore, high-strength steels or cold-drawn wires with a high yield point are the appropriate materials to use for prestressed-concrete construction. In practice, high-strength bars having a yield point of 75,000 to 80,000 psi are to be used, or cold-drawn wires with an elastic limit of something like 160,000 to 180,000 psi are suitable. These wires are sometimes in the form of wire ropes.

As for the concrete, this too must generally be able to resist a much larger compressive unit stress than that required for ordinary reinforced concrete. Therefore, concretes with a 28-day compressive strength of 4,000 to 8,000 psi are necessary. Such strength can be obtained with rich mixes, good aggregates, proper water-cement ratios, excellent compaction, and careful curing. Good control is needed for such work.

Obviously the prestressing must cause sufficient compression in the concrete to annul any future tension that loads may develop. If not, the concrete is likely to crack and to behave similarly to ordinary reinforced concrete for the portions of the loads that are excessive—exceeding those which can be considered to overcome the prestressing. Since the cross-sectional area of high-strength steel is generally relatively small, the deformations beyond the prestressing limit are likely to increase rapidly with excess loading. Therefore, this condition is not desirable, and prestressing is only partially effective.

In prestressing, one must remember that what he wants is a tensioning medium that will take up large strains before the tension is eliminated. A large pressure on the ends of a member is not effective if it is destroyed by a tiny shortening of the member. Furthermore, if a bar sticks out of the ends of a member into which it is bonded, a jack applied at each end to put a tension in the bar and a compression in the concrete is primarily a pull-out test. It does not stretch the whole bar but only the end portions before bond transfers the stress into the concrete. The effect is local. If the beam is then bent, the part of the bar in the central portion will behave as usual and the concrete will crack as for any similar reinforced-concrete member. That is why the idea is pictured with springs in Fig. 13-1, since they can apply pressure to the concrete to a considerable extent in spite of the latter's deformation. Thus the compression is useful throughout the beam, and cracks cannot open up until the local tension from bending overcomes the compressive stress.

Partial prestressing denotes the application of some amount of prestressing force but not enough to avoid some, or even considerable, tension in the member under maximum loads. For example, a beam may be prestressed enough to withstand the dead-load moments and perhaps

one-half the specified live load without causing tension on, or cracking of, the section. This is done with the idea that the maximum live load will be applied very seldom. The beam can crack if necessary under that condition but, when this load is removed, the crack will close again. Perhaps such a scheme will result in a more economical member in first cost than will a fully prestressed design. However, the long-term service and the ultimate safety of the member may be open to question. Until more is known about this question, one will be wise to avoid partial prestressing in most cases because the live loads are often increased in spite of the designer's original intentions.

13-3. Methods for pretensioning. Since pretensioning requires that large tensile forces in the steel must be resisted initially by the forms

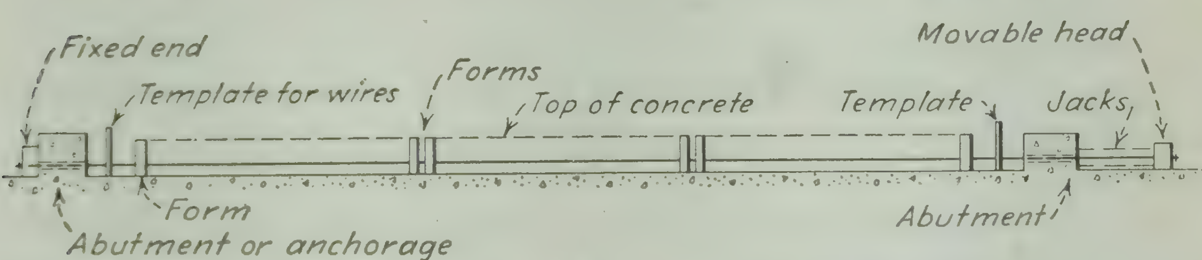


FIG. 13-6. Principles of multiple casting of pretensioned precast members, Hoyer method.

or by some strong anchorage, this method is more suitable for work done in the shop than it is for large jobs built in place in the field.

The manufacture of precast prestressed-concrete products seems to have tremendous possibilities. It is probable that one will soon be able to buy standard sizes of members with predetermined dimensions and strengths just as he can now purchase steel beams or heavy timbers. He can then design his structures to use these standard sizes, thus taking advantage of both precasting and prestressing.

One illustration of shop methods of production is given in Fig. 13-6. This is supposed to picture an arrangement for casting prestressed slabs about 2 ft wide. Concrete or other forms may be used as a base. When multiple units are to be cast with several in a series, the anchorages or abutments may be made strong enough by themselves to resist the pull without a complete intervening form to resist the compression. The ends are designed to permit connection of the wires to some sort of head. One end may be stationary and the stretching may be done by jacks at the other end with the reaction and bending resisted by the forms, or both ends may be equipped with jacks. The movable end is then blocked to hold the tension. Templates may be used to hold the wires in position beyond the end forms. Bulkheads or intermediate forms are used to separate the pieces into the desired lengths. The forms are oiled. The concrete is cast, compacted, and cured. Then the jacks are reapplied,

the tension is released, the wires are cut, and the end and side forms are removed. The slabs are then lifted off the bottom and stored.

By the use of high-early-strength cement and the vacuum process for removing excess water, and by steam curing, the attainment of proper strength in the concrete can be expedited so that reuse of the forms is made fairly efficient. Thus, even though the initial cost of the forms may be large, this is offset by their long service. Vacuum lifters are also exceedingly useful in handling these precast products, especially if the details are arranged so that the steel has some kind of end anchorage—like a plate or welded washer—that transmits the pressure to the green concrete through bearing instead of through bond alone, since this will allow quicker removal from the forms.

To a small extent, pretensioning may be done in the field by using the wires in loops so that wedges can be driven through the loops and against the forms. However, it is obvious that control of the prestress is difficult, the results may be variable, and the forms and labor may be unduly expensive.

One advantage of pretensioning is the thorough encasement and protection of the reinforcement.

Since bond is the primary means of the transfer of stress from the steel to the concrete, wires are preferable to bars because of their large surface area compared with their cross-sectional area. A diameter of 0.2 in. is a preferred maximum. Indented wires may provide better bond than plain ones, but the difference will not be large.

13-4. Methods for posttensioning. The posttensioning of the steel has many advantages. The concrete of the member can be used to resist the reaction from the tension in the steel. The losses from shrinkage are largely eliminated so that less steel or less highly stressed steel can be employed to obtain a given compressive force on the concrete. Members can be cast in the shop, on the ground, or in place in the field, and the forms can be removed as soon as the strength of the concrete permits it. The magnitude of the tension can be controlled without being influenced by shrinkage of uncertain extent. Large tensile forces can be used without danger of slippage at or near the ends of the reinforcement. The steel can also be curved as shown in Fig. 13-4(*b*), or inclined as in Fig. 13-20.

There are some disadvantages in posttensioning. The reinforcement may be coated with some material of the nature of grease to allow it to slip if it is installed prior to pouring of the concrete. This is troublesome and may not always be perfectly effective. If holes are made in the concrete during casting so that the steel can be inserted later, some type of form is needed. Cardboard, thin metallic tubes, rubber "bars" that can be pulled out, and hoses that can be expanded by pressure and later

released—these are some of the materials that may be used for forming these holes. The wires may be pulled through the holes. Again the wires may be enclosed as groups in light metallic or other sheaths before the concrete is poured. This is a good way to do the work. When the steel is finally installed and stressed, the clear space around and between the wires that constitute the reinforcement should be grouted for protection of the steel against corrosion.

The details of end anchorages and the spaces required for them are matters that need careful study. Figure 13-7(a) shows an upset threaded

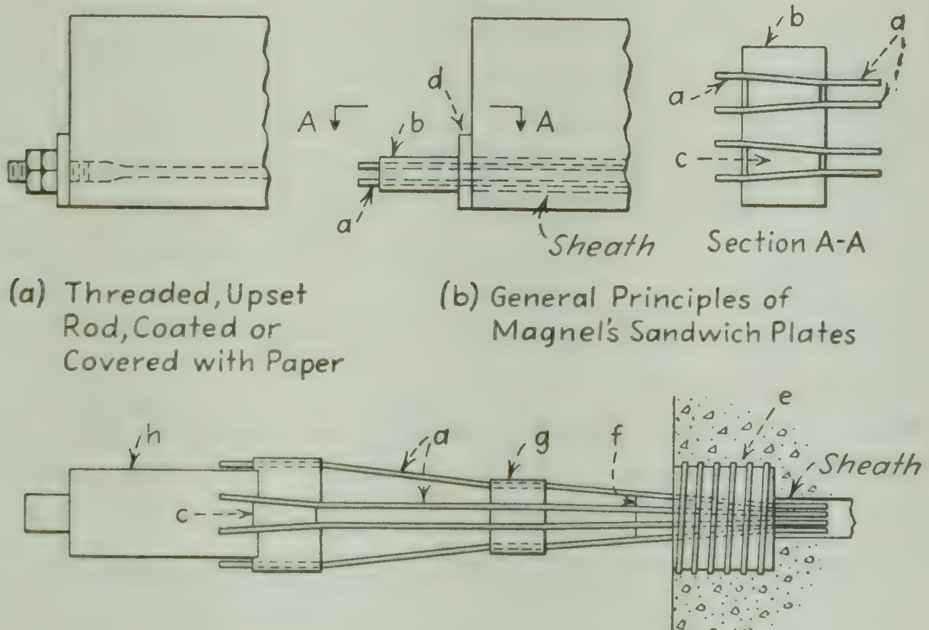


FIG. 13-7. Some methods of posttensioning.

bar of high-strength steel that is installed in the concrete and coated with grease. The posttensioning is secured by tightening the nuts and stretching the rod inside the concrete. This is not a practical arrangement for many cases.

In Fig. 13-7(b) is pictured the general idea of the sandwich-plate anchorages used by Magnel. These can anchor several wires simultaneously, and they seem to be very effective. The wires are placed in layers (four in a row in the illustration). Some desired number is determined by the load requirements or the tensioning equipment. These can be encased in a metal sheath or tube, the end of which may be flared to permit the wires a to spread apart so as to fit into the wedge-shaped slots in the plates b. The wires are long enough to be gripped by the jack head. The jacks stretch the wires as necessary, reacting through bearing plate d against the concrete of the member. The wedges c are driven or pressed

in to grip the wires and then the jacks are removed and the excess length of the wires is cut off. The inside of the sheaths around the wires is grouted through holes left for that purpose. Generally the anchorages are finally encased in concrete for protection.

The conical anchorages used by Freyssinet are indicated in simplified form in Fig. 13-7(c). Such details as these require considerable study. The manufacturers of such materials should be consulted regarding their use. However, the principles are simple. Only the general features will be explained.

Freyssinet uses a concrete cylinder e with steel spirals in it to prevent bursting. It has a conical hole through which the parallel wires are splayed over a guide g on the jack h. The wires are anchored temporarily by wedges c to the body of the jack. The jack reacts against the concrete of the member. When the wires are tensioned, the jack holds them while a secondary jack plunger inside rams the conical plug f into the conical hole in the cylinder. The wires are then gripped between the two surfaces, one or more of which has the equivalent of "teeth" to help hold the wires. The jack is then released, the wires are cut, and the end may now be encased in concrete.

13-5. Shear and web reinforcement. In standard reinforced-concrete beams, the diagonal tension can become a serious problem when the shearing forces are relatively large. The situation is very different in prestressed members that always have compression on the entire area. Under such conditions, even though the members deflect and curve, there should be no cracks, and not even tensile forces tending to cause cracks. Therefore, the shearing resistance is that of uncracked concrete subjected to transverse shearing stresses and to something approaching the longitudinal shearing stresses that exist in beams of homogeneous material. The magnitude of the maximum permissible unit resistance of uncracked concrete to shearing action is rather uncertain, but experience seems to show that it is relatively great.

Therefore, prestressed-concrete beams can be designed with relatively light thin webs. Such sections as those in Fig. 13-8(a) may be used. The saving in weight is obviously considerable. This is likely to be more important than the saving of the cost of the extra concrete, because prestressing is generally most helpful when long spans and shallow depths are needed and for which the minimizing of dead load is very important. The thin webs are also an advantage because, if solid rectangular sections were used, the extra concrete near the center would not be needed for bending resistance but it would require more prestressing steel in most cases in order to secure the compressive stress (or compressive strain) that is needed in the member to counteract the expected or possible tensions for which the member is designed.

To illustrate this saving in prestressing reinforcement, assume the section of Fig. 13-8(a). Contrast it with that of (b) if the stress condition under the initial prestressing is to be as shown in (c) before the dead load is applied to the beam. Approximately, the total force to be applied to cause this pressure on the I section is 228,000 lb; for the rectangular member, it is 540,000 lb.

The elimination of cracking under diagonal tension is one of the great advantages of prestressing. Web reinforcement as ordinarily considered is not necessary even though a little steel to tie across the member may

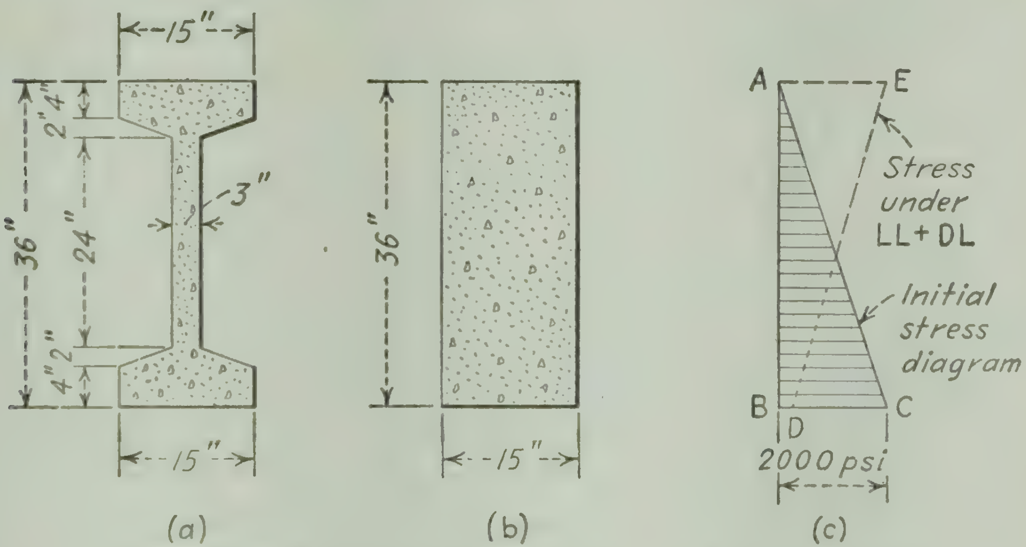


FIG. 13-8. Comparison of thin webs and thick sections.

be desirable. The best source of information as to the permissible proportions of the webs of I sections or of channel shapes and box sections is the experience of others with structures that have been built or tested.

Until some other specification is adopted, it is probably safe to use a theoretical transverse shearing resistance of $0.10f_c'$ for working loads. This is to be confined to the web and to its projection in the flange areas. For example, in Fig. 13-8(a), the resisting area may be assumed to equal a strip 3 in. wide and 36 in. deep unless there are holes in the web that should be deducted, or unless the holes for posttensioning reinforcement render the area beyond them ineffective.

Theoretically, the shearing stress in a prestressed beam is a maximum at the neutral axis. Its magnitude may be computed by the formula

$$v = \frac{VQ}{Ib} \tag{13-1}$$

where v = unit shear at the plane being investigated, V = external transverse shear acting upon the section, Q = static moment of the area of the cross section one side of the plane being considered, I = the moment of inertia of the entire concrete (uncracked), and b = the width of the

member at the point in question. However, the results of the computations generally have little or no significance because of the great shearing resistance of uncracked concrete. If the member is stressed to the extent that it cracks, then the thin web will probably be in difficulty, and such a condition should not be permitted. At any rate, diagonal tension is seldom troublesome in prestressed members that are not overloaded. Comparisons of the results found from Eq. (13-1) with allowable values set for $v_L = V/bjd$ in ordinary concrete members mean little.

A possible weakness in pretensioned members is a slippage of the reinforcement at the ends of beams. For instance, if the wires in Fig. 13-9 were to be ineffective in their bond in the distance AB, cracks might open up as indicated, and the ends of the beam might break off. This danger does not exist in the end anchorages of posttensioned reinforcement because the steel cannot possibly slip. However, one must be careful that the concrete under the bearing plates does not crush or spall off. The webs near the ends are usually enlarged for this purpose.

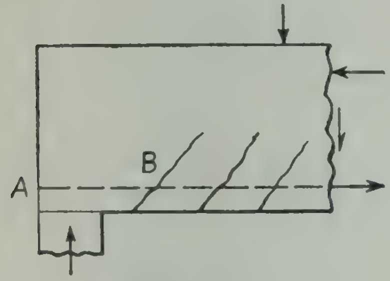


FIG. 13-9.

It is desirable to "tie" the flanges of thin-webbed members together with wires or light bars. Special care should be used to have them at and near the ends of pretensioned members. However, the amount of steel to use seems to be a matter of practical judgment rather than of theory alone.

If some of the prestressing wires are curved or sloped upward, they will have a component that assists in the resistance to transverse shearing.

13-6. Computations for a pretensioned beam. Assume that a straight precast pretensioned beam of I section is to be designed to support a uniformly distributed load of 500 plf in addition to its own weight. The span length is to be 30 ft, and the beam is to be simply supported. Let $f'_c = 5{,}000$ psi, $f_c = 2{,}000$ psi, the ultimate strength of the wires = 250,000 psi, and f_s for the wires = 150,000 psi. Allow 20 per cent reduction of f_s because of shrinkage, compression, and creep. No tension is to be allowed on the section. Assume that the diameter of the wires is to be 0.196 in., and $A_s = 0.03$ in.^{2} per wire.

The approximate methods[1] recommended by the Portland Cement Association will be used because they seem to be sufficiently accurate. They illustrate the principles very well, and the procedures are easily understood.

Assume the section shown in Fig. 13-10(a). The ratio of span to depth is $30/1.5 = 20$, which is much greater than the customary ratio for standard reinforced-concrete beams.

[1] Design of Prestressed Concrete, *Portland Cement Assoc., Bull.* 25.

Being a symmetrical member, the center of gravity (neglecting the steel) is at the mid-depth. The area of the section

$$A = 10 \times 3 \times 2 + 3.5 \times \tfrac{1}{2} \times 4 + 3 \times 12 = 103 \text{ in.}^2$$

Next compute I:

$$\text{Rectangles 1: } \frac{10 \times 3^3 \times 2}{12} + 30 \times 7.5^2 \times 2 = 3{,}415$$

$$\text{Triangles 2: } \frac{3.5 \times 1^3}{36} \times 4 + 1.75 \times 5.67^2 \times 4 = 223$$

$$\text{Rectangle 3: } \frac{3 \times 12^3}{12} = 432$$

$$I = 4{,}070 \text{ in.}^4$$

The beam itself weighs $^{103}\!/_{144} \times 150 = 107$ lb per ft. Call it 110

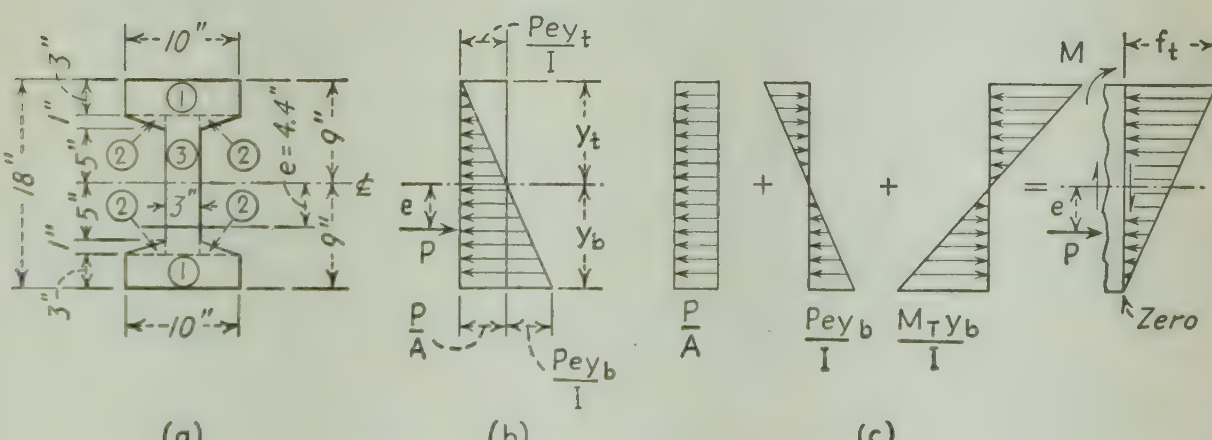


FIG. 13-10. Combination stresses.

lb per ft. Then the bending moment M_B for the dead load of the beam alone is

$$M_B = 110 \times \frac{30^2}{8} \times 12 = 148{,}000 \text{ in.-lb}$$

The dead- and live-load moment M_F to be superimposed in the field is

$$M_F = 500 \times \frac{30^2}{8} \times 12 = 675{,}000 \text{ in.-lb}$$

$$\text{Total moment } M_T = 675{,}000 + 148{,}000 = 823{,}000 \text{ in.-lb}$$

Before proceeding further, check the section for the total moment to see if f_t at the top is within the allowable 2,000 psi.

$$f_t = \frac{M_T y_t}{I} \tag{13-2}$$

$$f_t = \frac{823{,}000 \times 9}{4{,}070} = 1{,}820 \text{ psi}$$

This is a little on the side of safety and will be accepted tentatively.

If the prestressing of the beam alone is to cause zero stress at the top, as indicated in Fig. 13-10(*b*), the stress diagram is found from

$$f = \frac{P}{A} \pm \frac{Mc}{I} \qquad \text{or} \qquad f_b = \frac{P}{A} + \frac{Pey_b}{I} \tag{13-3}$$

and

$$f_t = \frac{P}{A} - \frac{Pey_t}{I} \tag{13-4}$$

Notice that, if the member is unsymmetrical, the distances from the centroidal axis to the extreme "fibers" are different. In this case, remembering that the radius of gyration $r = \sqrt{I/A}$, and $I = r^2A$, the required prestressing force would be, from Eq. (13-4),

$$0 = \frac{P}{A} - \frac{Pey_t}{I} = \frac{P}{A} - \frac{Pey_t}{r^2A} = \frac{P}{A}\left(1 - \frac{ey_t}{r^2}\right) \tag{13-5}$$

Therefore,

$$1 - \frac{ey_t}{r^2} = 0 \qquad \text{or} \qquad e = \frac{r^2}{y_t} \tag{13-6}$$

Thus the eccentricity of the prestressing force is independent of the magnitude of that force.

$$r^2 = \frac{I}{A} = \frac{4{,}070}{103} = 39.5 \text{ in.}^2$$

Therefore,

$$e = \frac{39.5}{9} = 4.4 \text{ in.}$$

Next, assume that the final combination of M_T and P, with its lever arm, is to make the stress at the bottom equal to zero, as shown in Fig. 13-10(*c*). Then

$$f_b = 0 = \frac{P}{A} + \frac{Pey_b}{I} - \frac{M_Ty_b}{I}$$

Then

$$\frac{P}{A} + \frac{Pey_b}{Ar^2} = \frac{M_Ty_b}{Ar^2}$$

$$P\,\frac{(r^2 + ey_b)}{r^2} = \frac{M_Ty_b}{r^2}$$

Therefore,

$$P = \frac{M_T}{(r^2/y_b) + e} \tag{13-7}$$

By substituting numerical values, Eq. (13-7) gives the required compressive force to make $f_b = 0$.

$$P = \frac{823{,}000}{(39.5/9) + 4.4} = 94{,}000 \text{ lb}$$

The maximum stress produced by this force with its eccentricity is therefore

$$f_b = \frac{94{,}000}{103} + \frac{94{,}000 \times 4.4 \times 9}{4{,}070} = 1{,}820 \text{ psi}$$

This agrees with the result from Eq. (13-2), as it should in this case.

This force P is supposed to be the final compression (or tension in the steel) after the 20 per cent losses from shrinkage, compression, and creep are deducted. Therefore, to get the area of wires needed, multiply the allowable f_s by 0.8. Then

$$A_s = \frac{P}{0.8f_s} = \frac{94{,}000}{0.8 \times 150{,}000} = 0.783 \text{ in.}^2$$

Since the eccentricity of the resultant prestress is to be 4.4 in. in order to

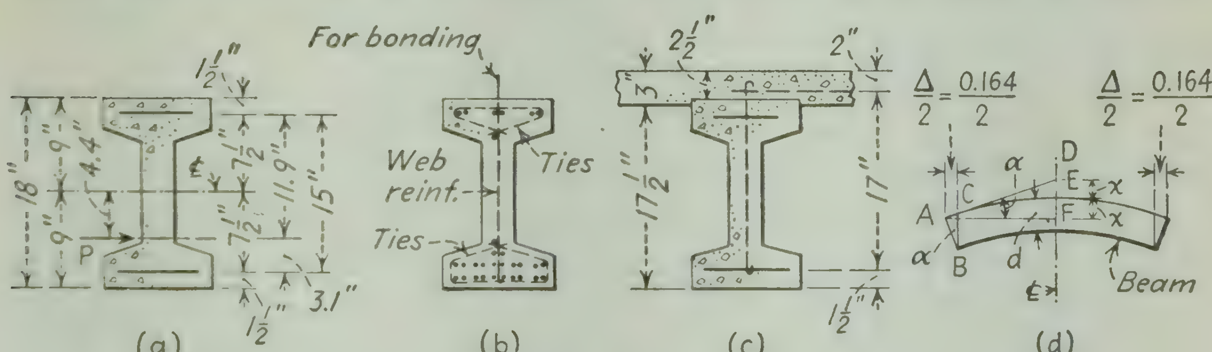


FIG. 13-11.

have zero stress in the top initially, some of the wires will have to be in the top flange to counterbalance those in the bottom flange because the wires have to be placed where there is space to put them.

Assume that the centers of gravity of the top and bottom groups of wires are as pictured in Fig. 13-11(a). The area of the top steel times its lever arm from the bottom steel = the resultant P (all of A_s) times its distance from the bottom steel. Therefore,

$$\text{Top wires} = \frac{0.783 \times 3.1}{15} = 0.162 \text{ in.}^2$$

$$\text{Bottom wires} = 0.783 - 0.162 = 0.621 \text{ in.}^2$$

This requires $0.162/0.03 = 6$ wires at the top and $0.621/0.03 = 21$ wires at the bottom. In order to use them in pairs, place 6 in the top and 22 in the bottom.

The details of spacing and arrangement of the wires will depend upon such things as the size of aggregate, jacking equipment, and cover of concrete desired. A general picture is shown in Fig. 13-11(b). The ties and web reinforcement are nominal. They might be No. 8 or 10 wires.

Additional wires should be used in the web near the ends of the beam to ensure proper shear resistance.

The tops of the transverse wires in the web project so that they can be bonded into the joint between precast panels or into thin slabs because the beams should have lateral support. If no other lateral support is provided, holes may be left in the web so that a central precast or poured-in-place diaphragm can be used and tied into the entire floor or roof.

If the final 120,000 psi in the prestressing wires is to be developed by bond, with $u = 0.045f'_c$, the required length L_s for average bond stresses is

$$L_s = \frac{f_s A_s}{(\Sigma o)u} = \frac{120{,}000 \times 0.03}{0.615 \times 0.045 \times 5{,}000} = 26 \text{ in.}$$

Actually, the bond may be much higher than the average close to the end, but this shows the danger of slippage of plain wires in this vicinity. It illustrates again the desirability of some type of mechanical anchorage and of a little web reinforcement.

As the beam of Fig. 13-11(b) deflects under prestress only, it will bend upward slightly. When the final load M_T is acting, the beam will be bent back downward. These curvatures change the lengths of the wires a little, and they affect the stresses in them correspondingly. However, these changes are small. At the most, they cannot be more than nf_c. In general, they may change the stress in the wires by 5 or 10 per cent. In this simply supported beam, the curvature due to prestress tends to relieve the stress in the bottom wires, but dead and live loads tend to bring it back again.

Another question is what the beam can resist before it will crack. The allowable tension is uncertain. If it is assumed to be $0.1f'_c$, and if the prestress at the bottom is completely used up by M_T, then

$$M \text{ at cracking} = M_T + f\frac{I}{y_b}$$

$$M = 823{,}000 + 0.1 \times 5{,}000 \times \frac{4{,}070}{9} = 1{,}050{,}000 \text{ in.-lb}$$

Notice that this increase is not large and that the tension in the concrete cannot be relied upon once the crack has formed. Thereafter, the member depends upon the steel for tensile resistance since the crack will probably progress, once it gets started, until the steel alone stops it. The moment of inertia of the cracked section will be very much less than that of the uncracked one as soon as the crack has progressed to any appreciable extent. Therefore, the cracked condition is a limit to be avoided.

As the moment increases further, the steel stress approaches the ultimate, but the large strain that accompanies the increase of 100,000 psi or more in the wires will probably cause such a concentration of pressure

at the top of the beam that the flange may fail in compression before the steel gives way. Anyhow, the deflections at this stage will be excessive.

As far as the transverse shearing stress at the end of the beam is concerned, it can be called

$$\frac{(110 + 500)15}{3 \times 18} = 170 \text{ psi}$$

The "cracking moment" divided by the total "working moment" may be said to be a measure of the safety factor. This would be

$$\text{S.F.} = \frac{1{,}050{,}000}{823{,}000} = 1.27$$

It seems to the author that the original design moment might well be increased a little so that some reserve will be provided before any tensile stresses are required in the concrete. This applies when the live loads are uncertain, and it is recommended largely because of the weakness in shear and moment if cracks do occur.

Therefore, it might be a desirable policy to use the philosophy of ultimate-load design in the analysis of prestressed members, basing the design upon no tension (or little tension) in the concrete. One might use about 75 to 85 per cent of the ultimate strengths of the concrete and of the wires as the allowable stresses. Then he might multiply the dead and live loads by the desired safety factors to obtain the design moments and shears. The tension in the concrete can be investigated later, and perhaps some small allowance can be made because of it when one selects the final factors by which to multiply the working loads in the final analysis.

What happens if this beam has a poured-concrete slab placed on it as shown in Fig. 13-11(*c*)? First it is subjected to prestress. Then it has to support its own weight. Assume that it also supports the plastic concrete of the 3-in. slab and the forms, and that the beams are 5 ft c.c. Since the forms will be removed, their weight will be excluded as far as permanent effect upon the stresses is concerned.

$$\text{Total } w = 110 + 38 \times 5 = 300 \text{ plf (approx)}$$

$$M = \frac{300 \times 30^2 \times 12}{8} = 405{,}000 \text{ in.-lb}$$

$$\text{Resulting stress} = \frac{My}{I} = \frac{405{,}000 \times 9}{4{,}070} = 895 \text{ psi}$$

The poured slab is to be tied into the precast beam and the top of the beam is to be serrated or roughened so as to secure adequate resistance to longitudinal shear at the junction. Then the hardened slab and the

beam are assumed to act together in resisting live loads as a sort of T beam. However, in this case, do not be too optimistic in counting upon the slab. A width b of four times the thickness of the slab each side of the top flange of the precast beam seems to be great enough. If so, the width b of the T beam will be $10 + 2 \times 4 \times 3 = 34$ in.

Remember that prestressing is a device to eliminate cracking of the concrete and that its effect is that of enabling the beam to withstand bending without having much, if any, tension on the section. In this

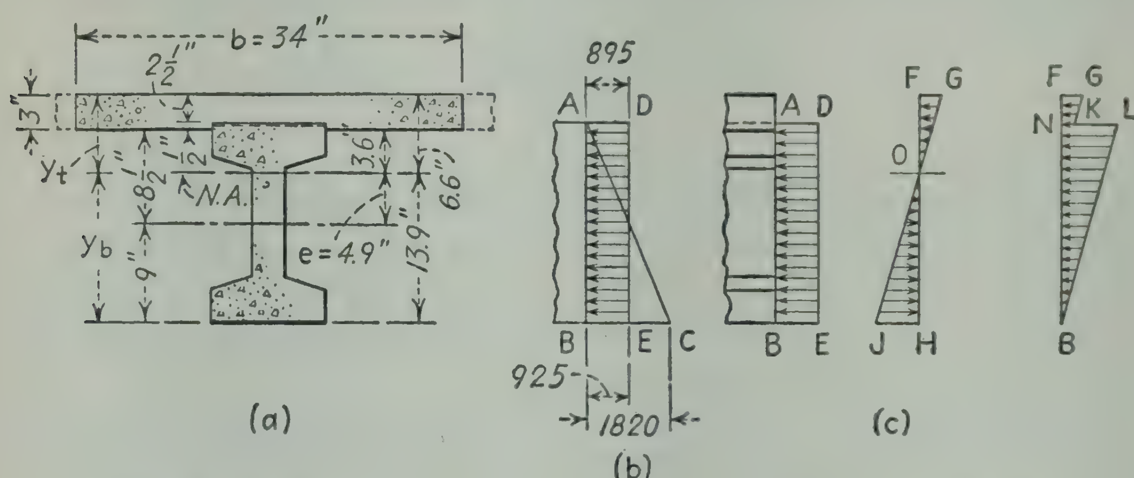


FIG. 13-12. Combined action of prestressed beam and poured slab.

case, then, assume that the effective member is the section shown in Fig. 13-12(a). This, as a monolithic member, has an area of 200 in.^{2}. The center of gravity and distances to top and bottom edges are as shown in the sketch. I about the neutral axis = 9,200 in.^{4}.

Figure 13-12(b) shows the stress condition in the precast beam. The diagram ACB is for prestress alone; $ADEB$, for prestress plus the bending caused by the dead loads of the beam and slab. The latter diagram is reproduced in Sketch (c).

Now, if the compressive stress BE (925 psi) is to be annulled by the bending moment M_L applied to the combined member, the tension JH of Fig. 13-12(c) = f_b, or

$$f_b = 925 = \frac{M_L y_b}{I} = \frac{M_L \times 13.9}{9{,}200}$$

$$M_L = 605{,}000 \text{ in.-lb}$$

The final stress diagram is pictured by $FNBLKG$ of Fig. 13-12(c). Thus the combined member can withstand a total moment of 405,000 in.-lb for dead load plus 605,000 in.-lb for live load, or M_T = 1,010,000 in.-lb. This is naturally larger than the 823,000 in.-lb computed for the precast beam alone. However, one must be careful how he depends upon such combinations. Precast roof units set on the beams and mortared

together, even with the ties shown in Fig. 13-11(b), should not be trusted to increase the bending resistance of the member.

How much live-load deflection should be allowed in a prestressed-concrete beam? Some set a limit of $\Delta = L/800$, where L is the span length. Such statements are dangerous. Prestressing has considerable stiffening effect through the availability of the uncracked concrete section and the resulting larger moment of inertia compared with that of the cracked section of ordinary concrete members. The permissible deflection depends largely upon the use to which the member will be put and upon the engineer's judgment in the matter. If the computed deflection under live load even approaches $L/800$, vibration may become objectionable.

Until cracking occurs, the customary formulas for deflection may be used for computations of deflection. However, the reversed deflection caused by prestress is to be considered. In this case, with a prestress of 1,820 psi at the bottom and zero at the top, for the entire length of the beam, the shortening of the bottom will be about

$$\Delta = \frac{fL}{E_c} = \frac{1{,}820 \times 30 \times 12}{4{,}000{,}000} = 0.164 \text{ in.}$$

where E_c is assumed to be as shown. Where the curvatures are so small, even circular arcs can be treated as though they were parabolas without resulting in serious errors for the purpose of these computations. Therefore, assume that the deflected beam is pictured in Fig. 13-11(d). The ends are rotated an angle α, and the distance AC is assumed to be $\Delta/2$. Then

$$\tan \alpha = \frac{AC}{AB} = \frac{\Delta}{2d} = \frac{0.164}{2 \times 18}$$

Angle ABC = angle DAF. For a parabola, $DE = EF$. Then

$$2x = AF \tan \alpha$$

or

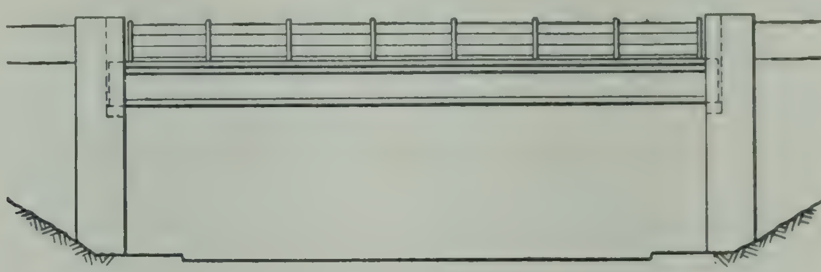
$$x = \frac{180}{2} \times \frac{0.164}{2 \times 18} = 0.41 \text{ in.}$$

The downward deflections caused by D.L. and L.L. can be considered to start from this theoretical elevated position.

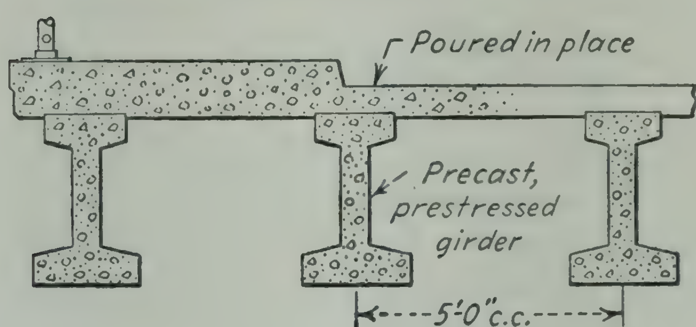
The transformed section based upon the full concrete area plus the "transformed area" of the steel might be used in computing the moment of inertia I, but this does not seem to be desirable in prestressed members. In the first place, referring to Fig. 13-1, how can the springs that are the cause of the impressed compression change the properties of the section any further than through the effect of the compression itself? The wires or cables seemingly should not affect I until the member is stressed

sufficiently to produce tension in it, which is not the desired situation. The inclusion of the steel would increase the computed I, and therefore its omission will be on the side of safety. However, the steel area is so small that the additional effect may not be more than 5 per cent.

If cables in sheaths cut out several square inches of the concrete, as they usually do in posttensioned members, reductions in I should be made accordingly. In pretensioned beams, it is usually advisable to use the full concrete area when computing I.



(a) Side Elevation



(b) Cross Section

FIG. 13-13. A highway overpass.

13-7. Computations for a posttensioned girder. Perhaps the best way to study posttensioning is to work out the design of a problem using it. Therefore, assume that a prestressed concrete highway bridge of the character shown in Fig. 13-13 is desired. The spacing of girders is tentatively made 5 ft c.c. The span is 56 ft. The girders are to be simply supported. Use the equivalent uniform loading of the American Association of State Highway Officials for H-20 trucks. Assume $f'_c = 5{,}000$ psi, $f_c = 1{,}800$ psi, and a safety factor of 2.0 for live load plus impact with a tension of 500 psi. Also assume that the live loads and impact loads are to have a safety factor of 1.25, and the dead load 1.0, with no tension produced in the bottom flanges of the girders. A temporary tension of 200 psi will be allowed in the top flanges during prestressing if this is necessary. Use Roebling prestressing cables.

One of the most difficult parts of the job is that of getting a reasonably good trial section to start with before making a lot of detailed computations. One way to do this is to make a rough preliminary design.

The 5-ft spacing of girders is equivalent to one-half of a highway lane. Therefore, the specified uniform load of 640 plf and the 18,000-lb concentration are to be divided by two for each girder. Then the live-load bending moment is

$$M_{\text{L.L.}} = \frac{320 \times 56^2}{8} + \frac{9{,}000 \times 56}{4} = 251{,}000 \text{ ft-lb}$$

$$\text{Impact } M_I = 251{,}000\left(\frac{50}{L + 125}\right) = 251{,}000\left(\frac{50}{56 + 125}\right)$$

$$= 251{,}000 \times 0.276 = 69{,}000 \text{ ft-lb}$$

$$M_{\text{L.L.}} + M_I = 320{,}000 \text{ ft-lb}$$

Then the design bending moment M_L is $1.25 \times 320{,}000 = 400{,}000$ ft-lb.

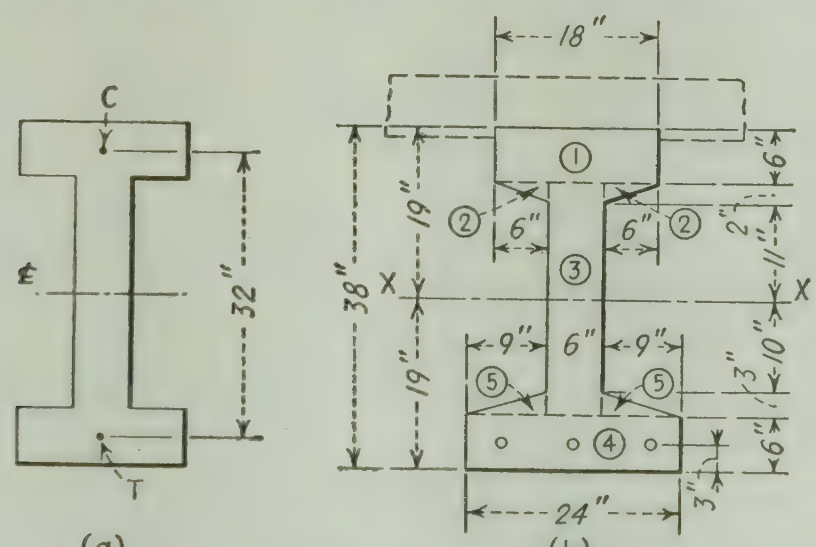


FIG. 13-14.

The slab, which is designed first, will be assumed to weigh 88 psf. Assume that the girder weight is about the same as that of the slab. Then a trial average load for dead load is 180 psf, approximately. A trial M_D is

$$M_D = (180 \times 5)\,\frac{56^2}{8} = 353{,}000 \text{ ft-lb}$$

Then the total bending moment M_G is

$$M_G = (400{,}000 + 353{,}000)12 = 9{,}000{,}000 \text{ in.-lb (approx)}$$

Try the approximate section shown in Fig. 13-14(a).

$$C = T = \frac{M_G}{32} = \frac{9{,}000{,}000}{32} = 282{,}000 \text{ lb}$$

Assuming that the deck slab will participate in the eventual compression flange, the bottom flange is likely to be critical because of the pre-

stressing. For this flange to withstand the compression from a prestressing force at least equal to T, with a guessed average unit stress of 1,600 psi, the area of concrete needed is

$$A_B = \frac{282,000}{1,600} = 176 \text{ in.}^2$$

Add a little to this because the prestressing force may be larger, and a few square inches will be lost in holes for the cables.

Try the section shown in Fig. 13-14(b), with a gross area of 189 in.^{2} in the bottom flange. Make the top flange of the precast beam a little

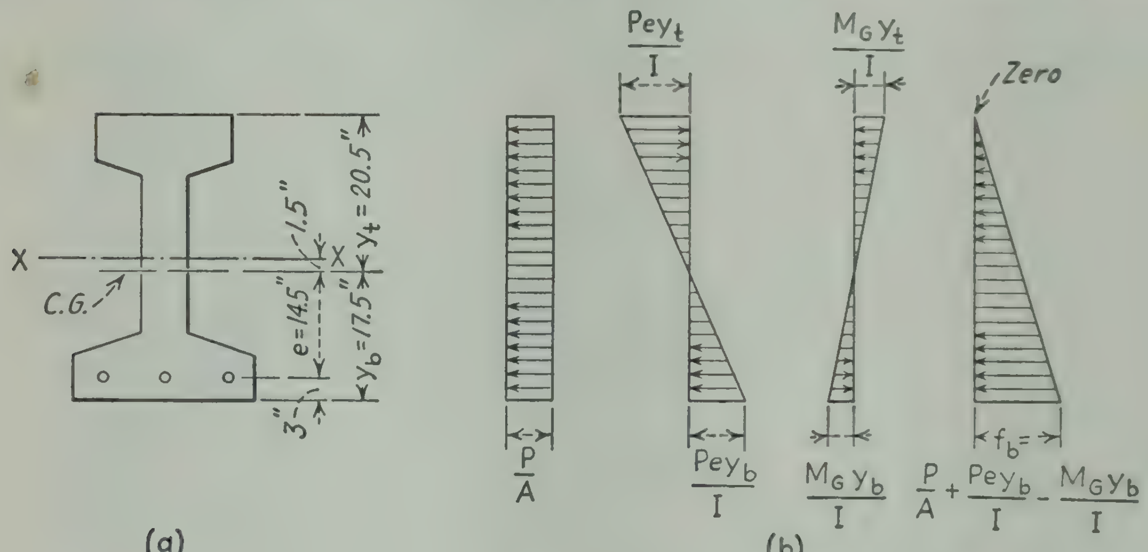


FIG. 13-15.

smaller than the bottom one. Find the center of gravity of the beam about the mid-depth axis X-X. The area of the section shown, with 6 in. deducted for holes for the cables, is 441 in.^{2}. The center of gravity of the member is 1.49 in. below X-X. Call it 1.5 in. Then the moment of inertia of the girder about the center-of-gravity axis is approximately 77,500 in.^{4}. From these,

$$r^2 = \frac{I}{A} = \frac{77,500}{441} = 176 \text{ in.}^2$$

With posttensioning, advantage can be taken of the fact that the prestressing force tends to curve this precast girder upward but the weight of the member opposes this action. The section is pictured in Fig. 13-15(a). It weighs $^{441}/_{144} \times 150 = 460$ plf. Then its dead-load moment at the center is

$$M_B = \frac{460 \times 56^2 \times 12}{8} = 2,160,000 \text{ in.-lb}$$

Now assume that the bottom fiber stress f_b is to be a maximum and the stress at the top is to be zero under the action of the prestressing load P

and the dead-load moment of the girder. Then, for the top fiber, as indicated in Sketch (*b*),

$$f_t = 0 = \frac{P}{A} - \frac{Pey_t}{I} + \frac{M_B y_t}{I}$$

or

$$\frac{P}{A} = (Pe - M_B)\frac{y_t}{I} \tag{13-8}$$

When Ar^2 is substituted for I, this gives

$$P = \frac{M_B}{e - (r^2/y_t)} \tag{13-9}$$

The desired eccentricity of P at the center of the girder is 14.5 in., as shown in Fig. 13-15(*a*). Substitute the numerical values in Eq. (13-9). Then

$$P = \frac{2{,}160{,}000}{14.5 - (176/20.5)} = 366{,}000 \text{ lb}$$

This does not need to be modified for shrinkage to any great extent, because the girder is supposed to be well cured before the prestressing force is applied. Furthermore, compressive shortening of the member will take place as the cables are tightened. However, the magnitude of P might be increased slightly to allow for creep and some small subsequent shrinkage. In this case, call $P = 375{,}000$ lb in order to have a little reserve.

If P is equally divided between three Roebling cables, $P_1 = 125{,}000$ lb. For this, use a 1⅜-in. cable, as shown from Table 13-1.

TABLE 13-1. Roebling Galvanized Prestressed Concrete Strand for Posttensioned Design

Diameter, in.	Weight per ft, lb	Area, in.²	Min guaranteed ultimate strength, lb	Design load, lb
0.600	0.737	0.215	46,000	26,000
0.835	1.412	0.409	86,000	49,000
1	2.00	0.577	122,000	69,000
1⅛	2.61	0.751	156,000	90,000
1¼	3.22	0.931	192,000	112,000
1⅜	3.89	1.12	232,000	134,000
1½	4.70	1.36	276,000	163,000
1 9/16	5.11	1.48	300,000	177,000
1⅝	5.52	1.60	324,000	192,000
1 11/16	5.98	1.73	352,000	208,000

These strands are fabricated from hot-dip galvanized wire, which assures complete protection against corrosion without further treatment.

For the top fiber,

$$f_t = \frac{P}{A} - \frac{Pey_t}{I} + \frac{M_By_t}{I}$$

$$f_t = \frac{375{,}000}{441} - \frac{375{,}000 \times 14.5 \times 20.5}{77{,}500} + \frac{2{,}160{,}000 \times 20.5}{77{,}500}$$

$$f_t = 20 \text{ psi tension} \qquad \text{(satisfactory)}$$

For the bottom fiber,

$$f_b = \frac{375{,}000}{441} + \frac{375{,}000 \times 14.5 \times 17.5}{77{,}500} - \frac{2{,}160{,}000 \times 17.5}{77{,}500}$$

$$f_b = 1{,}590 \text{ psi compression} \qquad \text{(a bit conservative)}$$

The girder alone must withstand the weight of the concrete deck and any diaphragms that are used. The effect of the latter will be small.

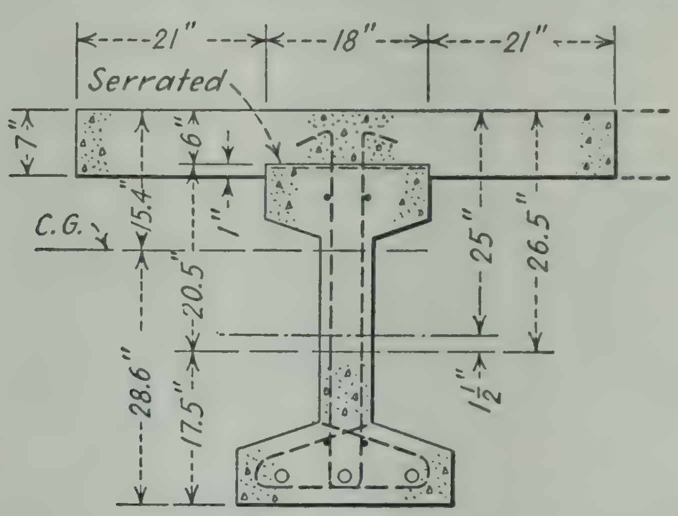


FIG. 13-16.

This bending moment for the deck alone is

$$M_D = \frac{(88 \times 5) \times 56^2 \times 12}{8} = 2{,}070{,}000 \text{ in.-lb}$$

The resulting bending stresses are

$$f_t = \frac{2{,}070{,}000 \times 20.5}{77{,}500} = 550 \text{ psi compression}$$

$$f_b = \frac{2{,}070{,}000 \times 17.5}{77{,}500} = 470 \text{ psi tension}$$

The resultant stress at the bottom is $1{,}590 - 470 = 1{,}120$ psi compression, and that at the top is $550 - 20 = 530$ psi compression. Obviously, there is plenty of reserve strength for forms and other construction loads.

After the deck has cured, the shear keys and ties pictured in Fig. 13-16 will enable the floor to participate in resisting compression. How-

ever, the full development of such a large area may not be trustworthy, but try it to see what happens. The width of 18 in. plus $4t$ each side would give $b = 18 + 2 \times 4 \times 7 = 74$ in., but 60 in. c.c. of girders is naturally an upper limit. The center of gravity and other data for this assumed T beam are shown in Fig. 13-16.

The live-load-plus-impact bending moment for design purposes was previously computed as 400,000 ft-lb. Therefore,

$$f_{tL} = \frac{400{,}000 \times 12 \times 15.4}{192{,}000} = 385 \text{ psi compression}$$

$$f_{bL} = \frac{400{,}000 \times 12 \times 28.6}{192{,}000} = 715 \text{ psi tension}$$

Therefore, the total compression in the top of the precast member is obviously well within the 1,800 psi allowed. The final stress in the bottom is

$$f_b = 1{,}120 - 715 = 405 \text{ psi compression}$$

This is unnecessarily conservative.

Now test the precast girder if it supports M_L without help from the slab.

$$f_t = \frac{400{,}000 \times 12 \times 20.5}{77{,}500} = 1{,}270 \text{ psi compression}$$

$$f_b = \frac{400{,}000 \times 12 \times 17.5}{77{,}500} = 1{,}080 \text{ psi tension}$$

The resultant stresses will then be

$$f_t = 530 + 1{,}270 = 1{,}800 \text{ psi compression}$$
$$f_b = 1{,}120 - 1{,}080 = 40 \text{ psi compression}$$

Thus, even when acting alone, the girder probably can support the load safely. However, the design is somewhat too conservative if the deck is relied upon, as it may be to a considerable extent.

The original prestressing force of 366,000 lb can be reduced a little, or the bottom flange might be decreased. Try using $P = 330{,}000$ lb with the girder otherwise unchanged. For the prestressing,

$$f_t = \frac{330{,}000}{441} - \frac{330{,}000 \times 14.5 \times 20.5}{77{,}500} + \frac{2{,}160{,}000 \times 20.5}{77{,}500}$$
$$= 50 \text{ psi tension}$$

$$f_b = \frac{330{,}000}{441} + \frac{330{,}000 \times 14.5 \times 17.5}{77{,}500} - \frac{2{,}160{,}000 \times 17.5}{77{,}500}$$
$$= 1{,}340 \text{ psi compression}$$

Combined stresses for the top, with the deck acting as part of the flange,

need not be recomputed. The final stress in the bottom flange, with the deck acting, is

$$f_b = 1{,}340 - 470 - 715 = 155 \text{ psi compression}$$

This will be accepted.

Having this revised force $P = 330{,}000$ lb, check the combined member to see what safety factor is provided for the live load at the cracking moment. As computed previously, the compression in the bottom flange under prestress and deck loads is $1{,}340 - 470 = 870$ psi compression. This, plus a tension of 500 psi allowed, gives a usable fiber stress of $870 + 500 = 1{,}370$ psi. Then

$$f_b = 1{,}370 = \frac{M'_L \times 28.6}{192{,}000}$$

$$M'_L = 9{,}200{,}000 \text{ in.-lb}$$

This gives

$$\text{S.F.} = \frac{M'_L}{M_{\text{L.L.}+I}} = \frac{9{,}200{,}000}{320{,}000 \times 12} = 2.4$$

This exceeds the specified value of 2.0, and it will be accepted.

One arrangement of reinforcement in addition to the cables is pictured in Fig. 13-16. The longitudinal bars are for the purpose of tying the other reinforcement together as a cage before placing it.

The cables must support a tension of 110,000 lb each. Therefore, use 1¼-in. Roebling cables.

At the ends of the girders, the prestressing force will be acting alone. Equation (13-6) gives the required eccentricity if the stress at the top is to be zero.

$$e = \frac{r^2}{y_t} = \frac{176}{20.5} = 8.6 \text{ in.} \qquad \text{Call it 8.5 in.}$$

The two cables in the outer portions of the bottom flange are to remain straight, but the central one can be bent up in the web toward the ends. This cable can be located as follows:

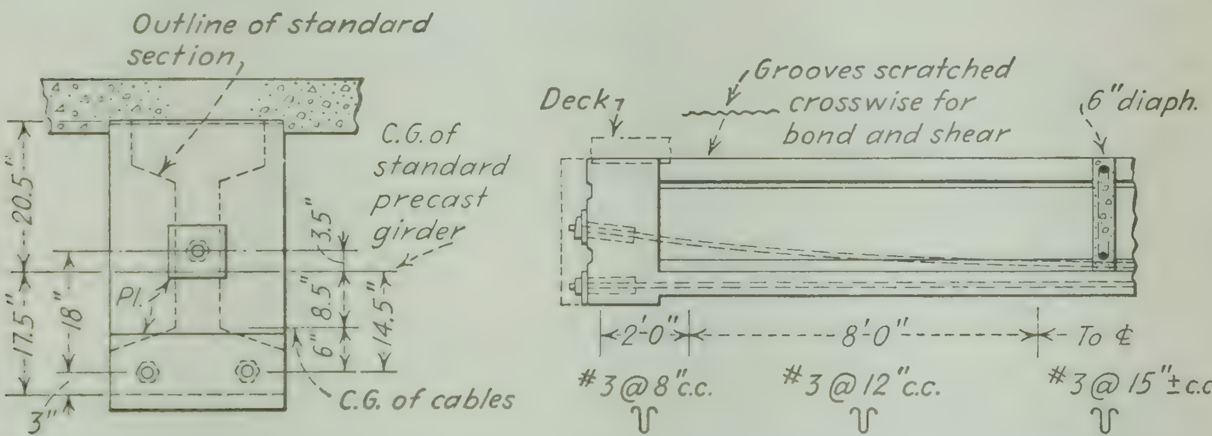
$$2 \text{ cables} \times (14.5 - 8.5) \text{ in.} = 1 \text{ cable} \times x \text{ in.} \qquad (x = 12 \text{ in.})$$

Therefore, locate the central cable as shown in Fig. 13-17(*a*). Curve it as a parabola, as indicated in Sketch (*b*). Check the stresses at the quarter point of the span in order to make sure that they are satisfactory. Change the slope of the cable if necessary.

The end of the girder should be thickened to accommodate the anchorage details and to prevent crushing, buckling of the web, and excessive vertical or longitudinal shearing stresses. Add web reinforcement and

ties as indicated in Fig. 13-17(*b*). It is important to tie together the end under and around the cable anchorages so as to prevent bursting or spalling of the concrete. This reinforcement may be spirals or circles around the anchorages together with a cage of small bars to knit the end together.

The cables are to be in metallic sleeves about $1\frac{3}{4}$ in. in diameter. After prestressing, the space between them and the cables is to be grouted. Holes or connecting tubes are to be left for this purpose. As indicated in Fig. 13-17(*b*), the ends of the anchorage details are to be encased in concrete.



(a) End View (b) Partial Side Elevation

FIG. 13-17.

The curved cable actually tends to lift up the central part of the girder with respect to the ends as the cable tries to straighten out. Sometimes this principle can be utilized as a direct support for the transverse loads on a member. If the cable is made to follow a parabolic curve, as in Fig. 13-17(*b*), it acts somewhat like a self-anchored suspension-bridge cable.

In this case, assume that the pull equals 110,000 lb, neglecting the effect of friction on the curved sheath when the cable is tightened. With a mid-ordinate of 1.5 ft, the slope of the cable at the end is

$$\tan \alpha = \frac{2 \times 1.5}{28} = 0.107$$

Then, approximately, the vertical component of the cable pull is

$$P_v = 110{,}000 \times 0.107 = 11{,}800 \text{ lb}$$

This tends to relieve the vertical shear in the concrete near the end of the girder.

In some cases, the cables can be made straight and sloping from the end to a point part way along the member—as at the diaphragm of Fig. 13-17(*b*)—then kinked at this point and run horizontally to a similar

kink at the opposite side. This cable exerts an upward force at the kink, and it also relieves the shear in the concrete near the end of the member by the amount of the vertical component of the cable tension.

Ordinarily, an estimate of the friction produced by stretching a curved cable within its sheath may be made by assuming that the curve is circular and that $T = pr$, where T = the cable tension for design at the center, p = the normal pressure in pounds per linear foot, and r = the radius of the curve in feet. With a coefficient of friction $f = 0.4$, one may estimate the extra tension needed to move the cable in its sheath. In this case, the mid-ordinate $m = c^2/8r$, where c is the chord length, 56 ft. Then

$$r = \frac{c^2}{8m} = \frac{56^2}{8 \times 1.5} = 261 \text{ ft}$$

$$p = \frac{T}{r} = \frac{110{,}000}{261} = 420 \text{ lb per ft}$$

The frictional force for one-half is

$$F = 420 \times 0.4 \times 28 = 4{,}700 \text{ lb}$$

This assumes jacks operating simultaneously at both ends of the cable.

Another way to approximate the frictional resistance to tightening of a curved or kinked cable is to multiply the vertical component of the cable tension at the end by the coefficient of friction. Of course, if the cable is jacked at one end only, the computed resistance should be doubled. Therefore, the jacks at the ends of this curved cable might well be made to apply a force of 110,000 + 5,000 = 115,000 lb if both ends are jacked simultaneously.

Roebling cables were purposely chosen for these girders because they can be stretched as units, even when curved. The jacking equipment and anchorages are shown in Fig. 13-18. In the case of Magnel cables, with the wires about $5/16$ in. c.c., and with the wires tightened two at a time, some type of spacer is needed to hold them in position. Even when the wires are tightened in series from the inside of the curve outward, there is likely to be considerable resistance to the stretching of the individual pairs when curved.

Furthermore, it would be desirable to tension all cables simultaneously. If one in this girder is tensioned, it applies 110,000 lb pressure to the concrete, which compresses accordingly. As the others are tightened the compressive deformation increases, thus tending to relieve the tension in the first cable. Either they should be tensioned a second time or they should be tightened enough more at first so that the desired tension will remain after the compressive shortening of the concrete has occurred.

Notice again that bond and shear problems in prestressed concrete are

not the same as in conventional reinforced concrete until the concrete has cracked, which is an overload condition to be avoided. The members are treated as though they were homogeneous. Furthermore, although not strictly accurate, it seems to be satisfactory to treat the concrete as though it were elastic within the range of the working stresses used.

The plans for casting the girders are to be made in accordance with the requirements of the particular situation. The girders may be cast on

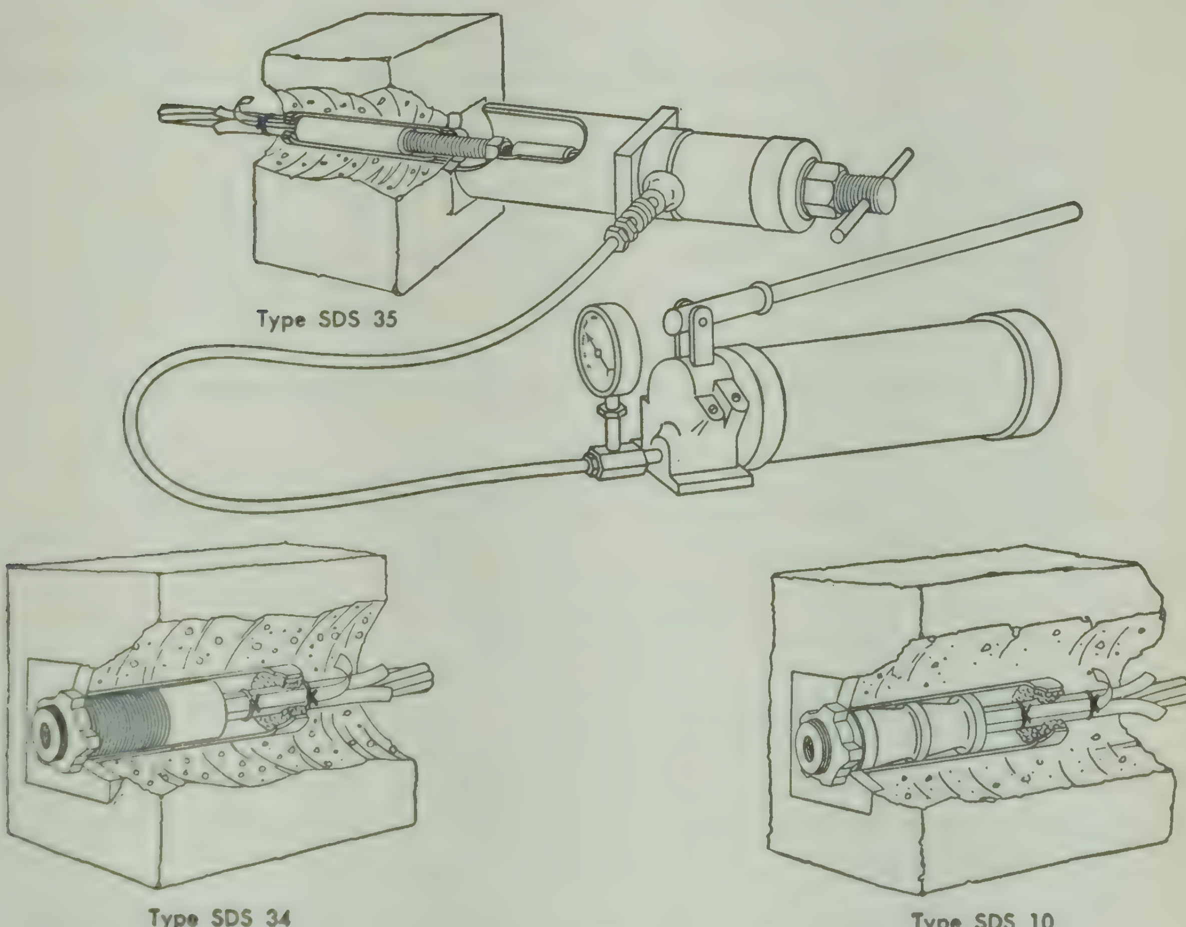


FIG. 13-18. Jacking equipment for Roebling simplified posttensioning method.

one approach, then skidded or lifted into place. They may be cast below the structure and lifted into position. They may even be manufactured elsewhere and transported to the site. In any case, the stresses caused by erection procedures are to be investigated and provided for.

As with ordinary reinforced-concrete construction, many beams of varying dimensions and make-up might be used for this particular service. The one designed here is shallow but not unduly so. It might be made a little lighter and shallower, but the saving, if any, would not be great.

13-8. Miscellaneous construction. Many adaptations of the principles of prestressing can be made. Some will be explained briefly.

Beams of considerable length and strength can be made by using a

series of accurately faced concrete blocks with prestressing wires or cables somewhat as shown in principle in Fig. 13-19. The blocks are assembled and then posttensioned. Here the blocks are of three kinds. The end ones *a* have some kind of detail that will accommodate the anchorages, and they are strong enough to resist the local pressures. The intermediate blocks *b* are hollow so that the cables can slope down to the "kink point" at block *c*, from which they extend straight across to a corresponding block beyond the center. Generally the joints between the blocks of the precast beams are not mortared. Friction due to the pressure resists most of the transverse shear. In this case, the cables are bare

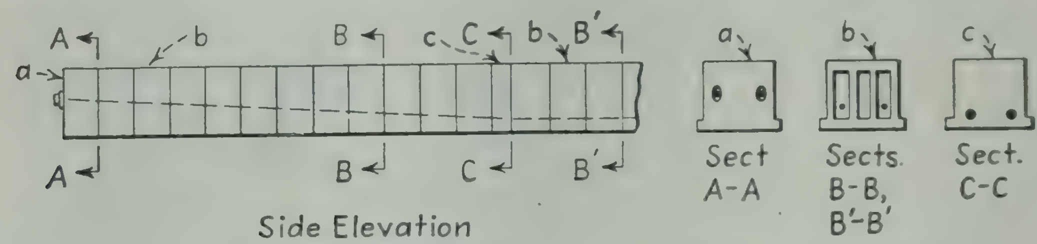


FIG. 13-19. One arrangement for a prestressed block beam.

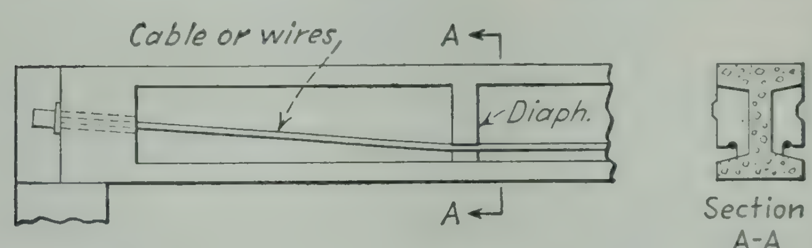


FIG. 13-20. Precast posttensioned unit with unencased steel.

inside the blocks. The lips on the bottoms of the blocks provide for mortaring or concreting between adjacent beams when a concrete topping with wire mesh is poured over the erected units.

A different arrangement for a precast prestressed-concrete beam is pictured in Fig. 13-20. This is a posttensioned unit. The cables are not encased but are alongside the web, as pictured in section *A-A*. The units are to be set side by side with the diaphragms interlocked; then a 2-in. layer of concrete with wire mesh is to be placed across the top. If desired, the diaphragms and ends could have a hole in them transversely with respect to the beam so that, after erection, rods or cables could be threaded through them and all could be tensioned so as to pull the beams tightly together in order to make them act more like a monolithic unit.

Whether or not the use of bare cables, bars, or wires is desirable will depend largely upon the particular case in which they are to be incorporated. Is there danger of serious corrosion?

Poles, piles, roof slabs, wall panels, and many similar small units are especially well adapted for manufacture as precast prestressed units,

especially with bonded wires. Although a faint tinge of rust may be beneficial for the bond on wires in such units, be careful that no serious rusting has occurred before they are used. The bond resistance of embedded galvanized wires should be ascertained thoroughly by tests and experience before it is trusted too much.

Prestressed-concrete tanks, and even prestressed pipes, are very useful. The bars, cables, or wires are generally posttensioned and then

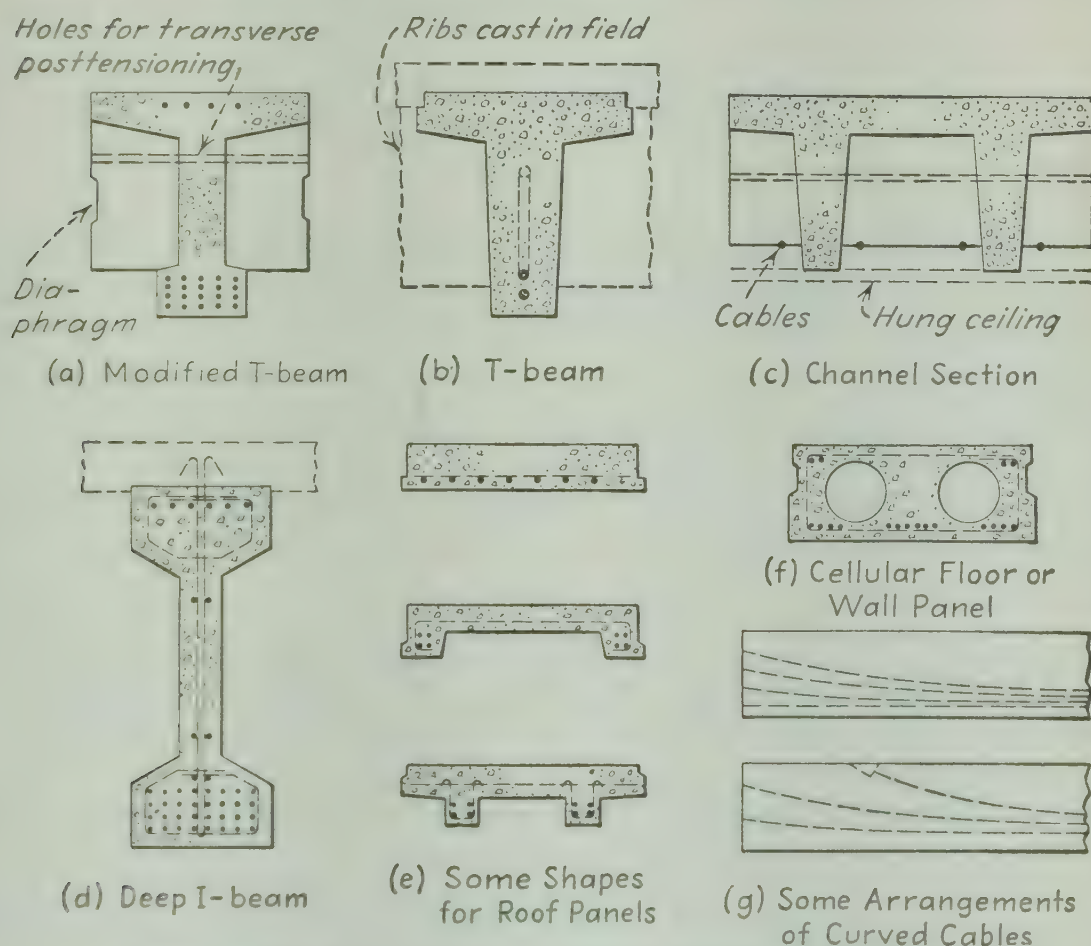


FIG. 13-21. Various types of precast prestressed members.

covered with gunite, mortar, or concrete for protection. The purpose is not to resist bending but to squeeze the concrete so that shrinkage, temperature changes, and ring tension cannot cause even minute cracks that would permit leakage through the concrete.

Some other sections that are practicable as prestressed-concrete beams are shown in Fig. 13-21.

Continuous beams and girders are more difficult to prestress than are simply supported ones. Komendant[1] shows some European practice for such structures. In general, they are posttensioned. Some principles

[1] August E. Komendant, "Prestressed Concrete Structures," McGraw-Hill Book Company, Inc., New York, 1952.

that may be used are indicated very sketchily in Fig. 13-22. It is probable that continuous members should be poured in place; then the cables can be tightened. However, since reversal of stress, fatigue, and rapid variations of stress are likely to occur in continuous structures, prestressing of the particular kind of structure desired and the details and pro-

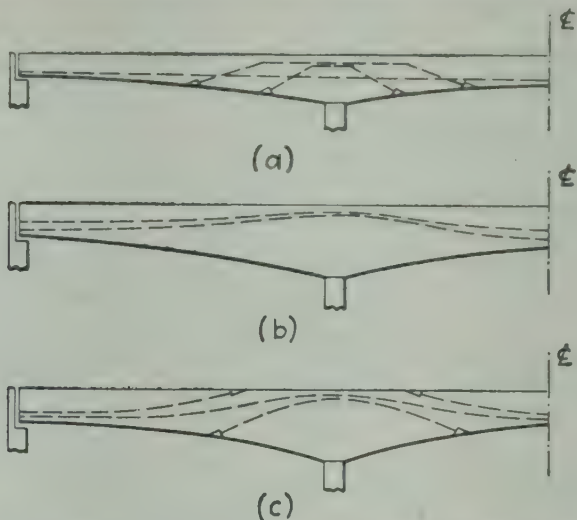


FIG. 13-22. Some studies of schemes for posttensioning a three-span girder.

cedures to be used in building it should be studied very carefully before prestressing is utilized in them.

Practice Problems

For the following problems, assume these data: f'_c = 5,000 psi, f_c = 1,800 psi, yield-point stress of reinforcement other than Roebling cables = 250,000 psi, f_s in wires = 150,000 psi, size of wires = 0.196 in., and all members are simply supported. Design for the condition of no tension in the bottom flange.

13-1. Design a precast pretensioned roof panel 24 in. wide and 10 ft long to support a superimposed uniform load of 60 psf.

13-2. Design a pretensioned T beam like that of Fig. 13-21(a) to have a top width of 24 in. and a span of 45 ft. It is to support a superimposed uniform load of 125 psf and a concentration of 500 lb at its center.

13-3. Design a pretensioned I beam similar to that of Fig. 13-10(a) for a span of 32 ft, for a spacing of 5 ft c.c., and to support a 4-in. slab and a uniform live load of 125 psf.

13-4. Design a posttensioned I beam of the type shown in Fig. 13-20 for a span of 32 ft and superimposed load of 600 plf. Use Roebling cables.

13-5. Design a girder for a highway bridge like that of Fig. 13-13 if the span is 64 ft, the deck is a 7-in. slab, the girders are 5 ft c.c., the average uniform live load is 64 psf, a concentrated live load of 9,000 lb is at the center of the span, and the impact factor is 0.3. Use a safety factor of 1.2 applied to the live load.

14
ARCHITECTURAL AND MISCELLANEOUS DETAILS

14-1. Basic principles underlying architectural treatment. The design of the architectural features of any reinforced-concrete structure must be based upon the nature of concrete itself and upon the processes that are to be used in the building of that structure. In other words, concrete is a substance that, in a plastic state, is placed against forms that give to it whatever shape the forms possess so that, after the concrete has set, these outlines and shapes are permanently maintained by the concrete. Therefore, the details of the whole subject must be thought about in terms of forms, simplicity, pouring schedules, and other practical matters of construction so that the finished structure will become one harmonious entity whose parts automatically blend to produce the desired architectural ensemble. Figure 14-1 is an example of this.

Furthermore, the general architectural conception of the structure should be based upon these considerations, along with those of beauty, proportion, surface texture, permanence, color, economy, and functional effect. The designer should abandon many of the ideas that have been developed through the past in using stone and brick masonry which emphasize moldings, cornices, decorative carvings, and other features for which concrete is not suited. Concrete is a special material, and the architecture of a concrete structure should be adapted to it. For instance, Fig. 14-2 shows a four-ribbed reinforced-concrete arch bridge which has been designed upon this basis.

Much of the architectural detail of a concrete structure should also be based upon a consideration of light and shadows. Excessively large flat surfaces should be avoided; they should be broken up by horizontal or vertical markings or moldings, by moldings in both directions, or by offsets. Lines should be straight and distinct, avoiding intersections at large obtuse angles. The special detail features should be coordinated with the necessities of the building operations so that the construction

FIG. 14-1. Los Angeles County Hospital, Los Angeles, Calif. (*Courtesy of Portland Cement Association.*)

FIG. 14-2. Dyckman St. Arch, Henry Hudson Parkway, New York City. (*Courtesy of Portland Cement Association.*)

joints can be placed at markings or offsets, the joints themselves thus being indistinguishable.

An interesting treatment of large surfaces is shown in Fig. 14-3. With the base and panel treatments, one hardly realizes that this might have been an unattractive retaining wall.

Another manner of breaking up large surfaces is shown in the treatment of the abutments of Fig. 14-2. The surface gives a sort of clapboard effect with a multitude of small horizontal shadows.

FIG. 14-3. Retaining wall at portal of Figueroa Tunnel, Los Angeles, Calif. *(Courtesy of Portland Cement Association.)*

In planning details to accommodate formwork, one should remember that plywood is a common lining for the forms for architectural concrete. However, the greatest reasonable length of plywood is about 8 ft (maximum 10 ft). If this is the limit between moldings, offsets, or other details, it will avoid the marks that might otherwise be visible in the concrete at the junctions of the ends of abutting pieces. Also, with the panels of plywood laid with their long dimension horizontally, joints must often occur between the upper and lower pieces, but these joints can be braced thoroughly. If slight marks occur in the concrete at such points, they are not usually objectionable because of their horizontal position. However, care should be used to line up these joints across or around the structure. A patchy effect should be avoided.

14-2. Moldings and minor details of forms. The moldings or markings that are used to break up flat surfaces should be detailed properly. It is very easy to nail strips on the insides of the forms, thereby causing recessed moldings or markings in the finished concrete, but it is rather difficult to make recesses in the forms themselves. It is also better to recess these cuts in the concrete, because small projections are likely to be damaged.

The following points, some of which are illustrated in Fig. 14-4, should also be considered:

1. Wide, shallow recesses do not cast sharp shadows.

2. Horizontal, recessed moldings or markings should be V-shaped, beveled outwardly at the top and bottom, or horizontal at the top and

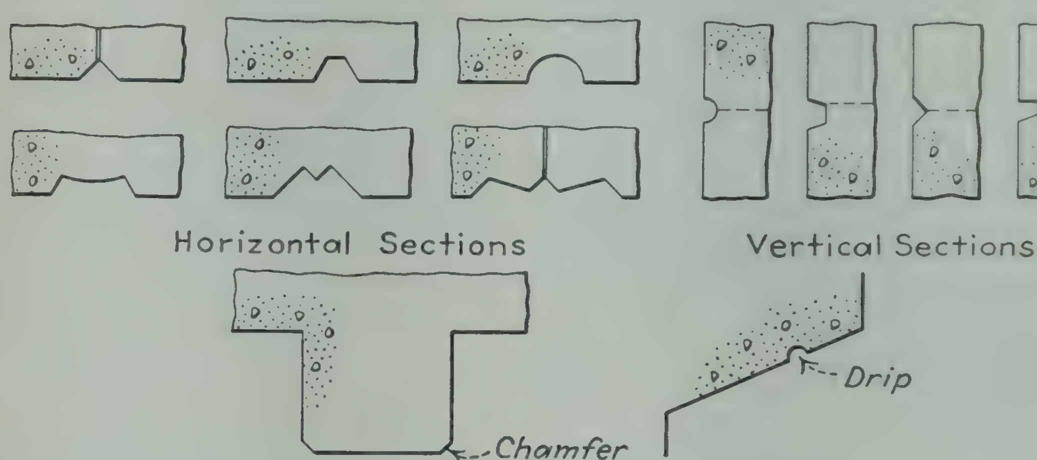


FIG. 14-4.

beveled at the bottom. Horizontal ledges are likely to collect and hold dirt; they are also more difficult to fill so as to produce sharp edges, unless the hydrostatic head of the concrete is considerable.

3. Sharp acute angles in moldings are difficult to fill properly and may spall off easily.

4. Chamfering of projecting corners is desirable, but filleting is difficult at reentrant angles.

5. Small, recessed V strips or halfrounds (1 to 1½ in. deep) should be used at construction joints. If this is not permissible, a straight finish strip should be attached to the forms at the line of the joint so as to avoid a ragged line in the concrete. Sometimes the top of the forms can be a straight edge along which the concrete can be finished with a trowel for a depth of about 1 in.

6. Lining up of moldings in successive pours is very important; so is proper mitering of the form strips at intersections.

7. Vertical keyways should be made, if possible, by fastening the key strip on the inside of the forms of the first pour.

8. Moldings must be sloped along their edges sufficiently to facilitate the removal of the forms.

9. The depths of moldings and V cuts automatically affect the location of the reinforcement, because the rods should have 1½ to 2 in. of cover at the deepest part of the recess. When the cuts are too deep, the reinforcement that crosses behind them has to be bent in order to keep the rods near enough to the main surface.

A very simple but effective treatment at a contraction joint in a retaining wall is shown in Fig. 14-5. The grooves are only 1½ in. deep, but they are sufficient to cast sharp shadows.

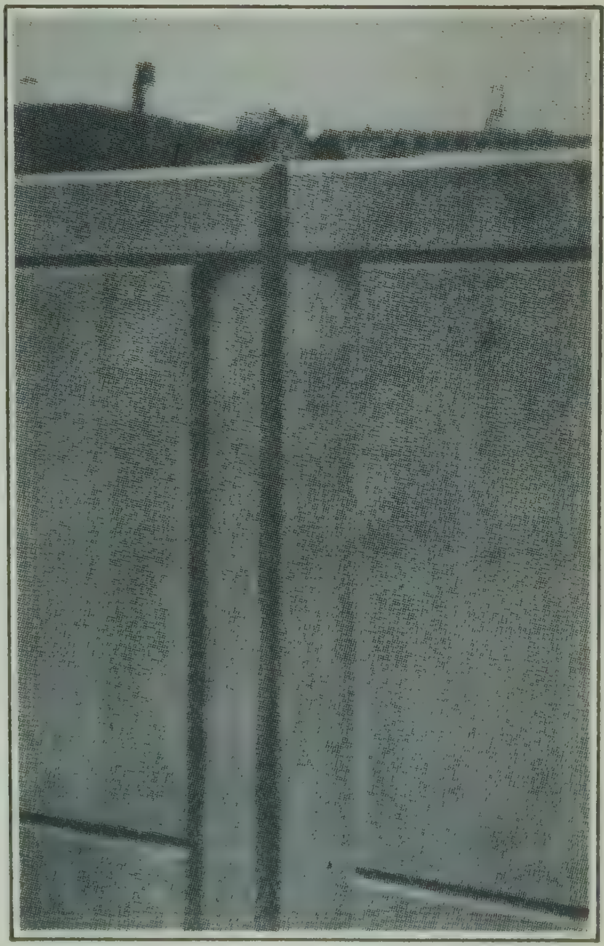


FIG. 14-5. Details at expansion joint.

14-3. Surface finish and appearance. It is ordinarily desirable to have a smooth finish on the surface of the concrete. Plywood or similar materials generally serve this purpose very well. However, in the case of buildings or other structures that will be seen at close range, it is often desirable to rub the concrete with a carborundum stone and water so as to remove all objectionable irregularities and variations in the pattern or the texture of the surface.

Bushhammering or other treatment that destroys the mortar coating over the aggregate is sometimes used on massive structures, but the effect upon durability is open to question. However, this type of finish has the advantage of being very difficult to deface with chalk and similar materials; it also can be used in securing paneled effects, but the aggregate should be suitable in color, about ¾ to 1 in. in size, a good gravel or crushed stone. The general layout must be simple, and the bushhammering must be done thoroughly if the effect is to be satisfactory.

When any painting of exterior surfaces is desirable, a wash or paint of cement is recommended by the Portland Cement Association. Consult the manufacturers for other suitable paints.

The finishing of horizontal surfaces should be done by screeding, using a wood float, or troweling moderately. Excessive troweling or other

working that flushes too much of the mortar to the surface will be likely to cause "crazing"—very fine cracking of the surface.

The builder should be very careful to use aggregates that are properly graded and that are uniform in color. Otherwise, variations between pours will be distinguishable. Also, he should be careful to avoid segregation of the aggregates during the placing of the concrete, or else the bottom of a pour will appear coarse while there is an accumulation of finer material at the top of the previous one. This requires careful placing, the use of a concrete that is not too dry or too wet, and careful spading of the surface. Generally, the use of vibrators which are applied to the forms or in the body of the wet concrete is not sufficient to guarantee a good surface regardless of all other conditions. Great care must be exercised when using vibrators to make sure that the forms do not become bulged, displaced, or leaky at joints.

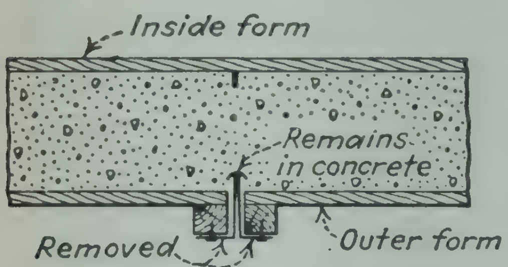


FIG. 14-6.

Another thing to be avoided in the surface of concrete is cracking due to shrinkage or variations in temperature. Adequate reinforcement will prevent this usually if the pours are not too large and if the details are such that the necessary deformation can occur at vertical and horizontal contraction joints along the lines of the moldings. Doors, windows, and other openings in walls are likely to cause such cracking of the concrete because they are points of relative weakness. In these cases, "dummy joints" such as that of Fig. 14-6* may be used together with a decrease in the reinforcement across the joint, thus inviting the crack to occur at a predetermined concealed point. In very long buildings, it is even desirable to divide the structure into units about 200 ft or less in length by using expansion joints which completely isolate adjacent units.

Other features that are desirable in order to maintain a satisfactory appearance of concrete surfaces are the following:

1. Waterproofing of the back of the concrete when it is subjected to the penetration of moisture—as in retaining walls.

2. Avoidance of metallic accessories which are almost certain to cause streaking.

3. Provision of a wash (or slope) on the tops of parapets so that drainage will carry dirt back away from the surface which one wishes to maintain in a satisfactory condition.

4. Provision of a "drip," or groove, under cantilevered construction

* A. M. Young, Crack Control in Concrete Walls, *Eng. News-Record*, Aug. 11, 1938.

as shown in Fig. 14-4. This is very important to prevent streaking of the surface due to rains.

14-4. Temperature reinforcement. If a piece of concrete could be rigidly connected to immovable supports, and if this piece could be subjected to a fall in temperature of 1°F, the resultant tension in the concrete due to the latter's attempt to contract would be

$$f_c = E_c \frac{\Delta L}{L} = E_c \omega$$

where ω = the coefficient of thermal expansion or contraction. Then, if E_c = 3,000,000 psi and ω = 0.000006, f_c = 18 psi. However, since the range of temperature variations is frequently ± 60°F, the corresponding maximum stresses in this concrete would be $f_c = \pm 18 \times 60 = \pm 1{,}080$ psi. From this, it is clear that the compressive stresses alone would not be critical, but the concrete would surely crack if it were subjected to such a tensile stress. The amount of steel that would be needed to carry this tension is clearly excessive.

In practice, structures are not often fully restrained this way unless they are keyed to rock foundations. However, if they are very long, there may be sufficient restraint or frictional resistance to cause them to crack in tension. Therefore, the engineer should build his structures in short units so that these cracks cannot develop.

When one considers that ω for steel is practically the same as for concrete, he realizes that the use of rods to prevent cracks due to temperature variations is not for the purpose of stopping the expansion and contraction but merely to knit the structure together and to avoid localized cracks. The amount of steel that is required to do this is not definite, but it may be assumed to be about 0.0025 times the cross-sectional area of the concrete. However, parapets and similar relatively thin parts which are attached to more massive structures should be reinforced more heavily.

The temperature reinforcement should be placed near the surfaces of concrete structures, and especially near edges where cracks may start. When only one side is exposed, about 60 to 70 per cent of the steel should be near the exposed side, unless the walls are thin. Even when the foundations are on rock, the parts of the structure that are above the footings should be provided with adequate joints and reinforcement.

Special attention should be given to concrete aprons at doorways; pavements of areaways; sidewalks that adjoin foundation walls; basement walls of poured concrete that are largely or entirely exposed to the atmosphere, as at the back of a structure on a lot that slopes down and toward the rear; and long monolithic walls like that of Fig. 14-7(a), par-

ticularly when they have southern exposure in the northern temperate zone.

An apron should be seated on the foundation wall, and perhaps doweled to it, when heavy loads will pass over it. The apron should be cut by contraction joints into sections about 10 to 15 ft wide. Pavements of areaways on earth should be free from main structures, and they should

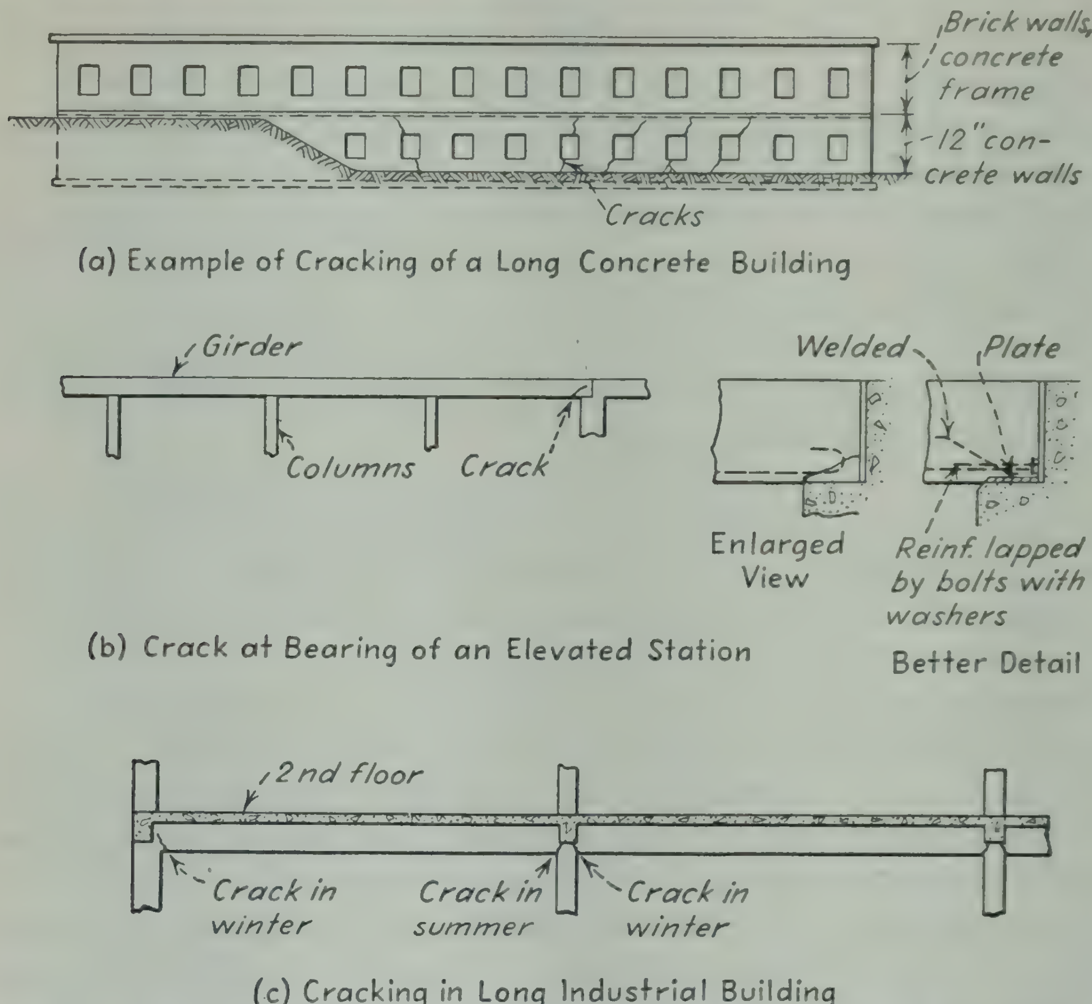


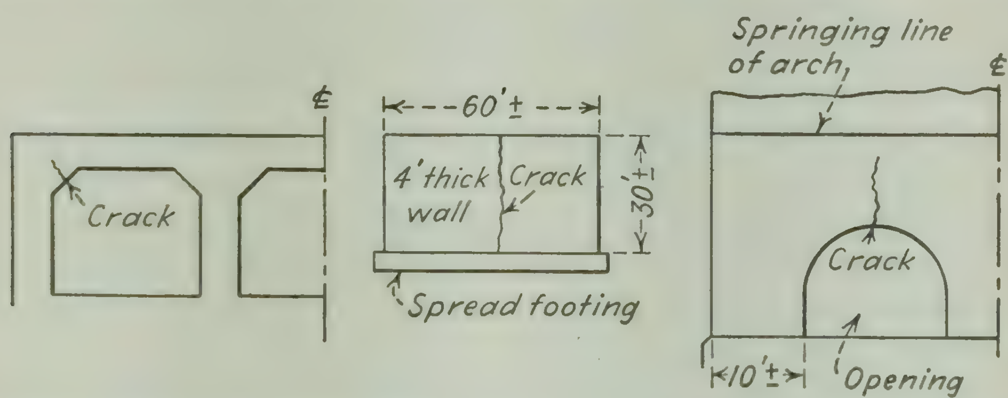
FIG. 14-7. Examples of cracking apparently caused by changes of temperature.

be cut into sections approximately 10 by 20 ft in size. Adjoining sidewalks should be cut free from foundation walls, and they should have contraction joints 5 to 10 ft c.c. Long monolithic walls of basements, and such walls of first stories without basements, when in cold climates, should have contraction joints at 20- to 30-ft intervals, even though this is troublesome in construction. Such joints should also be near corners, at offsets, and at points of weakness where cracks may occur.

Structures that are narrow and high—such as piers—do not need steel to resist cracking due to temperature to the same extent as do long ones. They are free to expand or contract vertically. Their dead load will not let cracks open up.

When concrete is used in places that are subjected to very high tem-

peratures,[1] there is a far different problem. Ordinary concrete may be weakened by dehydration above 500 to 600°F; at 1200°F it may be almost worthless. No amount of reinforcement will stop this action. On the other hand, concrete made with Lumnite and low-silica aggregate may withstand about 1000°F. If ladles of molten metal are to be placed on or alongside concrete surfaces, these should be protected by replaceable materials such as bricks. Unless ventilation can remove the heat, even these are inadequate; *e.g.*, an uncooled concrete foundation slab for a



(a) Crack in Bridge Abutment (b) Crack in End of Heavy Bin Support (c) Crack in Arch-Bridge Pier

FIG. 14-8. Examples of shrinkage cracking of heavy structures.

reverberatory furnace, if placed on earth, will have its temperature gradually increased to somewhere near that of the furnace in spite of the latter's lining because the heat has no way of being dissipated. Surface spalling may occur when one side of a section is subjected to sudden high temperature locally applied.

14-5. Shrinkage. Shrinkage of the concrete in setting is somewhat like a drop in temperature of 30 to 80°F (depending upon the "richness" of the concrete), except that the reinforcement does not shrink simultaneously. The shrinkage actually sets up compressive stresses in the rods so that, if the area of the steel is too great, the rods will be stronger in compression than the concrete is in tension, thereby producing the cracking that the reinforcement was supposed to eliminate.

The best way to handle shrinkage is to provide joints so that the concrete can shrink without causing trouble. With proper planning, the work may be arranged so as to build long structures in alternate sections, pouring rather long portions first, then filling in the shorter intermediate sections later—preferably allowing the first ones to set for 1 or 2 weeks.

[1] Alfred L. Miller and Herbert F. Faulkner: A Comparison of the Effect of High Temperatures on Concretes of High Alumina and Ordinary Portland Cements, *Univ. Washington, Univ. Expt. Sta., Bull.*, Series 43, Sept. 15, 1927; also Concrete Subjected to High Temperatures, *ACI Proc.*, Vol. 35, p. 417, 1938–1939.

In multistory structures with heavy rather solid walls and intermediate floors or in massive ones poured in complete horizontal lifts, the lower portions necessarily shrink first. When a higher lift or a floor is poured, its shrinkage is restrained so that it may develop cracks. Proper jointing is the best remedy, but full-height wall reinforcement inclined upward toward the center of shrinkage may minimize cracking. However, this is seldom practicable. Buildings with reinforced-concrete framework and filled-in walls do not have this difficulty because the columns can safely deform if full-height expansion joints are used about 300 ft apart.

Cooling of concrete by using ice in the mixing water and by means of refrigeration during the period of setting are sometimes thought to be desirable for massive structures such as dams. The purpose is to prevent too high a temperature resulting from the chemical reaction of the cement, and thereby to avoid cracking from subsequent cooling. Such means are costly and may not eliminate the trouble. Proper planning of pours and joints, construction of alternate sections, and provisions for future grouting may secure satisfactory results.

When shrinkage causes bending moments in such structures as arches and rigid frames, the resulting stresses should be considered in the design.

14-6. Construction joints. Construction joints must be located so as to cause no serious weakness in the structure. It is therefore desirable to place them in regions where the shearing stresses and the bending moments are small or where the joints will be supported by other members. However, they must be located so as to facilitate construction.

Such joints must be adequately keyed in order to transfer the necessary shearing forces. Figure 14-9 shows various arrangements. The numbers in the circles denote the sequence of the pours.

The following comments should be noted, the letters referring to the various sketches in Fig. 14-9:

(*a*) This key is easy to form, but it holds water when it is horizontal. The water should be removed. When a keyway is vertical, the form for it should be attached on the inside of the forms for the first pour.

(*b*) The raised key causes objectionable formwork.

(*c*) These intermittent precast blocks set by hand in the wet concrete are not very strong. They are easily forgotten, and they require extra hand operations. This intermittent key idea can be used to advantage in vertical joints that must withstand vertical and horizontal shears by placing pieces of 2 by 6 planks about 8 in. long and 18 in. c.c. on the inside of the end forms of the first pour.

(*d*) This type of key can be used only when the shearing forces are as shown, but it is good for such cases.

(*e*) This is theoretically better than (*d*); but when the rods are close together, it is difficult to finish properly. It is also easily forgotten by the workmen.

(*f*) This V can be made by hand after the concrete is poured, thus eliminating form strips which would interfere with the pouring of the concrete. It is good for thin walls and often for others (if it is not forgotten).

(*g*) If the shears are small or the direct compressive loads are large, this hand-roughened surface is often sufficient. Trowel the edge to get a straight line for appearance.

(*h*) This type of key is good for arches or other structures that it is desirable to pour in sections so as to minimize shrinkage stresses.

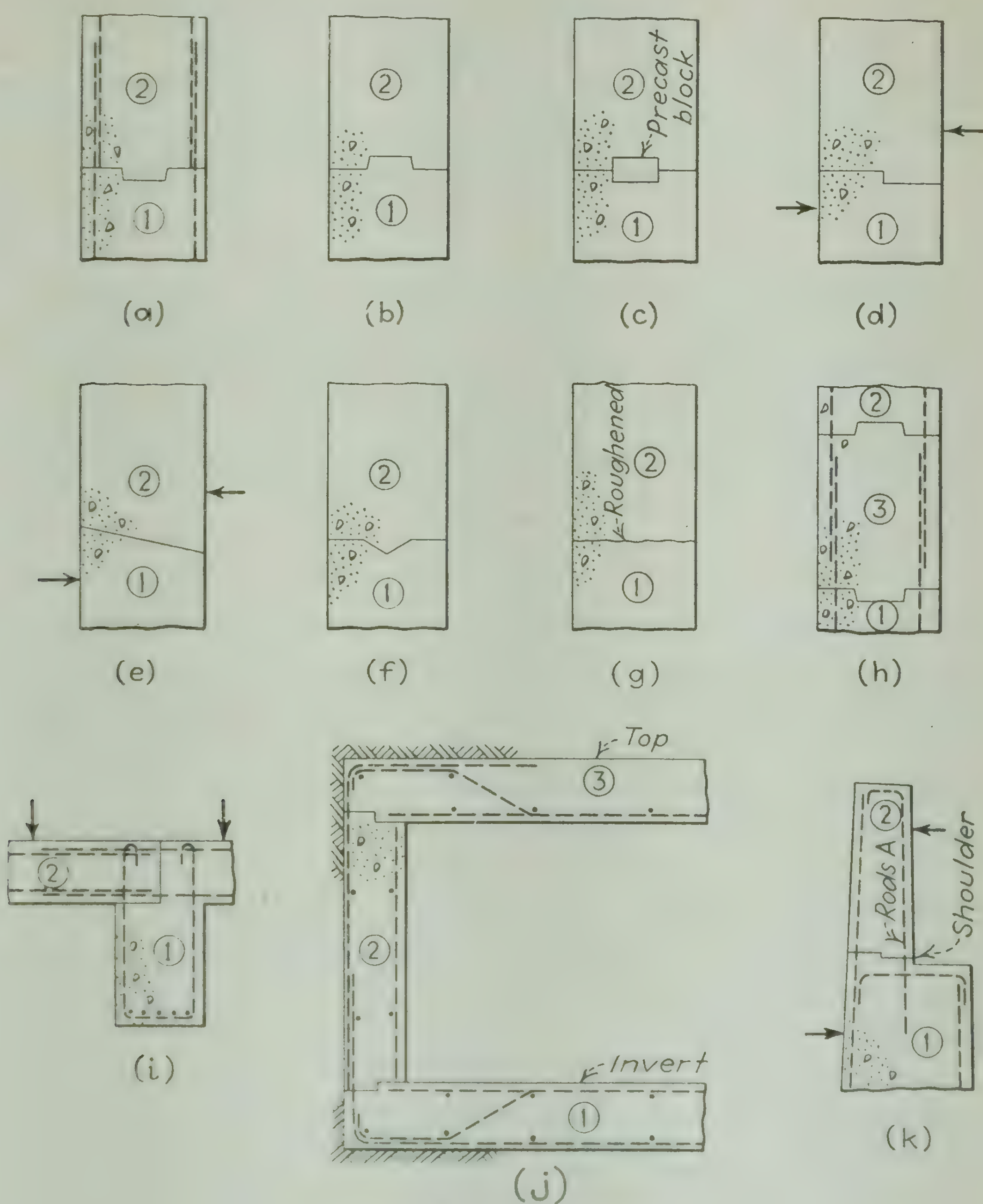


FIG. 14-9.

(*i*) This is a possible arrangement. There is no need for a true key when the joint is supported. This may weaken the T beam

(*j*) This shows suggested keyways at corners—such as for box culverts where the invert must be troweled or screeded. The wall forms can be braced on the inside. It is assumed that the shears in the walls are small compared with those which are in the top and bottom slabs. The vertical pressure on the walls causes friction that helps the keying action, whereas, if the full width of the wall were recessed into the

horizontal slabs, the effective thickness of the latter for resisting shears would be seriously decreased.

(k) This illustrates the provision of a shoulder for use in setting forms.

A key should be designed so that its width w [Fig. 14-10(a)] is sufficient to transfer the shearing force, using ordinarily a shearing unit stress of about $0.1f'_c$. When the shearing force is reversible, the width of the key should be $t/3$ or slightly smaller, so that all three parts of the joint will have approximately equal strength. The thickness t' should provide sufficient area in bearing along the edge at a unit stress of not over about $0.2f'_c$. A thick but narrow key (large t') is likely to break off.

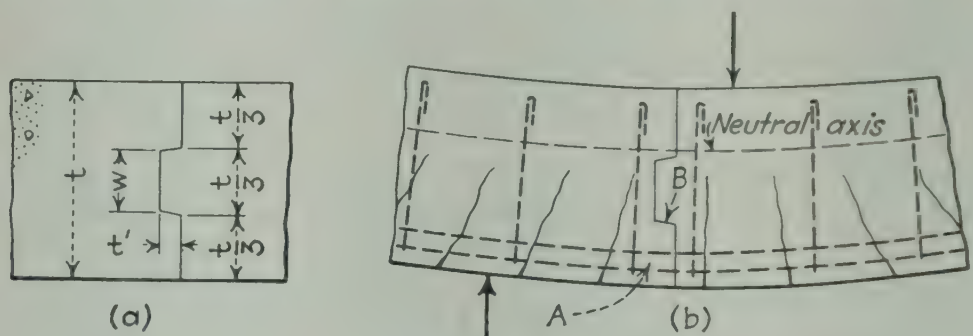


FIG. 14-10.

Figure 14-10(b) shows why a key at a point of large bending stresses may have very little value. The tensile forces may cause cracks which destroy the ability of a piece like A in the figure to resist a pressure applied at B. In such cases, it is advantageous to use vertical or inclined stirrups both sides of the joint, as shown in the figure.

Sometimes keyways are called for in places where they are not necessary; *e.g.*, at the junction of a column with a floor or the junction of a tall bridge pier with its footing. When no large shearing forces exist there is no sliding action that requires a key. Bond and friction are sufficient to hold the members in position even without the dowel action of the reinforcement.

It is usually necessary to have reinforcement which passes through construction joints. If so, the rods from the first pour should project through the joint enough to secure the desired bond, as in Figs. 14-9(a), (h), (i), (j), and (k). In this way, the rods in the later pour can rest upon or bear against the concrete of the previous pour. In some special cases, separate dowels like rods A of Sketch (k) can be set by hand in the wet concrete of the first pour. However, rods or dowels should not be relied upon for resisting the shear at the joint, because they will crush the concrete locally before they will withstand much shearing force, unless special provision is made to avoid this action.

In all cases, construction joints must be cleaned thoroughly before the next pour is made. All laitance must be removed, using wire brushes,

water under high pressure, or other means. It is often desirable to coat the joint with a little mortar just prior to the placing of the concrete upon it. The exposed edges of the joint should be finished straight, or they should have a small V strip on the forms ($\frac{1}{2}$ to 1 in. deep).

The volume of concrete that can be deposited in one continuous pour will influence the locations of construction joints in massive structures. These should be planned far in advance. The other extreme occurs in the case of very thin walls—4 to 6 in. thick. These cause difficult pouring if they have any great height. They must be built in short lifts, by the use of pumping or by depositing through a small spout and hopper called an "elephant's trunk."

The maximum number of cubic yards of concrete for one continuous pour depends upon the equipment available, the distances that the concrete has to be transported, and the character of the structure. In general, 300 yd^3 will be enough for one pour in 8 hr on even a big job.

Needless construction joints should be avoided, especially in retaining walls and other structures in which such joints may be a cause of the seepage of water.

14-7. Expansion and contraction joints. An expansion joint is used, generally with a premolded mastic or cork filler, when an elongation of adjacent parts and a closing of the joint are expected. A contraction joint is usually made without a filler except for a paint coat of asphalt, paraffin, oil, or some other material to break the bond. Its purpose is to prevent cracking. In general, the contraction caused by shrinkage will offset a large part of the subsequent expansion due to a rise of temperature, and a little pressure on the joint is seldom harmful anyway.

One of the first things to consider about expansion and contraction joints is that of the best locations for them from the standpoint of their proper functioning. They should be at points of change in thickness, at offsets, and at other points where the concrete will tend to crack if shrinkage and temperature deformations are restrained or prevented. The engineer must study a structure carefully in order to discover these points. Ordinarily, joints should be about 30 ft c.c. in exposed structures.

The second consideration should be that of coordination with the pouring schedule and avoidance of extra construction joints.

The third matter is that of satisfactory details. For these, the following points are mentioned:

1. The keyways should be of the types shown in Figs. 14-9(*a*), (*b*), and (*d*) if there is a considerable shearing force at the joint. Figure 14-11(*a*) shows a vertical expansion joint which has been used in some of the retaining walls of the approaches to the Lincoln Tunnel. The compressible material can be fastened to the first pour by tacking it to the forms

and by having nails protruding so as to bond into the concrete. However, one must be careful to use materials that will not squeeze out, slump when heated by the sun, or stain the surface of the concrete. In the space outside the key itself, beveled strips may be used (instead of fillers) and later withdrawn, but this is difficult when the walls are thick.

2. The edges of the keyways should be beveled slightly; they should be coated with mastic paint or with some material that will break the bond but that is not thick enough to destroy the bearing value of the key.

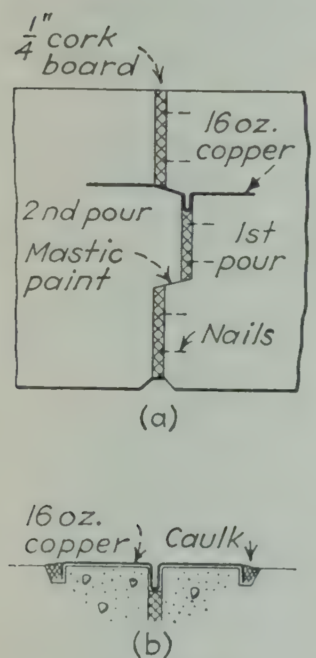


FIG. 14-11.

3. When the joints are likely to leak, they should be sealed in some way. Copper flashing is sometimes used as in Fig. 14-11(*a*). This copper should be folded into the joint so as to permit it to open slightly without rupturing the flashing; it must also be strong enough to hold its position during the placing of the concrete—an operation that must be done very carefully.

A second method of flashing, when the back of the joint is accessible, is pictured in Fig. 14-11(*b*). This is expensive because it is a sort of roofing job, but it is sometimes more reliable.

Copper water stops are likely to become crumpled during the concreting. The depositing must be done carefully. Steel plates ⅛ to ¼ in. thick with bolted and gasketed joints can be used at construction joints, but the accordian action of the fold shown in the sketches is needed whenever movement can occur.

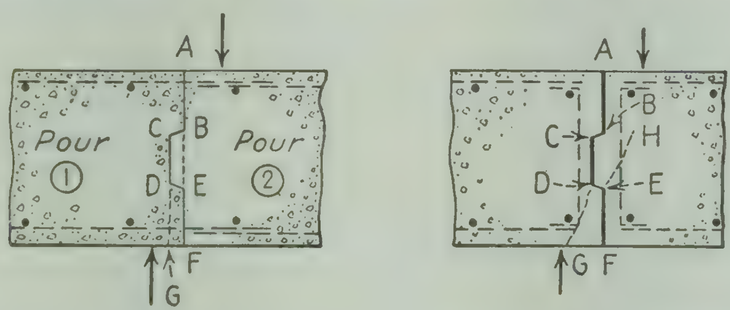
4. When it is possible to do so, expansion joints should be entirely open with an air space of 1 to 2 in. between the concrete sections. This is especially desirable in bridges where considerable motion occurs. However, to avoid visibility through parapets and similar parts, the joints may be offset in plan instead of being straight. In any case, one should be able to clean out such joints.

5. V cuts should be used at the joints, or the joints should be placed at moldings, reentrant corners, and other suitable points. The V cuts should be 1 to 2 in. deep or large enough (and the joint fillers thick enough) to guard against spalling of the edges because of the compressive forces which might be caused at the joint by expansion.

When a keyway is used at a construction joint AF to resist shearing forces as pictured in Fig. 14-12(*a*), the bearing on the surface DE is confined, and the unit pressure can safely be large because the concrete cannot get away. The key may shear along BE or DG but, since this is

ordinary shearing action without diagonal tension, the unit resistance will probably be great.

On the other hand, assume a contraction or expansion joint like that of Fig. 14-12(b). Here the faces AB, CD, and EF are not in direct bearing. The pressure on DE may cause a crack along DG because of combined shear and bending. The reinforcement is not close enough to resist this diagonal tension. The key might crack along EH but this is less likely. The section should then be proportioned so that the shearing stress across BE and DF does not exceed $0.03f'_c$ to $0.05f'_c$.



(a) Key at Construction Joint

(b) Key at Contraction or Expansion Joint

FIG. 14-12. Action of keys in shear.

In comparing Figs. 14-12(a) and (b), notice that $GDEF$ of the former cannot rotate clockwise under the load on DE because the adjoining concrete will not let it do so, whereas $GDEF$ of Sketch (b) may do this since an opening exists along EF.

"Dummy" joints like that which is shown in Fig. 14-6 are useful in some cases as contraction joints that will avoid unsightly shrinkage cracks. The reinforcement should be weaker at these joints than it is elsewhere.

14-8. Waterproofing. Retaining walls, basement walls, spandrels of earth-filled arches, subways, tunnels, and similar structures must be watertight if they are to present a pleasing appearance. The denser the concrete itself is the more impervious it becomes, so that too lean a mix is likely to facilitate leakage. However, it is exceedingly difficult—or almost impossible—to keep construction joints from leaking when there is an appreciable pressure of water behind them.

There are three ways in which this problem of seepage may be attacked, *viz.*, integral waterproofing, inside surface coatings, and outside surface coatings.

Integral waterproofing denotes materials added to the concrete when it is mixed in order to make the concrete itself impervious. The admixture supposedly fills all the voids through which the water might pass. In any event, complete reliance upon integral waterproofing is dangerous, especially at the construction joints.

Coatings on the insides of the walls are used sometimes as a means of stopping leaks, but they are very likely to be unsatisfactory. Mortars with impervious mixtures in them, paints with water glass or other chemicals which evaporate or congeal and leave crystals or chemicals in the pores of the surface of the concrete, and asphaltic paints—these are some of the coatings used. However, these are expensive, and it is unreasonable to expect them to stop the water at the last line of defense—the inside surface—especially when the structures are subject to temperatures that cause the water to freeze behind the surfacing. If used, inside applications should be chosen and applied by experts in that line.

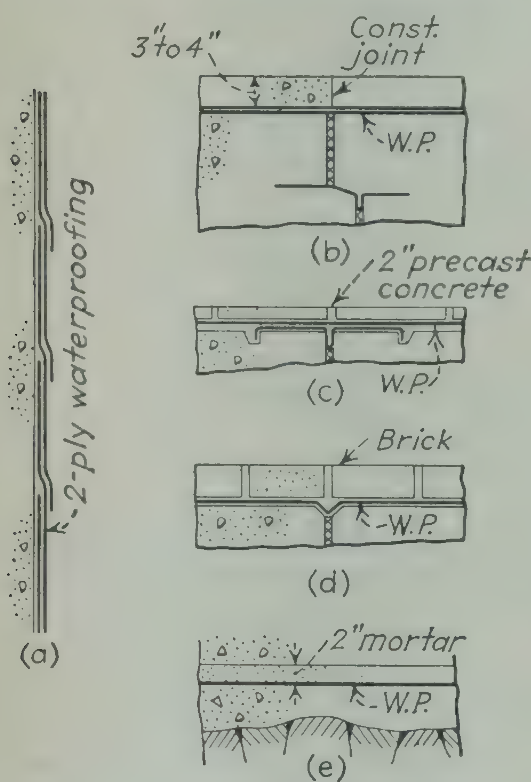


FIG. 14-13.

The best place to stop the leakage is at the outside surface—the point of entrance of the water. There are two customary ways of doing this. The first, and the most effective, is the use of membrane waterproofing which forms a continuous watertight sheet outside the structure; the second is the use of asphaltic emulsions or similar bituminous coatings forming waterproofing without membrane.

Membrane waterproofing is usually built up by coating the surface that is to be waterproofed with hot asphalt or coal-tar pitch, laying thereon successive layers of special fabric—placed shingle fashion and each layer coated with the mastic—until the desired number of layers or "plies" is obtained (two-ply, three-ply, etc.). Figure 14-13(*a*) shows this principle.

It is necessary to protect the membrane waterproofing against damage during backfilling; against penetration of oils, gasoline, or other solvents; and against the cutting tendency of sharp stones in the backfill. The first of these may be accomplished by the use of plywood which is laid against the membrane, but this is only a temporary material. Better protective coatings are poured concrete 3 or 4 in. thick and precast-concrete blocks or bricks set in mortar as shown in Figs. 14-13(*b*), (*c*), and (*d*), which picture details at expansion joints—always troublesome points.

It is best to place membrane waterproofing directly upon the outside of the wall, but this is not always economical, desirable, or possible. When one is waterproofing subways, tunnels, and other structures which are not accessible from the outside, it is necessary to provide a surface

on which the membrane may be placed before the concrete of the main structure is poured. A sample of this work is shown in Fig. 14-14, which pictures part of the bottom of the New Jersey shaft of the Lincoln Tunnel. A concrete lining, or "sand wall," is poured against the rock as shown above the line of black waterproofing. The membrane is then applied as shown, using a 2-in. coating of mortar on the invert or bottom to protect the waterproofing when the reinforcement and concrete are

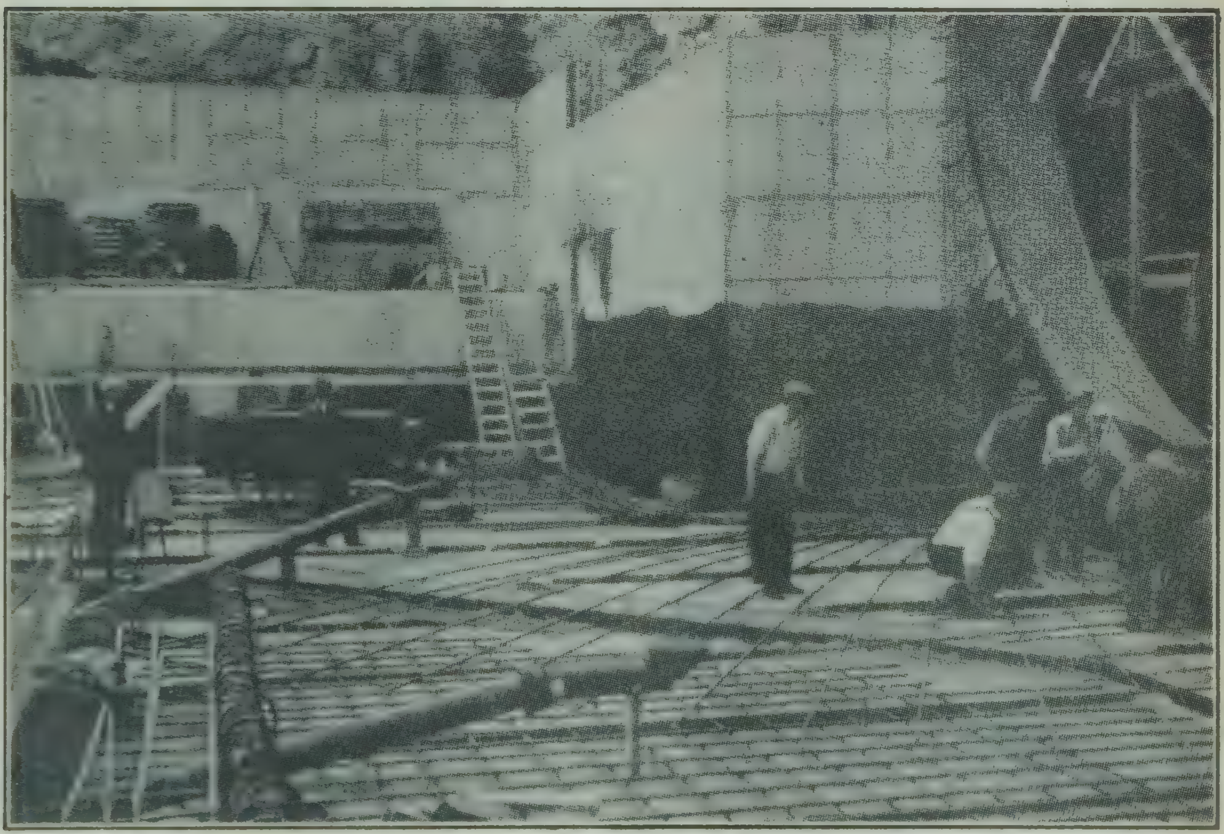


FIG. 14-14. Construction of invert of New Jersey shaft of the Lincoln Tunnel at New York City.

placed, as shown in Fig. 14-13(*e*). Of course, all roughness, projecting form ties, and other sources of damage are removed before the membrane is applied. It is necessary to keep the steel far enough away so that the subsequent spading of the concrete will not cause damage to the membrane. It is also obvious that such waterproofing work should not be attempted when any reinforcement is in the way.

The concrete must be dry when membrane waterproofing is applied. It is also necessary to keep water pressure from building up behind the sand walls, which are shown in Fig. 14-14, in order to avoid bulges in the membrane or displacement of the concrete. There is a shallow gutter-like drainage system behind the walls in the picture. It leads to a temporary sump.

Waterproofing without membrane—sometimes called "dampproofing"—is merely the application of bituminous materials to the concrete as a

paint coat or as multiple coats. It is beneficial, but it can be applied only from the outside; it is easily damaged; it has no particular elastic properties; and it is likely to be ineffective after a number of years. However, it is far better than no waterproofing at all.

It is less detrimental if waterproofing is omitted from stone-faced walls than from plain concrete-surfaced ones if the stones and the concrete are placed monolithically, because slight staining will not show very much, and the leakage is not likely to be serious. However, it is hardly

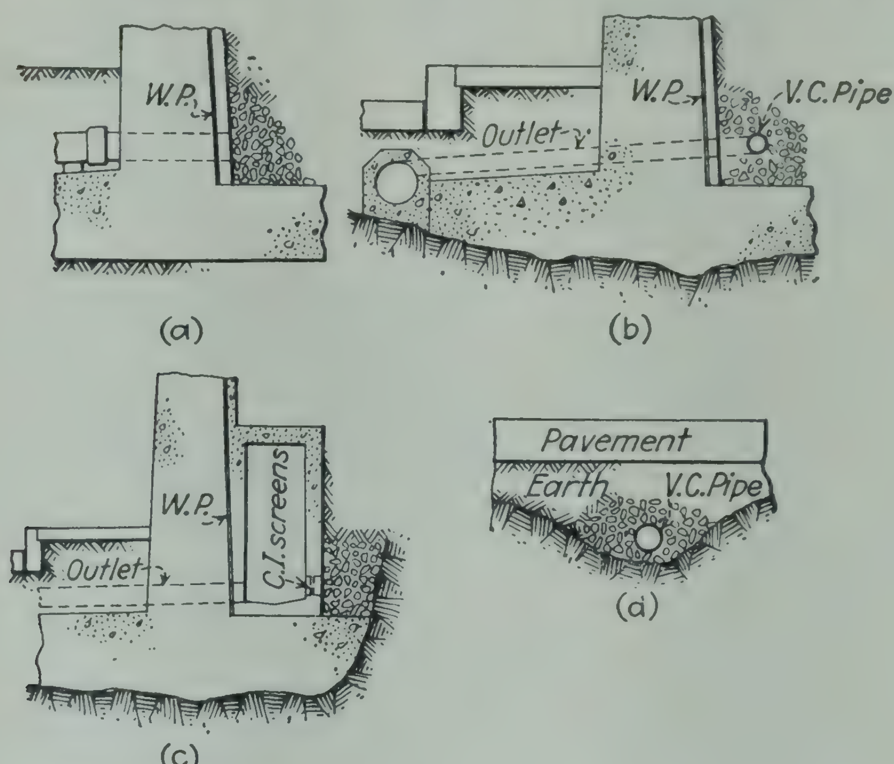


FIG. 14-15.

worth while to take such chances. At least, one should apply waterproofing without membrane.

14-9. Drainage. When the conditions are such that natural drainage can be provided readily, it is advisable to construct a drainage system whose function is to remove the ground water behind or outside the structure. This is desirable even with waterproofed structures, especially when they rest upon rock. Such installations vary from simple weep holes through the walls to elaborate interconnected systems.

Any such drainage system is subject to clogging by silt and to freezing. It should therefore be laid out so that it can be cleaned by rodding or flushing and so that it is below or behind the frost line, if possible. This can be done by a system of manholes, Y connections at intervals, or even galleries such as that of Fig. 14-15(*c*) which can be inspected.

A few means of drainage are shown in Fig. 14-15. Sketch (*a*) shows a simple "blind," or stone, drain behind a wall. The water passes through

the ground to this drain, thence to outlets through the wall, and finally through another system to a sewer or outlet. The addition of a vitrified-clay pipe line which has open joints wrapped with burlap or tar paper, as shown in Sketch (*b*), facilitates the removal of the water. The gallery drain in Sketch (*c*), which can be entered at manholes, has been used behind some important walls along the approaches of the Lincoln Tunnel. The adaptation of the same principles for use under pavements—called a "French drain"—is pictured in Sketch (*d*).

14-10. Deflections. The magnitude of the deflection of a reinforced-concrete beam is difficult to determine with accuracy. If the member is highly stressed, it will have many hair cracks; if lightly loaded, it may have very few. The former case is the one that usually concerns the engineer.

The big question is generally what to use for the moment of inertia I_c in the customary formulas and procedures for determining deflections. The I_c of the transformed section is probably the best one to use. It applies fairly well but not entirely in regions of large tensile stresses, but the beam is not all cracks. The member is probably much stiffer between cracks, near the simply supported ends, and near points of inflection. Probably it is satisfactory to use I_c for the transformed section with something like the following empirical correction factors applied to the computed deflections:

1. 0.9 for long simply supported beams.
2. 0.7 for continuous beams.
3. 0.8 for beams continuous at one end and simply supported at the other.

For E_c, use the design value or that found from tests. If I_c varies greatly from end to center, as it may for continuous beams, use a weighted average magnitude of I_c, considering the length over which a particular value applies.

It is probable that shrinkage of the concrete in heavily reinforced beams will increase their actual deflection because the steel will be trying to resist the shortening of the concrete. The steel will try to maintain its original length, whereas the concrete tries to shorten; thus they together will probably cause a curvature of the beam which is likely to be in the same direction as that produced by the design loads. However, this effect is not likely to be large. The increase in deflection may be estimated as 10 to 20 per cent of the computed deflection for loading.

Heavy long-term loads are likely to produce much larger deflections of beams than those computed by the ordinary formulas. This is the result of plastic flow of the concrete when highly stressed. Under such permanent large stresses in the concrete, the computed deflection, using E_c as determined by tests of 28-day cylinders or as $1{,}000f'_c$, should prob-

ably be doubled for loading applied for 1 year, and tripled if the loading is fairly permanent. Therefore, this feature should be estimated when the dead loads are heavy and deflections are important.

Although the magnitude of the deflection is questionable, a consideration of it qualitatively is very important. Otherwise, the designer may receive some rude shocks when his structures crack badly. Just three cases will be cited to illustrate this point and its consideration in the detail planning of structures, and in design matters.

Figure 14-16(a) shows one construction that was designed to hold a pair of heavy machines that would cause considerable shock. One

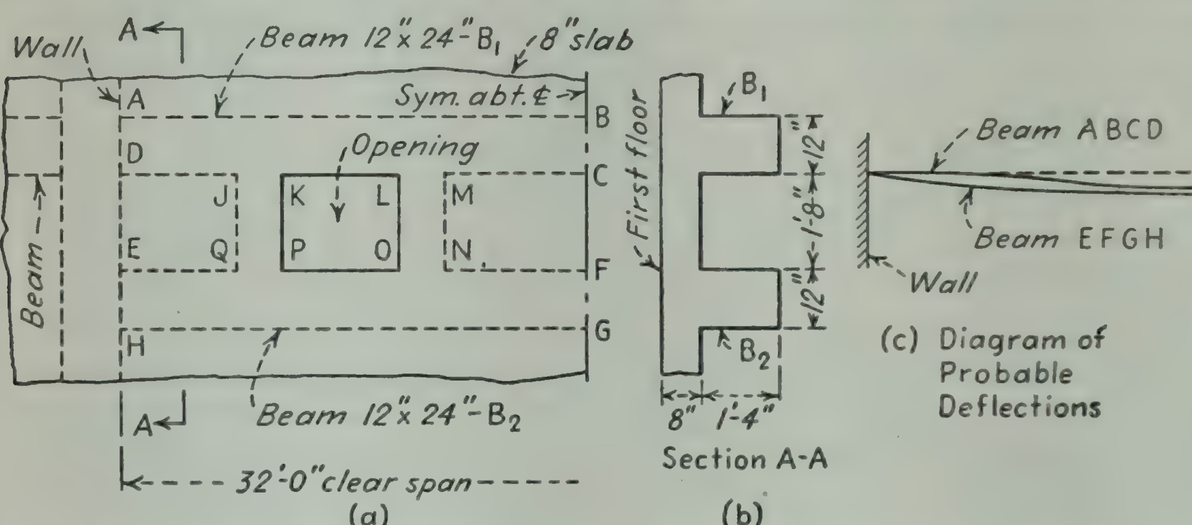
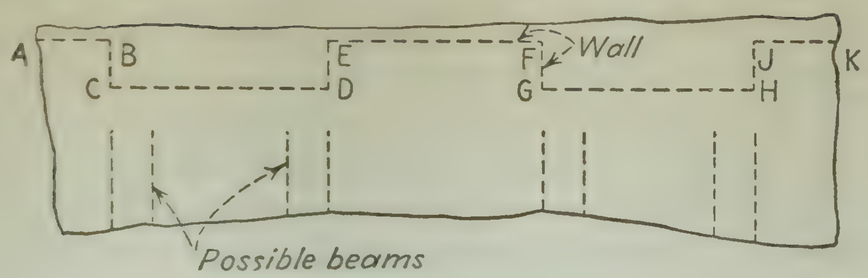


FIG. 14-16. Study of difference in deflections of beams under a machine.

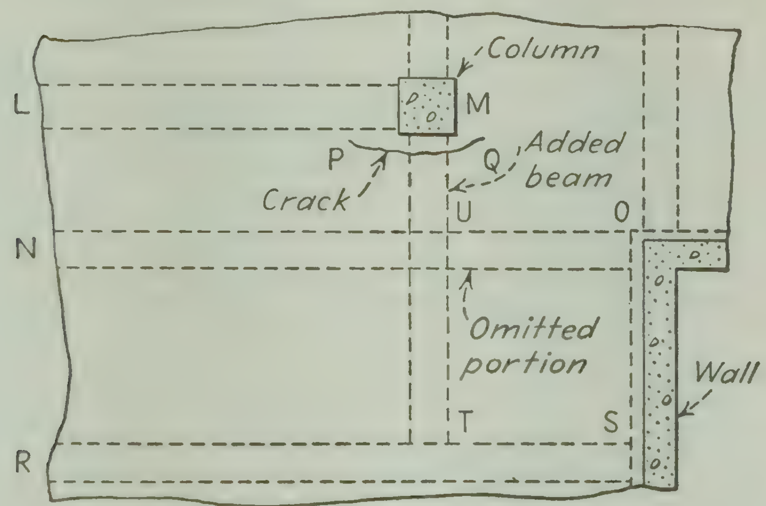
machine was centered over the opening $KLOP$. Beam $ABCD$ was continuous across the wall, whereas beam $EFGH$ was simply supported at the wall. An 8-in. floor slab was a common part of both T beams. Even with the dead load alone, these beams will deflect differently—somewhat as shown in Sketch (c). This, as well as the load of the machines and their vibrations, will tend to crack the slab and the diaphragms under the machines. When the situation was discovered, it was too late to redesign the structure. Therefore, posts were put under the centers of both beams so that deflections would be negligible.

If a long-span floor slab rests upon a wall that is serrated in plan, as in Fig. 14-17(a), the deflection of the slab tends to make it "ride" the edges CD and GH so that the load does not reach AB, EF, and JK satisfactorily. It may be desirable to use beams extending from the corners C, D, G, and H, then to span the slab across them.

If there is a jog in the supports, as pictured in Fig. 14-17(b), cracks may occur near the region PQ because beam NO may deflect enough with respect to the beam LM and the column to overstress the slab. It may be desirable to use a beam from M to T, make RS strong enough to hold it, and stop the first beam at point U.



(a) Floor Slab on Serrated Support



(b) Effect of "Hard Spot"

FIG. 14-17. Illustrations of effects of deflections.

14-11. Encasement of structural steel. Concrete is often used as an encasement around steel beams, girders, and columns in order to fireproof them or to protect them from corrosion. Gunite is used for the same purpose. However, it is necessary to do this work correctly so as to avoid unsightly cracking.

In the first place, if this encasement is placed on beams and girders before the major part of the dead load is applied, there is a probability that cracking of the encasement will occur, especially if the member has smooth flanges. For this reason, and in order to avoid spalling, the bottom flanges should be covered with wire mesh or beam wrapper, using 4- by 4- or 6- by 6-in. mesh and No. 8 or 9 gage wires—preferably welded. The minimum encasement should be about 2 in. for smooth flanges or $2\frac{1}{2}$ in. when there are rivet heads to be covered. The concrete on the sides of the webs should be fastened together by rods or wires which pass through the webs. These details are pictured in Fig. 14-18.

Stiffeners on the webs of encased girders are likely to cause cracks. Thin encasement is relatively light, but it must be carried out around these stiffeners, and that causes expensive formwork. With heavy encasement, the longitudinal rods along the web should be carried through

holes in the stiffeners. However, unnecessary stiffeners and other details that complicate the work of encasement should be eliminated.

When a bridge deck is poured against exposed steel, as in Fig. 14-18(*d*), the load of the slab should be carried on a shelf angle; welded ties should bond it to the metal; the surface should be pitched slightly away from the steel for drainage; and some good expansion joint cement or mastic should be placed in a small tooled groove at the junction with the metal

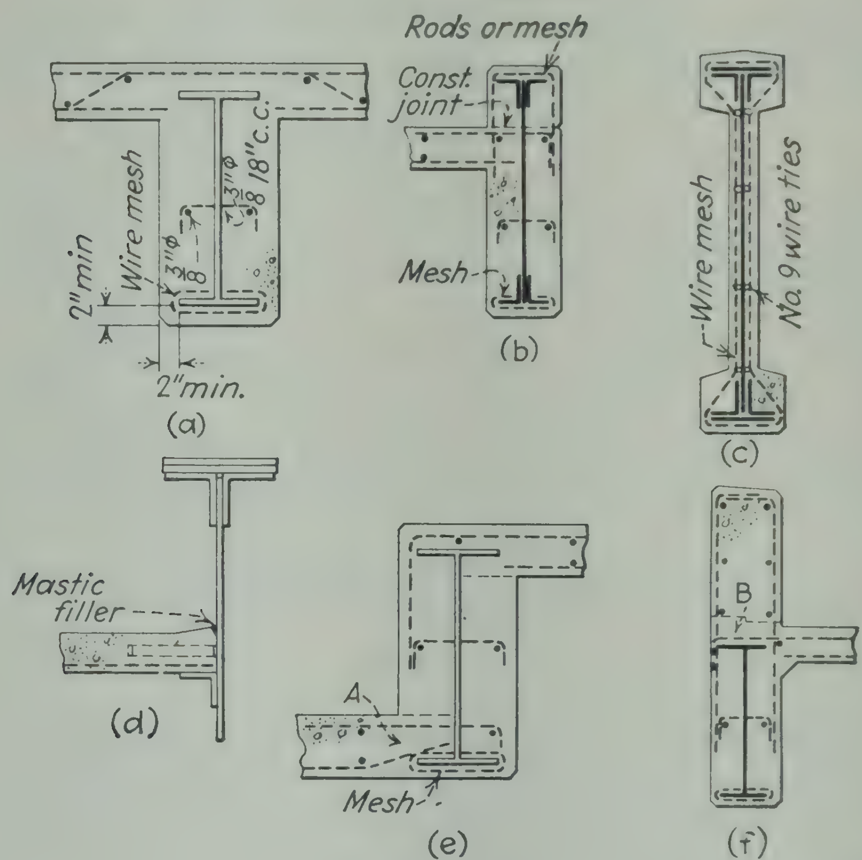


FIG. 14-18.

because the joint must be sealed to prevent rusting of the steel. Similar treatment is generally advisable in any case where bare steel enters concrete if it is exposed to the weather. In important cases, it is often advisable to keep the concrete entirely free from the steel so that the latter can be painted. It is obvious that channels with the flanges turned down, beams with horizontal webs, and similar steelwork causing air pockets are undesirable. They should have at least 3- or 4-in. holes about 18 in. c.c. in the webs to permit concreting or grouting the hollow space.

Figure 14-18(*e*) shows another problem which arises sometimes when an offset in levels occurs at an encased beam. The lower slab must be thick enough at *A* to deliver its reaction on to the top surface of the bottom flange of the I beam, because the part of the slab that is below this flange is almost valueless for transmitting shear.

When a parapet or projection is supported as shown in Fig. 14-18(f), the tendency of the steel beam to act by itself is likely to cause cracks at B due to longitudinal shear. The parapet should have joints at close intervals, or else the member should be designed so as to act as a composite beam. Furthermore, special care must be exercised to tie the parapet down to the encasement of the beam itself.

14-12. Reinforcement around openings. When a rectangular opening must be made in a wall or some other member that is not subjected to bending, it is advisable to add special reinforcement, such as rods A in Fig. 14-19(a), in order to avoid the formation of cracks due to shrinkage, settlement, and changes in temperature. Such corners are points of weakness. However, the number of rods and the size to use are matters of judgment. In general, rods A should be at least sufficient to replace the temperature steel that the opening has eliminated; in other words, the area of two sets of rods should be 0.0025 times the area of the cross section of the cut—or more.

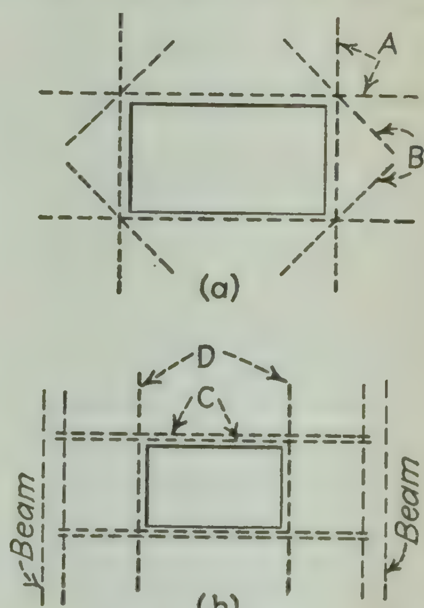


FIG. 14-19.

In cases where there are large shearing forces acting in the plane of the opening that is shown in Fig. 14-19(a), it is advisable to use additional diagonal rods like those marked B. If there is a series of openings in a long structure, it may be advisable to reinforce the solid portions above and below the openings as girders with special full-height rods designed as stirrups. The area of the steel must be determined by one's judgment of the seriousness of the particular situation.

When the opening occurs in a slab that carries bending, as in Fig. 14-19(b), rods C should be added to make up for the bars that have been eliminated by the opening—or these rods should be sufficient to carry all the loads. Rods D are used to secure lateral spreading. The area of metal in them should equal that of the missing lateral rods or 0.0025 times the cross section of the opening. When the opening is small, the slab may not need to be thickened; but if it is large, small reinforced-concrete beams should be used instead of rods C, extending the beams across to the nearest main supports. Of course, when the conditions are very severe, rods D should also be replaced by beams.

14-13. Torsion in concrete. There are two fundamentally different cases to consider regarding torsion in concrete members, *i.e.*, columns or other members that have compression over the entire section, and beams that have tension over part of the cross section. The former will be considered first

When a rectangular reinforced-concrete member fails in torsion, it generally starts to crack near the middle of the long side, the crack tending to follow a helix at approximately 45° with respect to the longitudinal axis of the member. This failure is primarily due to tension in the concrete. The longitudinal reinforcement should not be relied upon, because the rods are slender, and they depend upon the concrete for their torsional resistance. The most effective reinforcement for such a case is a spiral with its rods at an inclination of 45° to the member's axis so as to cross the possible cracks.

The maximum shearing stresses[1] in sections of isotropic material may be assumed to be the following:

1. *Edge of Circular Section.*

$$v_e = 0.637 \frac{M}{r_e^3} \tag{14-1}$$

2. *Middle of Long Side of Rectangular Section.*

$$v_m = \left(3 + \frac{2.6}{0.45 + \frac{B}{a}}\right) \frac{M}{8Ba^2} \tag{14-2}$$

3. *Middle of Short Side of Rectangular Section.*

$$v'_m = \left(3 + \frac{2.6}{0.45 + \frac{B}{a}}\right) \frac{M}{8aB^2} \tag{14-3}$$

where M = the twisting moment, r_e = radius of concrete section, B = one-half of the long side, and a = one-half of the short side. In general, B should not exceed $1.5a$. The diagonal tensions and compressions that result from these shearing stresses may be considered to be equal to the intensities of the shearing stresses themselves.

The designer should endeavor to avoid torsional stresses that, when computed by the preceding formulas, will cause in the concrete a net tension that exceeds the allowable tensile stress in the concrete (after overcoming the compressive stresses due to longitudinal loads, if any).

When the torsional moments cause excessive tensile stresses in a circular member with a 45° spiral, Prof. Andersen states that the following relation exists:

$$6Kv_e^3 = (v_e - t_c)^2(3v_e^2 + 2v_e t_c + t_c^2) \tag{14-4}$$

[1] See Paul Andersen, Rectangular Concrete Sections under Torsion, *J. ACI*, September–October, 1937. Equations (14-1) to (14-5), inclusive, are taken from this publication with minor changes in symbols.

where

$$K = \sqrt{2} \times N \times t_s \times A_s \times \frac{r_s}{\pi r_s^3} \qquad (14\text{-}5)$$

t_c = allowable tension in concrete, N = number of spiral bars 45° to the axis that are cut by a horizontal plane, t_s = allowable tensile stress in the steel, A_s = cross-sectional area of one spiral bar, and r_s = radius to the line of the spiral. Even when the member is square, it may be considered to be approximately the same as one whose cross section is the circle that may be inscribed in the square.

Professor Andersen's experiments also indicate that the ultimate torsional shearing unit stress in specimens that have no 45° spirals is approximately $v_m = 0.1f'_c$; spirals generally increase the magnitude of this ultimate stress; and the modulus of elasticity in torsion is about $0.45E_c$. The safe working stress in torsional shear should be about $0.03f'_c$ when no spirals are used.

When a beam that carries a severe bending moment is subjected to torsional action, the cracked concrete on the tensile side of the neutral axis should not be relied upon. The resisting moment should be computed from Eqs. (14-2) and (14-3), using only the uncracked part of the section *bkd*. A careful study of Fig. 2-3 will show the reasons for assuming it to be so. When the member is twisted, the cracks below the neutral axis prevent the transfer of shearing stresses across themselves, leaving only the uncracked portion of the beam to offer resistance. This seems to neglect the action of stirrups, but the stirrups themselves depend upon the uncracked portion for their holding power. In such a case as this, the torsional shearing stress on one side of the member adds directly to the transverse shearing stress which is due to beam action. The combined shearing stresses in the uncracked portion should not exceed about $0.2f'_c$ (even less if the torsion is relatively large).

When a beam supports a slab or another beam on one side only, the angular rotation of the end of the latter causes torsion in the edge support. In such a case, for a rectangular beam with a depth h which is not over 1.5 times the width b, the total angular rotation θ_T of the edge support may be approximated by assuming[1]

$$\theta_T = \frac{3.33ML}{E} \times \frac{b^2 + h^2}{b^3h^3} \qquad (14\text{-}6)$$

where M = the twisting moment and E = the modulus of elasticity in torsion. Of course, h is used in Eq. (14-6) in order to obtain a measure of the maximum torsional stiffness of the beam; h should be replaced by

[1] Taken from Mauer and Withey, "Strength of Materials," John Wiley & Sons, Inc., New York, with changes in symbols.

kd when one wishes to find the torsional resisting moment that can be developed safely by the member for any particular angle of rotation that is impressed upon it. However, the *exact* conditions in any given case are almost impossible to ascertain.

By finding θ_T for the edge support when $M = 1$, by finding the angular rotation of the end of the intersecting member as a simply supported beam that carries the given loads, and by finding the angular deflection of the end of this intersecting member when a unit moment is applied at its end, one can approximately balance the angular deflections and therefrom determine the torsional moment. However, this problem is generally neglected, and the effect of the twisting is supposed to be covered by the safety factor (not always a wise procedure).

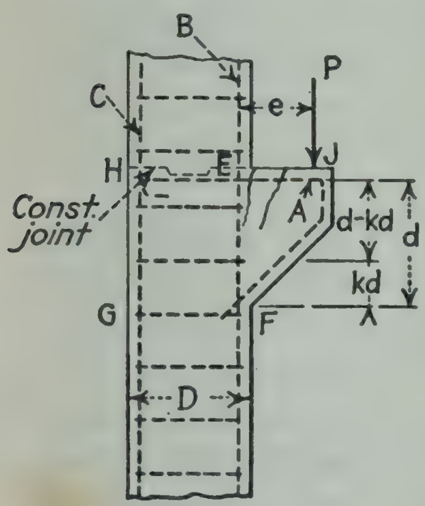


FIG. 14-20.

In Eq. (14-6), L is the length of the part of the member through which the torsion acts—the distance from the point of application of the torque to the reaction point. In case of an edge beam under a continuous slab, the magnitude of L is very uncertain; so also is the rotation of the end of the slab.

14-14. Reinforced-concrete brackets. When a load is applied upon a bracket of reinforced concrete as pictured in Fig. 14-20, the tendency is to cause tension in rods A, also to open up the column at E and G so as to produce tensile stresses (or reduced compressive ones) in rods B and C at these two points, with compressive stresses at F and H.

The eccentricity of the load, as far as the bracket is concerned, may be assumed to be e, the distance from the nearer row of steel to the load, because the point of compressive resistance is near F at the bottom. When e is less than d, it is sufficient to assume that the cracks will be about as shown in the figure so that the compressive stresses will be small and the rods A can be designed by the simple formula

$$M = Pe = A_sf_sjd = 0.87A_sf_sd$$

When e exceeds d, the bracket should be analyzed, as a cantilever beam, for compressive, tensile, and shearing stresses. The applied moment may still be called Pe.

For the column itself, the effect of the load P should be determined upon the basis of combined bending and direct stress (Chap. 7).

14-15. Planning reinforcement. As an example of some of the things that should be considered when planning reinforcement, examine the case illustrated in Fig. 14-21. Sketch (a) shows the cross section at

one portion of a twin intake tunnel for condenser water that is proposed for a power plant. It rests upon sand, and it supports a screen and pump house above it. The downward pressure on the bottom slab caused by the weight of the structure exceeds the hydrostatic uplift, even when the gates are closed and the tunnels are empty for cleaning. It is assumed that water can seep between the tremie seal and the slab. This empty

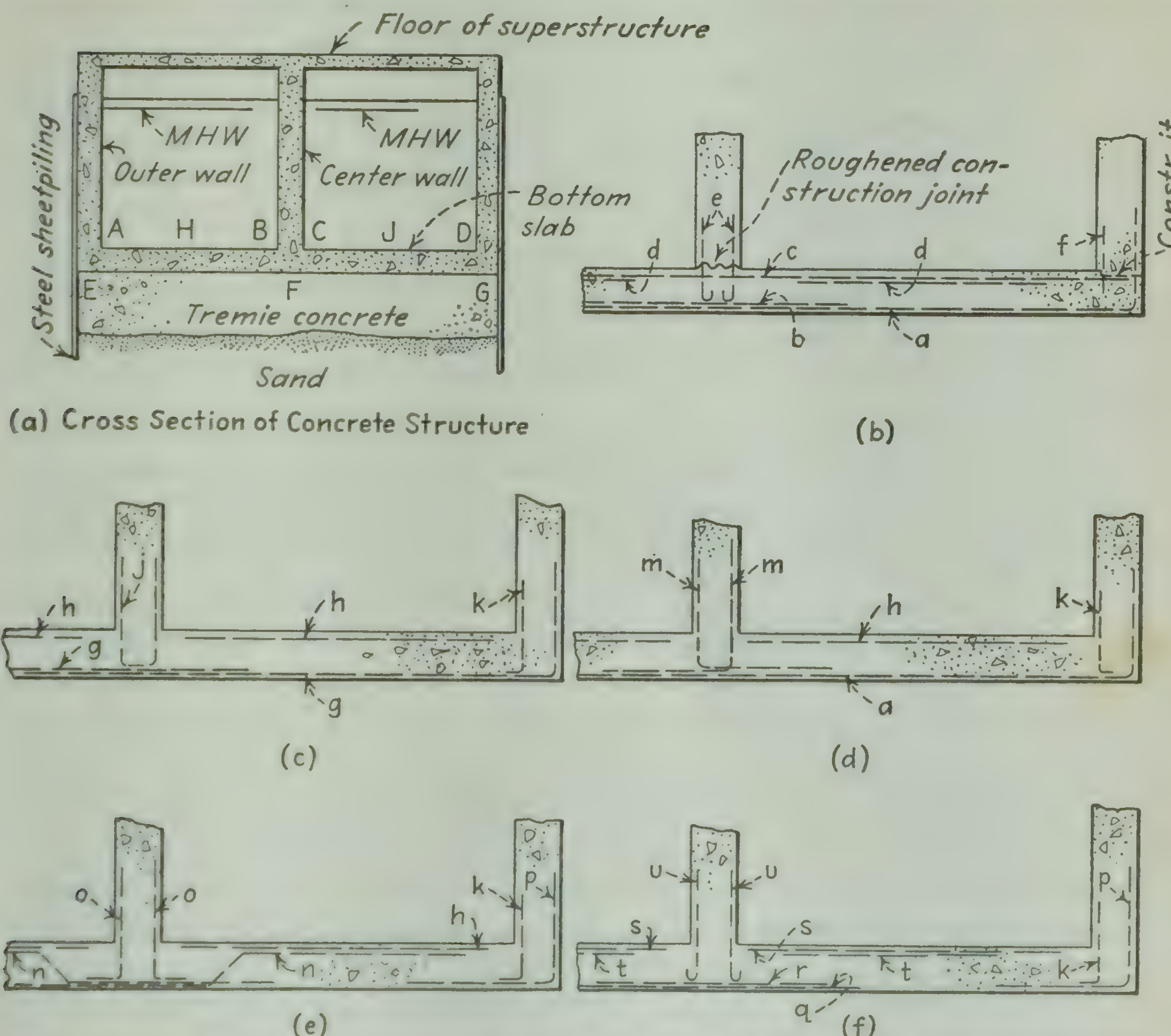


FIG. 14-21. A study of arrangements of reinforcement for a heavy slab subjected to hydrostatic uplift.

state is a critical load condition for the invert. In such a case, the bottom acts as a continuous slab with tension in the lower reinforcement near E, F, and G, and in the top rods near H and J. When one tunnel is emptied, the pressure of the water in the other one tends to cause tension near B or C in the center wall.

Assume that the bending moments and shears have been computed for design, that tentative thicknesses have been selected, and that a section requiring no web reinforcement is to be used. The reinforcement may be planned in various ways, but it is desired to find the best arrangement.

The following comments refer to the arrangements shown in the various sketches in Fig. 14-21:

(*b*) Rods *a* extend clear across the bottom and are bent up to resist the moments at the bottoms of the outer walls. Extra straight rods *b* are added under the center wall to resist the greater tension there. Rods *c* are extended clear across the top of the slab, with *d* added to reinforce the centers of the spans. To resist the unbalanced tension in the center wall, dowels *e* are used. Then a few light dowels *f* are added to hold the inner reinforcement of the outer walls during concreting.

(*c*) Here rods *g* in the bottom are lapped under the center wall to provide for the greater bending moment there. Bars *h* are made only as long as the tension in the top of the slab requires them to be. Single U-shaped dowels *j* are used for the center wall. They can then be rested on the lower mat during concreting. The bend at the bottom of *k* is for the same purpose.

(*d*) Bars *a* are used clear across the bottom but dowels *m* are bent alternately left and right to reinforce both the wall and the slab. However, the bond stresses may be undesirably high between the bends when the rods act in resisting tension in the bottom of the mat. When the rods are close together this tends to cause a screen effect during concreting.

(*e*) Rods *n* are bent up to serve as top reinforcement after they are not needed in the bottoms of the slab, thus helping bars *h*. When the slab is deep, this arrangement is not usually desirable. Dowels *o* are made with the bends turned outward. This is undesirable and is not the most effective anchorage against bending in the center wall. Bars *p* are extended to take care of the tension in the outer corners.

(*f*) This shows bars *q* and *r* of minimum length to provide for the bottom tension under the center wall and bars *p* to reinforce the outer corners. Rods *s* and *t* are the minimum for the tension in the top. Dowels *u* are obviously turned the wrong way.

Conclusions. Such a heavy important structure should be "knitted" together well. The author prefers rods *g* of Sketch (*c*) for bottom reinforcement, with extra rods *p* of Sketch (*e*) at the corners if needed. All are of moderate length and are easy to place, and the bottom is tied clear across. For the top of the slab, rods *c* and *d* of Sketch (*b*) seem to be the best since the slab is tied together well and there is some reinforcement crossing the center wall to resist the effect of tension near the corners. For the center, dowels *j* of Sketch (*c*) are preferable when the U is broad enough; otherwise *m* of Sketch (*d*) is preferable. A few light dowels *k* of Sketch (*c*) placed perhaps 3 ft c.c. are generally worth their cost.

14-16. Construction drawings. After the design drawings and specifications have served their purpose in the securing of bids and the letting of a contract for a structure, much still remains to be done in the line of *working drawings*—the ones used in the field in performing the construction. These may be prepared by the owner's engineer, by the contractor's organization, or by a firm that is paid to do that special job.

One part of the work is the preparation of *masonry drawings.* These are for the purpose of making excavations and for building forms. They are to show all the dimensions of the concrete members.

Another part is the making of *detail drawings* or *reinforcing drawings.* This is often done by the steel company that furnishes the bars. If so,

the rods may be marked, bundled, and shipped directly to the job ready for erection. In other cases, bars of stock lengths are ordered in advance from estimates of needed quantities. Then the cutting and bending is done in the field. The latter arrangement is often made in order to avoid delays in securing material because it may take considerable time to obtain the steel. In the meantime the detailing can proceed. Of course, this may be somewhat wasteful of steel because of short pieces that are left when the bars are cut and their length is not that of the ordered bars.

The detail drawings are prepared to show the numbers and positions of all bars, also their sizes, lengths, and shapes. Various offices are likely to have their own standards for detailing. The American Concrete Institute has prepared the Manual of Standard Practice for Detailing Reinforced Concrete Structures (ACI 315-51). This is to standardize such drawings and to simplify the detailing as much as possible. Figures 14-22, 14-23, and 14-24 have been prepared to illustrate the details for a portion of a floor system. They were made in accordance with the system used at the New York Office of the Anaconda Copper Mining Co. The ACI recommendations abbreviate the work a little.

Ordinarily, the bar schedule and summary of weights are on the drawing containing the details of the members, but, owing to lack of space, they have been assembled in Fig. 14-24.

The following points are worthy of note as a guide in making such details:

1. All rods are numbered differently if they vary in size, length, or detail.

2. All bars must be supported adequately. Such rods as *a*11 in Fig. 14-22, section *D-D*, appear to have no supporting ties, but they are actually tied to the rods which project from the wall below (already poured).

3. Rods should be of such size and length that they can be handled and placed with reasonable facility. Ordinary lengths for ordering are 30 to 60 ft. The last are often inconveniently long and heavy.

4. The thickness of the rods must be allowed for in the details. In Fig. 14-23, the top reinforcement in section *A-A* is placed 4 in. below the top of the slab so that it will pass under the rods shown 3 in. below the top of the slab in section *B-B*. Furthermore, the depth of the beam *B*1 is such that its lower rods will not hit those of beam *B*2. As an aid in visualizing such matters, it is enlightening for one to lay a few pencils on his desk to simulate the proposed arrangement of the bars.

5. Such figures as "3 × 13" shown for bars *a*3 in the upper left corner of Fig. 14-22 indicate that there are 3 similar sets of 13 bars each.

6. In detailing walls and slabs, as in Fig. 14-22, it is frequently advisable to make separate plans for top and bottom mats so as to clarify the details.

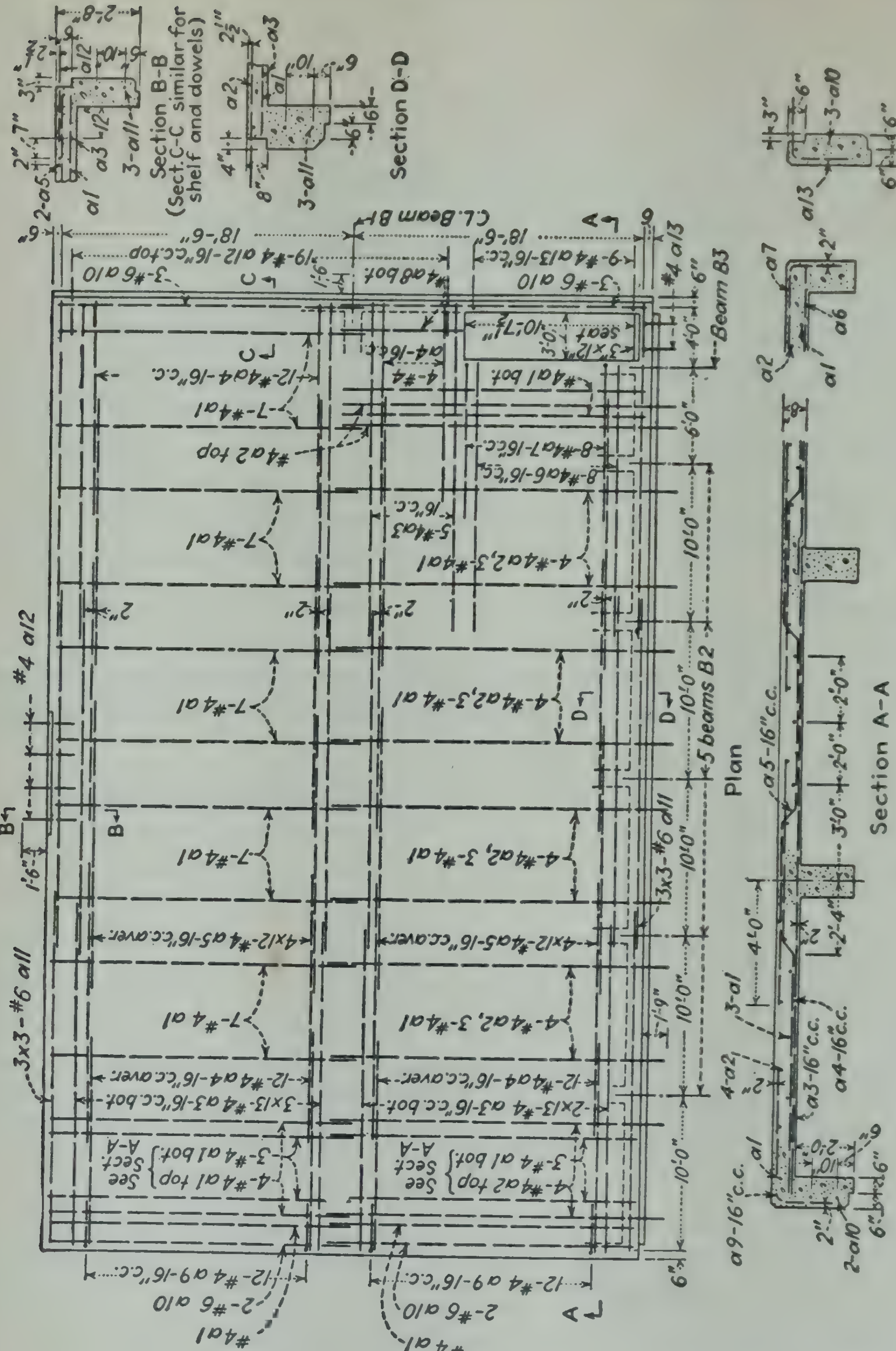


FIG. 14-22. Details of floor slab.

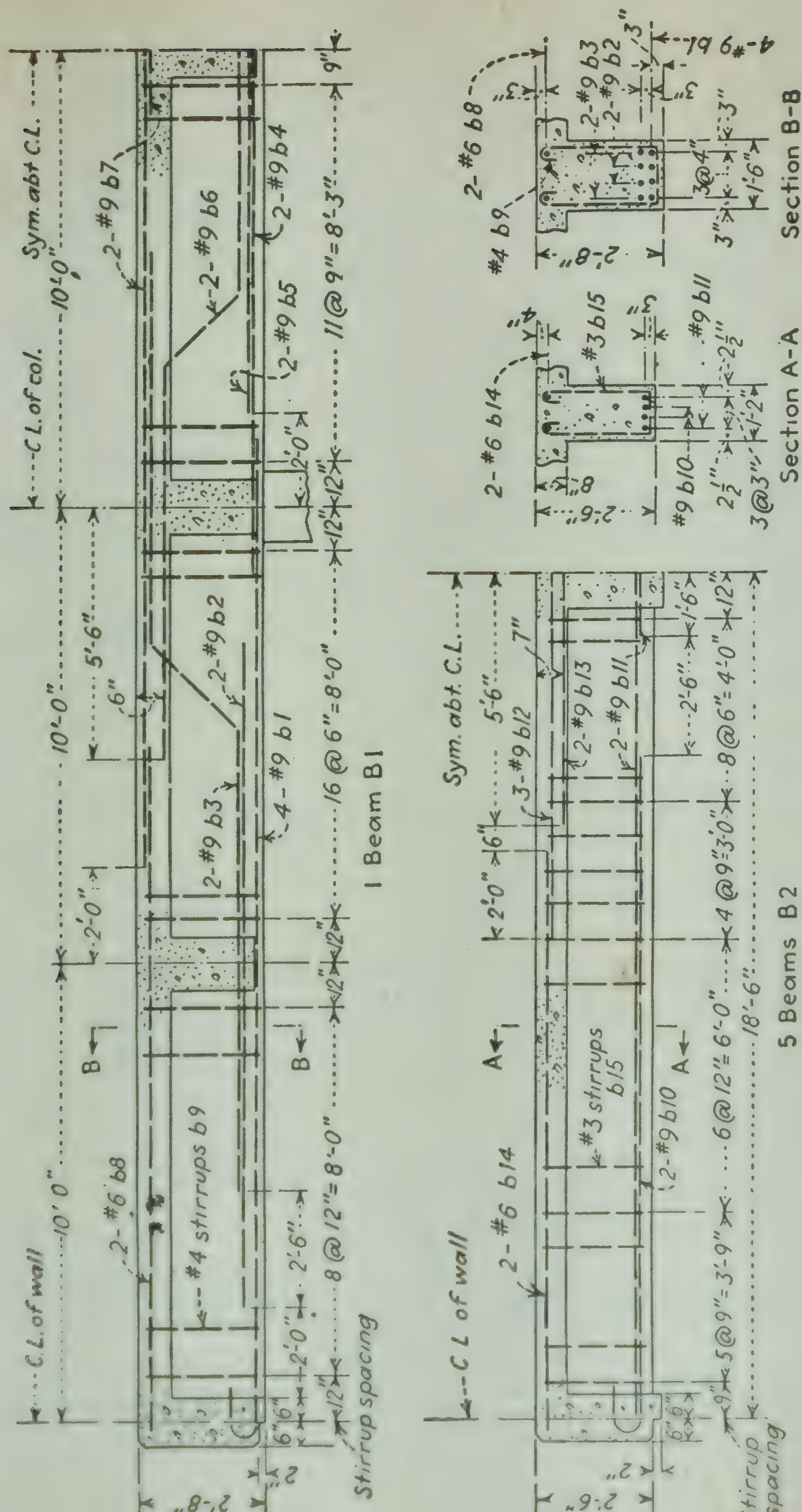
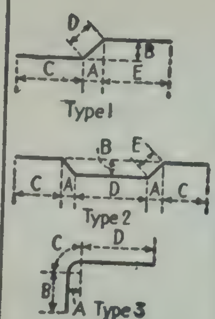


FIG. 14-23. Details of beams.

(a) Bar Schedule for Fig. 14-22

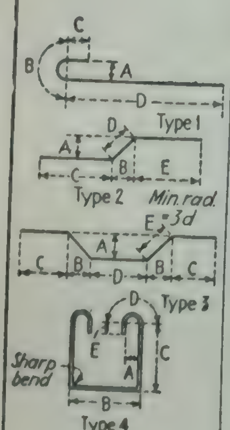


Mark	Size	No. reqd.	Type	A	B	C	D	E	Length	Total, lin ft	Remarks
a1	No. 4	61	Str.						19′9″	1,204.75	Lap at B1
a2	No. 4	22	Str.						21′9″	478.50	Lap at B1
a3	No. 4	70	Str.						21′0″	1,470.0	
a4	No. 4	40	1	4″	4″	7′10½″	5½″	6′0″	14′4″	573.33	
a5	No. 4	96	2	4″	4″	6′0″	5′4″	5½″	18′3″	1,752.0	
a6	No. 4	8	Str.						17′0″	136.0	
a7	No. 4	8	3	3¼″	1′6″	5″	10′1″		12′0″	96.0	
a8	No. 4	1	Str.						8′0″	8.0	
a9	No. 4	24	3	3¼″	1′7″	5″	4′6″		6′6″	156.0	
a10	No. 6	10	Str.						19′9″	197.50	Wire to wall bars
a11	No. 6	18	Str.						21′0″	378.0	Wire to wall bars
a12	No. 4	23	Str.						3′6″	80.5	
a13	No. 4	11	3	3¼″	1′7″	5″	2′0″		4′0″	44.0	

(b) Summary of Weights for Fig. 14-22

Size, No.	Lin ft	Wt per ft	Weight, lb
6	575.50	1.502	864
4	5,999.08	0.668	4,007
Total for dwg.......			4,871

(c) Bar Schedule for Fig. 14-23



Mark	Size	No. reqd.	Type	A	B	C	D	E	Length	Total, lin ft	Remarks
b1	No. 9	8	1	7″	11″	4″	21′6″		22′9″	182.0	
b2	No. 9	4	Str.						14′6″	58.0	
b3	No. 9	4	2	1′11″	1′11″	10′0″	2′8½″	8′6½″	21′3″	85.0	
b4	No. 9	2	Str.						23′0″	46.0	Outer
b5	No. 9	2	Str.						16′0″	32.0	Inner
b6	No. 9	2	3	1′8″	1′8″	8′6″	10′8″	2′4″	32′4″	64.67	
b7	No. 9	4	Str.						19′6″	78.0	Lap at C. L.
b8	No. 6	4	Str.						15′0″	60.0	
b9	No. 4	76	4	3″	1′2″	2′2″	5½″	2″	6′8″	506.67	
b10	No. 9	20	1	7″	11″	4″	14′6″		15′9″	315.0	
b11	No. 9	20	Str.						20′0″	400.0	Lap in vert. plane
b12	No. 9	15	Str.						16′0″	240.0	One in center
b13	No. 9	10	Str.						11′0″	110.0	
b14	No. 6	20	Str.						12′9″	255.0	
b15	No. 3	240	4	2¼″	10½″	2′0″	4⅛″	1½″	5′9″	1,380.0	

(d) Summary of Weights for Fig. 14-23

Size, No.	Lin ft	Wt per ft	Weight, lb
9	1,610.67	3.400	5,476
6	315.0	1.502	473
4	506.67	0.668	338
3	1,380.0	0.376	519
Total for dwg.......			6,806

FIG. 14-24. Illustrative bar schedules and summaries.

7. Bending diagrams for bars can be shown ordinarily by single lines, as in Fig. 14-24(c), type 1, but they are sometimes shown by the use of double lines, as in type 4, when the over-all dimensions are important for clearance. However, the latter arrangement often leads to errors in computing the required length of the rod; *e.g.*, for $b9$, $\frac{1}{2}$ in. must be deducted from B and $\frac{1}{4}$ in. from C to allow for the half diameter of the rod because the length is computed along the center line of the bar. When using single lines, dimensions such as A for type 1, Fig. 14-24(c), must allow for the thickness of the bar. Incidentally, bending of rods to form closed figures is usually inadvisable.

8. Avoid using detail dimensions with fractions less than $\frac{1}{8}$ in. because greater accuracy in cutting and bending is not practicable.

9. In bending stirrups, it is sometimes advisable to use sharp bends—radius to inner surface of rod = $\frac{1}{2}$ diameter of the main bar—when the main rods come in the corners, as in Fig. 14-24(c), type 4, because large curves tend to cause the main rods to crowd together.

14-17. Miscellaneous details. There are many practical and theoretical points which the engineer learns by experience. A few of these are pointed out here, because they sometimes cause trouble. Referring to the sketches in Fig. 14-25, note the following:

(a) When a beam parallel to a wall is poured against it, when it is keyed or bonded to it, or when it rests upon the edge of the wall, it will try to shift its load to the wall because its deflection is prevented. The result is a breaking of the junction or an eccentric load on the wall. The beam and the wall should be separated by a deflection joint (tar paper or similar isolating material), or the construction should be made wall bearing.

(b) When an expansion bearing is provided as shown in the sketch, the frictional resistance to motion will set up tensile stresses (at both fixed and expansion ends) which require special hooks in the rods.

(c) and (d). When rods are curved or offset, tensile stresses in them will tend to straighten them out and to spall the concrete unless they are tied back or otherwise detailed properly.

(e) When horizontal shelves, offsets, or brackets occur, like AB in the sketch, it is not advisable to try to pour the top surface against forms because of the uplift and the difficulty of filling the forms properly. When AB is sloped appreciably, forms can be used, but uplift must be guarded against.

(f) Corner reinforcement should be made so that one set of outside rods is bent around the corner. When there is a tendency to open up the inside corner, the inner rods A should be hooked near the outside of the wall so as to reinforce this corner. Without these hooks, rods A have very little effective anchorage. If the walls are thin, the continuity will be poor. There is insufficient room for the hooks so that it may be best to use a single rod for A, to bend it around 270° to the right so as to form an inside loop, the other end extending to the left.

(g) Rods at corners and in projecting parts—like A in the sketch—may cause trouble unless they are tied in properly. They should be placed so as not to interfere with the placing of the concrete.

(h) In cases of shelves where rods A are bent back, one must be careful to space

them far enough apart to avoid forming a screen effect that will interfere with the placing of the concrete. Small U-shaped rods which are placed by hand are often more advantageous than bending of the main rods. The vertical rods B should rest on the construction joint, with dowels to tie them into the lower pour. Any horizontal ties between the slab and the wall should be above the construction joint if the floor is poured last.

(*i*) Sharp corners such as at this expansion joint should be avoided because they may break off.

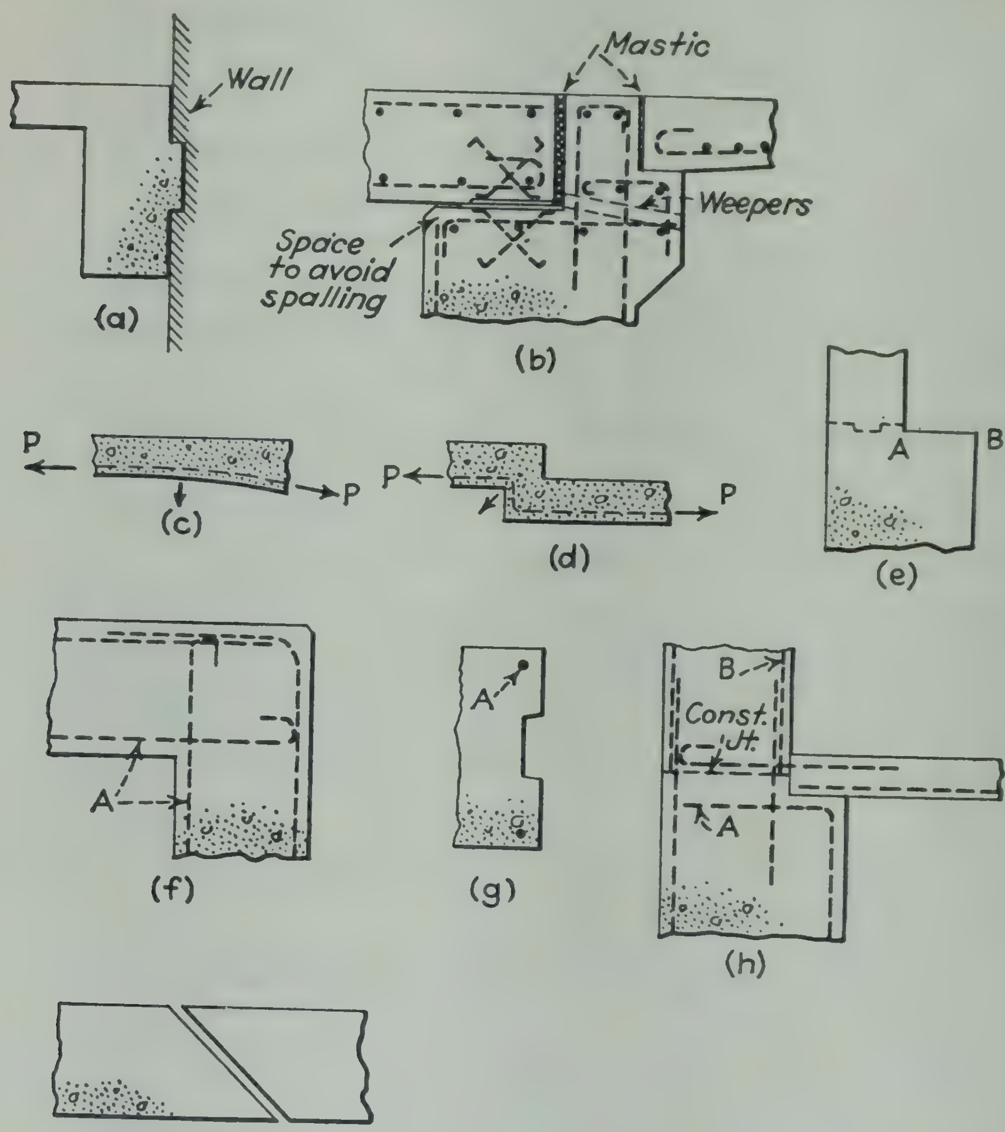


FIG. 14-25.

In planning all layouts of reinforcement, one must consider that wire chairs or mortar pads must be used to support rods that are above horizontal forms; vertical rods cannot hang in the air but must rest upon a support, such as the previous pour, or they must be wired to other rods that are so supported; horizontal rods must be wired to the vertical ones which act as small columns in supporting them; intersecting rods must be tied together thoroughly; tie rods or spacer rods must be used to hold

the main reinforcement in line; and multiple layers of rods must be held by ties and separators so as to make sure that their proper relative positions are maintained.

When surfaces have to be screeded, it is important to avoid projecting reinforcement that will impede the screeding. Small dowels or other reinforcement may be inserted after the finishing is complete. This often applies to bridge decks.

Excessively large groups of closely spaced rods may look well on a drawing, but they are very difficult to place accurately and to hold in position in the field. If they get out of line, if the upper layers sag too close to the lower ones, or if one of the rods is bent sideways so as to get too close to the forms or to an adjacent bar, they tend to act as a screen which makes the placing of the concrete very difficult—and the development of adequate bond very questionable. Honeycombing, segregation of the aggregate, and air pockets are likely to be the result. When such a group of rods is at the top of a large member, it is desirable to arrange the bars so that there will be one or two strips of clear space (about 5 or 6 in. wide) through which the concrete can be deposited and compacted.

These matters that have been discussed here may seem to be minor details, but careful attention to such practical things often makes the difference between a good job and an unsatisfactory one. Furthermore, good judgment, common sense, and the ability to supervise work carefully will always be among the greatest assets of the designer and the builder of reinforced-concrete structures.

These are some of the things that one should bear in mind when he is designing a concrete structure. Continued study by architects and engineers, more extensive experience by building contractors, and greater skill and care on the part of the men in the field—all these things together will bring still further advances in this great field of construction.

15
DESIGN PROBLEMS

15-1. Introduction. It is the purpose of this chapter to present several general layouts or plans of structures that are to be built of concrete. The plans will show the barest outline as it might be prepared by an architect, an owner, a mechanical layout man, or someone else who establishes the general dimensions required and the facilities to be housed. The student, as a structural engineer, is to develop the framework and details of the structure when built of concrete.

This method of approach is that required in practice. It is essential that the student learn to tackle such creative problems, not just to determine stresses in or sizes of particular isolated members. This is because an engineer does not design a bunch of beams and columns first, then try to fit them together to form a structure. He has in mind first a structure that he wants; then he determines how to make it strong enough. Practice with detail problems has been given with most of the preceding chapters in order to train the reader in the use of the tools of analysis and detailed designing. Now he should use these tools and practice building something with them.

The problems given here vary from small structures that are relatively simple to some that involve considerable difficulty. Thus the reader can practice upon whatever his abilities and the available time permit.

15-2. Assumptions. It is assumed that the student is not familiar with the analysis of statically indeterminate structures. Therefore, when continuity is involved, as it so frequently is in structures built of concrete, he has to make approximations. He may assume that the bending moments in ordinary continuous beams are the same as they would be if their ends were fixed, or if one end were fixed and the other were simply supported. Then he can use the data given in Figs. 1 and 2 of the Appendix. These are sufficient for the present purpose, which is to learn how to create designs for structures made of concrete. Similarly, the bending of columns can be neglected for the time being except where it is to be determined from causes other than that of frame action.

15-3. Deformed structures. When dimensioning structural members, especially when determining and locating the reinforcement, it is very helpful if the engineer will visualize the action of the structure under loads. This picturing of deformed structures to exaggerated scale is one of the things that Hardy Cross has emphasized so effectively. A good engineer will do it mentally even if he does not take time to put it

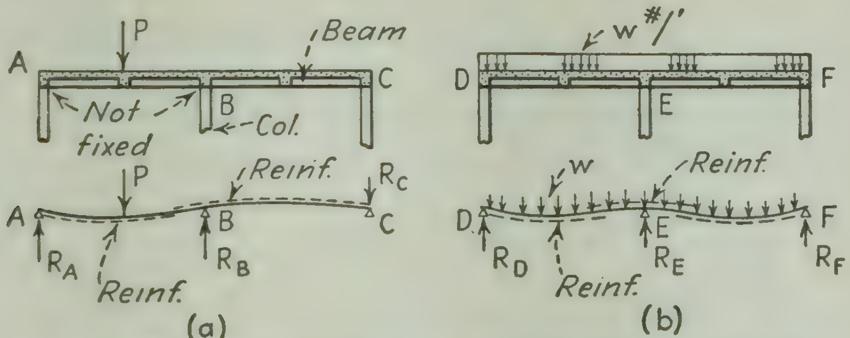


FIG. 15-1. Deformations of a two-span continuous beam.

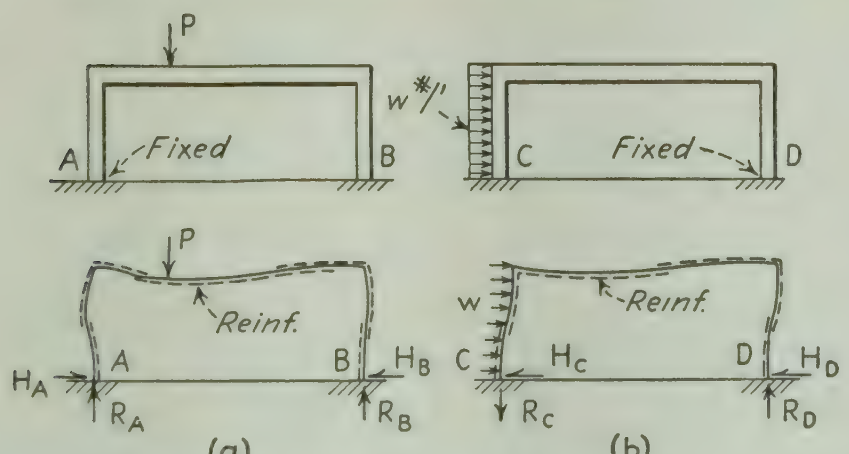


FIG. 15-2. Deformations of a rigid frame.

on paper. The beginner cannot afford to neglect this simple but effective aid to his designing.

Just a few illustrations are given to show the kind of thinking and the method of picturization. Figure 15-1 shows the deformations of a two-span beam. Figure 15-2 pictures a single frame under the action of two different loading conditions. In Fig. 15-3(*a*) is shown a loaded bin on columns, whereas (*b*) illustrates the action of a deep trench in the earth. The frames in Fig. 15-4 are given in order to picture the effects of some loads on certain continuous structures. In all the illustrations, the dotted lines show the location of the principal tensile reinforcement for the conditions pictured.

The reader should practice making such pictures until he develops a good sense of structural action. These problems will give him a chance to test his ability.

15-4. Concept of design. The student probably does not realize how largely the design of a structure is influenced by what the owner wants, by what the engineer wants, by what the architect wants, by what is good practice, by what is known to be satisfactory construction,

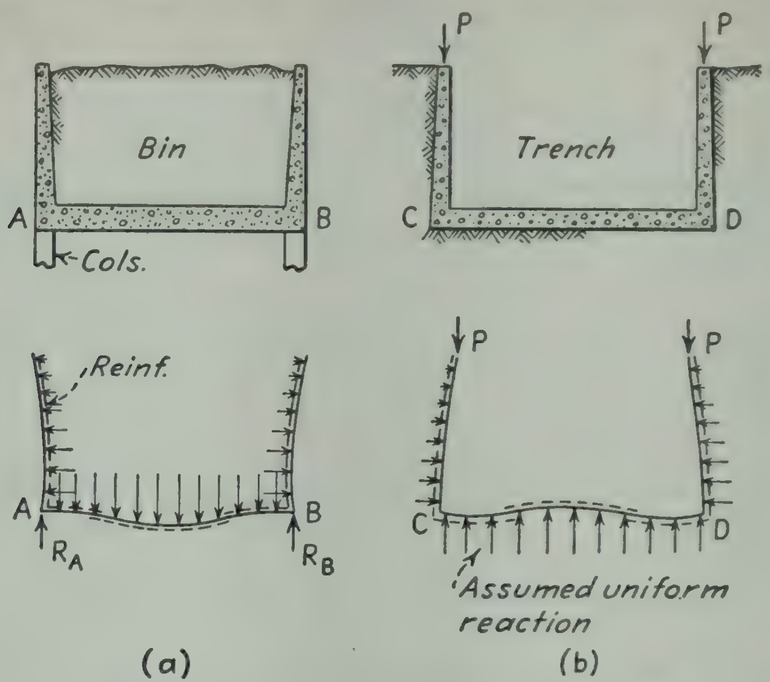


FIG. 15-3. Deformations of bin and trench.

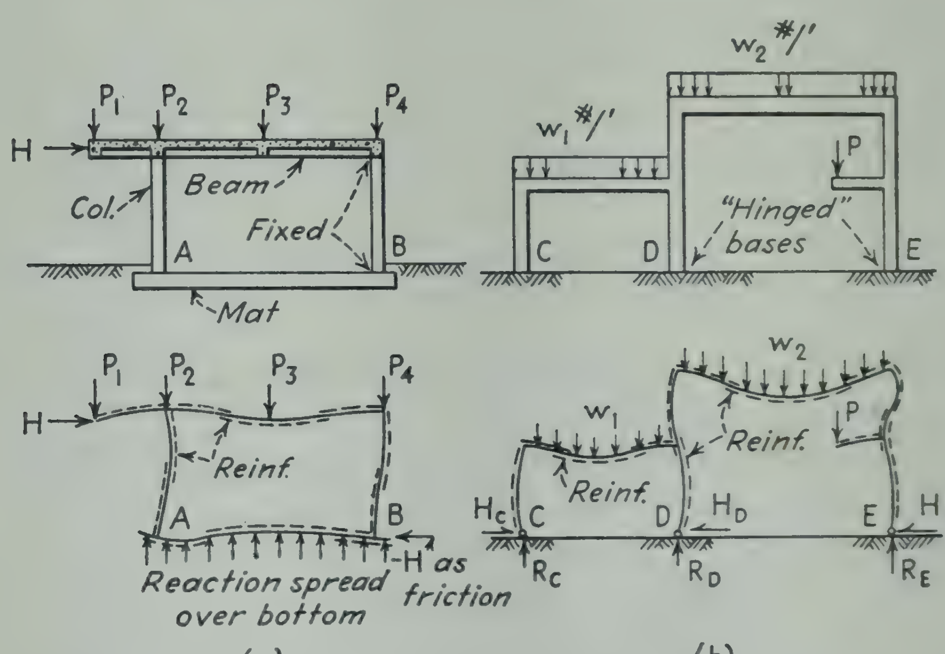


FIG. 15-4. Study of deformations of frames.

by what is most suitable for foundation conditions, by what will minimize operating and maintenance costs, and by what is economical construction in a particular region. In general, a structure is planned in considerable detail before the strength of its component parts is determined. These parts are then made strong enough to serve their pur-

poses. Not always are they the minimum sizes that will support the loads; they are what seem to be suitable and appropriate members for that particular structure. Thus good engineering judgment is rightly one of the assets of the planner of such structures.

In a broad sense, *design* means the planning, shaping, styling, and general proportioning of a structure. By custom, design is also used to denote *dimensioning*, which is the assignment of depths, widths, thicknesses, details, and reinforcement. These last have little effect upon economy after the general design has been determined. The student should realize this. Far too often he seems to think that a structure is or should be an assembly of members each having just the theoretical balanced design of steel vs. concrete, and that the structure must be made to use these members.

Stiffness is often one of the desired qualities of a structure. Admittedly it may be difficult or almost impossible to define or to state just what satisfactory stiffness means. It varies with different structures and their uses. Here again judgment and experience are of value.

As examples, two cases will be used to illustrate some of these ideas in an effort to show how that quality of judgment enters the picture.

A school was planned with a central corridor in each wing. The adjacent classrooms were 24 ft wide. The architect wanted to have a flat ceiling without beam haunches, yet he wanted it to be thin. He planned to use a lightweight floor spanning the 24 ft. Although it was only 9 in. deep, it was theoretically strong enough to resist the shears and bending moments. However, his engineer changed the depth to 15 in., used T beam (tin-pan) construction with a hung ceiling, and deliberately used overreinforced members. The purpose was to reduce the deflection under live load. Any apparent springiness of the floor, vibration of lights attached to the ceiling below, cracking of the junctions of floor and partitions above it, or crushing of partitions below were to be avoided. Hence the conservatism.

A beam-and-slab highway overpass was to be built at a metallurgical plant. Over the bridge was to pass a more or less continuous stream of ore trucks weighing 60 tons each when loaded. The bridge had been designed for a 45-ft span. Then it was decided to reduce the span to 35 ft by narrowing the roadway below. The designer reduced the sizes of the beams and widened their spacing to get what seemed to be a very efficient design in terms of the use of concrete and reinforcement. The supervisor ordered that the beams be spaced as before and that diaphragms be used between them in order to be sure that differential deflection of adjacent beams would not rupture the stiff floor slab. He also ordered that the T beam stems be deepened and widened, that the steel stress be limited to 18,000 psi in order to reduce cracking and to

provide an allowance for impact of unknown magnitude, that the stirrups be designed to withstand all the shear, and that part of the longitudinal reinforcement be bent up to give a cradled effect that would help to tie the simply supported T beam construction together. Was this wasteful? Probably it was wise action. How much did it cost to make sure that those heavy trucks would not cause fatigue and disintegration of the structure? The extra cost was really small since it merely added a little more steel and concrete to the deck but did not materially change the costly abutments, forms, finishing, and many other items of expense.

What is the most economical type of structure? That question is difficult to answer. In the case of small structures, any sensible type will probably be satisfactory, and the savings due to framing systems of various sorts will not be important. However, when one plans a large structure, and especially a multistory one, where considerable duplication exists, it may be wise to make comparative studies to determine what will be best for that particular case.

Again it should be called to the reader's attention that the cost of a concrete structure does not vary directly with the quantity of concrete used in it. Skimpy members may not be worth while even if they do yield small savings because a structure should be satisfactory as well as safe. Unsatisfactory structures mean money wasted or, at least, money spent unwisely.

For example, assume a beam-and-girder warehouse floor to hold a live load of 300 psf. The columns are 20 by 24 ft on centers. The beams are 24 ft long and 10 ft c.c. The girders are 20 ft long with one concentrated load in the middle. The quantities are estimated for a sturdy design and for another using skimpy sections. The same materials and allowable stresses are used for both designs. The estimated quantities are multiplied by the following assumed unit prices:

1. Transit-mixed concrete delivered and placed = \$15 per yd^3.
2. Reinforcing in place = \$0.10 per lb.
3. Forms = \$0.75 per ft^2.
4. Finishing of floor surface = \$0.15 per ft^2.

The results are the following:

Material	*Sturdy design*		*Skimpy design*	
Concrete	18.6 × 15 =	\$279.00	12.8 × 15 =	\$192.00
Reinforcement	3,460 × 0.10 =	346.00	4,120 × 0.10 =	412.00
Forms	736 × 0.75 =	552.00	722 × 0.75 =	541.00
Finishing	480 × 0.15 =	72.00	480 × 0.15 =	72.00
		\$1,249.00		\$1,217.50

Thus the costs are not much different. In the sturdy design, the slab is 8 in. thick; in the other, 5 in. These figures are given only for the purpose of showing some scale as to how costs may be affected by different features.

For the work of this chapter, the student is urged to invent a structural system that is as simple as possible, one that transmits the loads to

the foundation as directly and efficiently as it can be done, and one that is structurally reliable. It should also be easy to build.

It is natural for one to ask, "Why do we want this thing?" and "Why not make it some other way?" In general, the owner wants a structure to serve a special purpose, and he wants the engineers and contractors to design and build it for him. They should do so but, if better and more economical things can obviously be done, they very properly should discuss the matter with him. On the other hand, difficulties in the engineering work are seldom sufficient reason for not giving the owner what he wants when operating economies and special service are to be secured for him. Structural safety is naturally essential because an owner should not have to worry about the safety of his structure. If he is willing to pay for it, and if he prefers it to some suggested alternate, he should be given what he wants. Therefore, changing the general scheme is not the solution for any of the problems here.

Where not shown specifically in the problems, the dimensions, locations, and details are to be assumed for doors, windows, elevators, electric lighting, heating, etc. Sweet's catalogues, books on architectural standards, and the publications of manufacturers are suitable sources for such information. In practice, all these matters have to be worked out and provided for in the design. However, in these problems, the principal emphasis must necessarily be upon the structural part of each one.

15-5. Problems. In order to have uniformity in the qualities of the materials used in the problems in this chapter, assume the following unless stated otherwise in a particular case: $f'_c = 3{,}000$ psi; reinforcement is intermediate-grade A 305 bars; and the allowable unit stresses are to be as given in the Code, Table 1-8.

In many cases, the drawings show the live loads and other pertinent data needed for the design. Graphical scales are given when they seem to be necessary.

The problems purposely cover a wide range of ordinary concrete construction. They are all based upon practical cases, but many have been simplified in some minor details. It is believed that enough problems are provided to suit the skill and available time of individual students and of groups working as engineering squads.

Problem 15-1. Figure 15-5 shows the floor plan and some other pertinent data for a pump house at an industrial plant. The entire structure is to be built of concrete. The crane rails are to be supported upon concrete beams that rest upon pilasters. The crane has two wheels 8 ft apart at each end. The maximum wheel loads are 15,000 lb each.

Problem 15-2. Figure 15-6 shows the general plans for a small warehouse to be supported upon 30-ton piles. The floor is to be poured on

the ground but it must be self-supporting. The structure is to have a concrete roof, concrete columns, and spandrel beams. The walls are to be 4 in. of brick veneer with 8 in. of cinder blocks as backing.

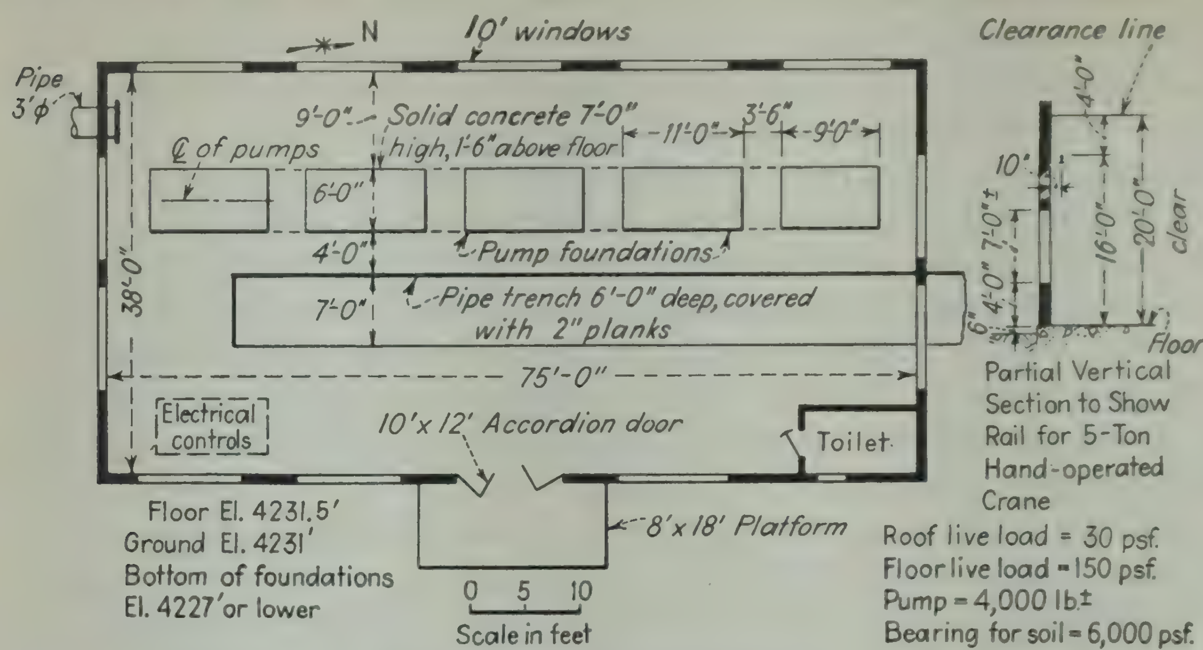


FIG. 15-5. Plan for a pump house.

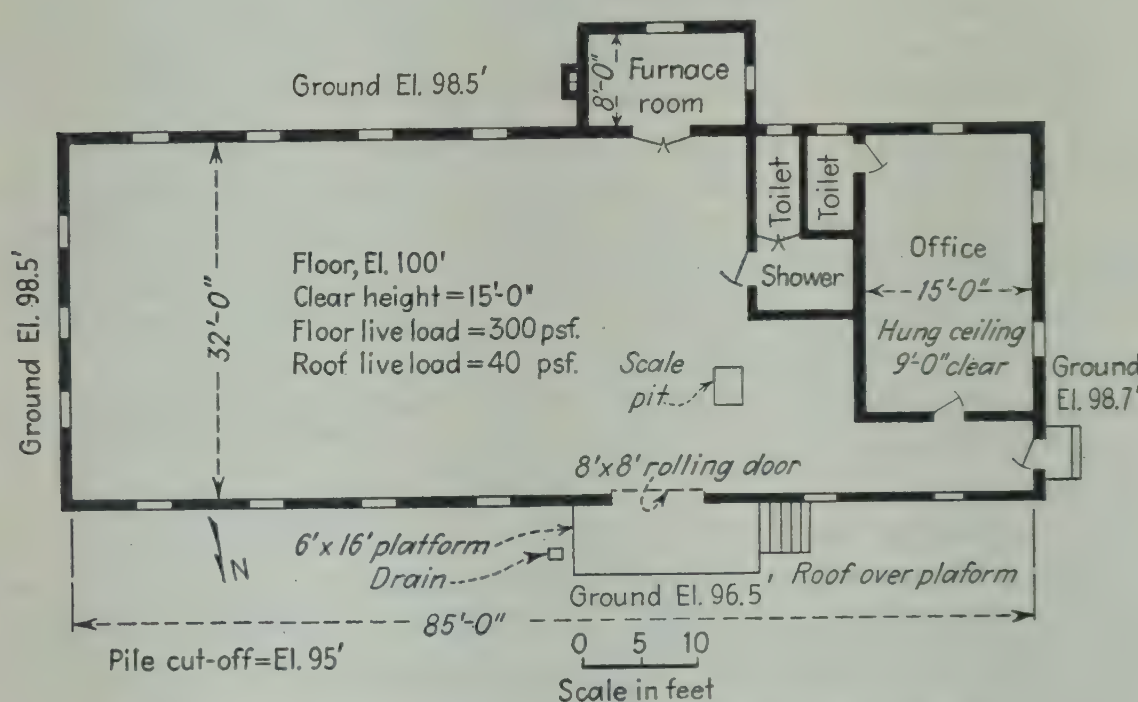


FIG. 15-6. A small warehouse on poured-in-place concrete piles.

Problem 15-3. Figure 15-7 shows the floor plan and other data for a small office building at an industrial plant. Roof, floors, and skeleton framework are to be of concrete. Use hung ceilings except in the basement. Other materials are optional.

Problem 15-4. Figure 15-8 pictures the plan for a multistory warehouse. Type and materials are optional except that the structure is to

be built with concrete framing or flat-slab construction. The layout is symmetrical about the center line of the building except for the offices, vault, and furnace room in the southeast corner. There are no similar facilities in the northeast corner but the toilets (T) are duplicated.

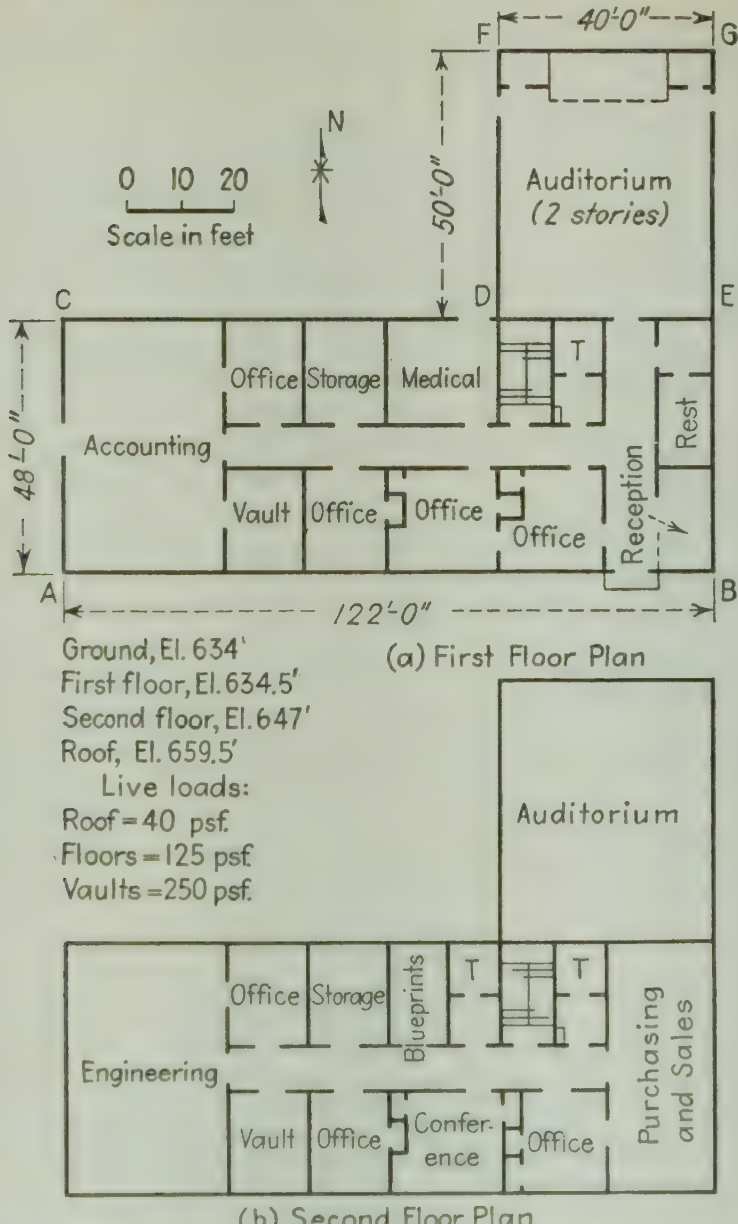
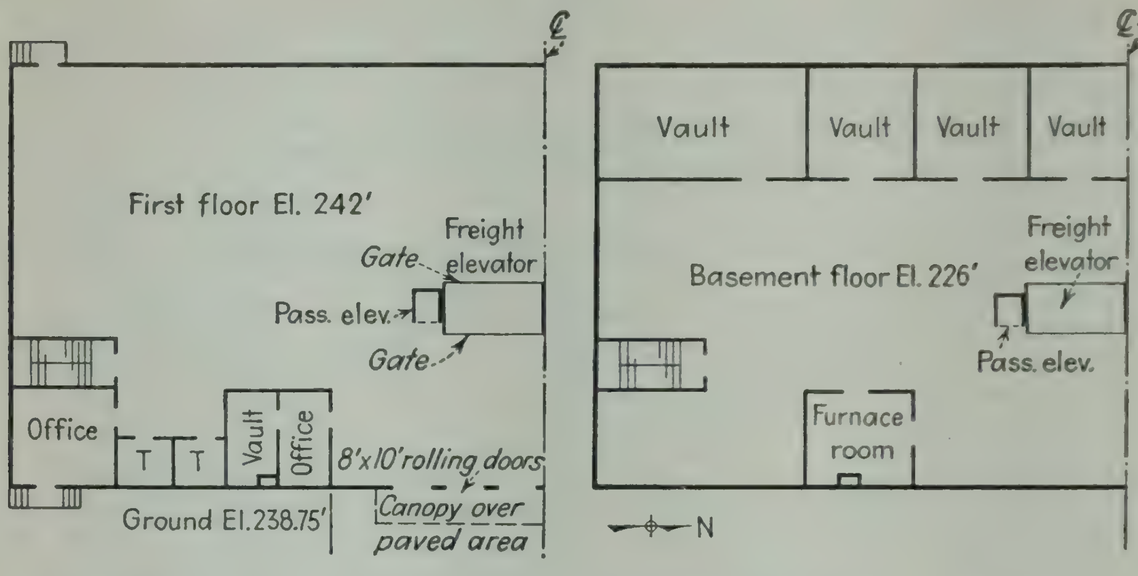


FIG. 15-7. Plan of an office building at an industrial plant.

Problem 15-5. The highway embankment in Fig. 15-9 is to have a box culvert extending through it as shown. Design the culvert, including its floor and end walls. Assume that the culvert supports all the fill above it plus a live load of 200 psf. The fill weighs 110 pcf. The soil under the culvert is sand of good bearing value.

Problem 15-6. Design the tunnel shown in Fig. 15-10. Consider the varying depths of earth and ore. Assume $w = 110$ pcf for ore and earth, $\phi = 35°$, $f'_c = 4{,}000$ psi, $f_s = 24{,}000$ psi, and p for soil $= 12{,}000$ psf.

This problem seems to be simple, but the two sections under the ore pile must support very large loads. Assume that the tunnel is built in open cut and that it must support all the material over it. The structure is built in short units in order to avoid cracks if unequal settlements occur.



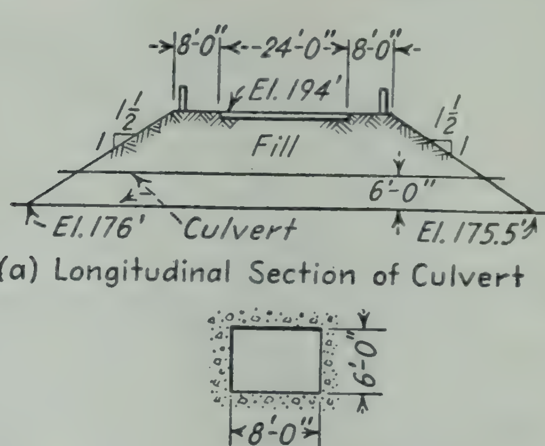
(a) First Floor Plan
Other floors similar except corner office omitted
2nd floor, El. 260'; 3rd floor, El. 278'; Roof, El. 294'

(b) Basement Floor Plan

Live loads:
Roof = 30 psf.
Floors = 300 psf.
Basement = 200 psf.

Safe soil bearing value = 6,000 psf.

FIG. 15-8. Partial plans of a large warehouse approximately 80 by 200 ft.



(a) Longitudinal Section of Culvert

(b) Transverse Section of Culvert

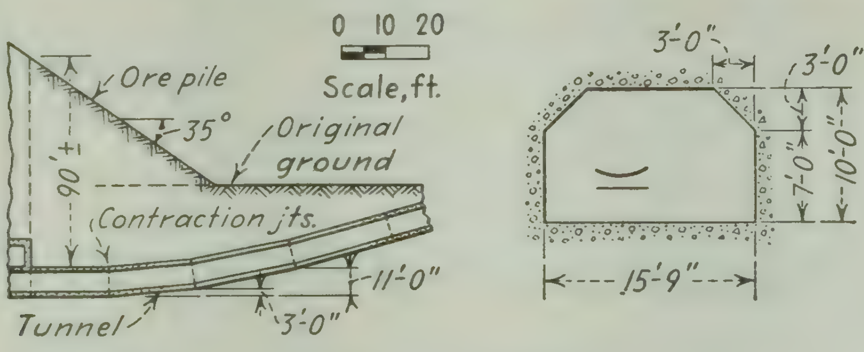
FIG. 15-9. A culvert under a highway embankment.

Problem 15-7. Figure 15-11 pictures the general dimensions for a three-span highway overpass. Assume a uniform live load of 250 psf. including impact. The lower roadway is in a cut. The soil is sandy, and it will support a pressure of 3½ tons per ft² safely.

Design the superstructure and draw pictures of the piers and abut-

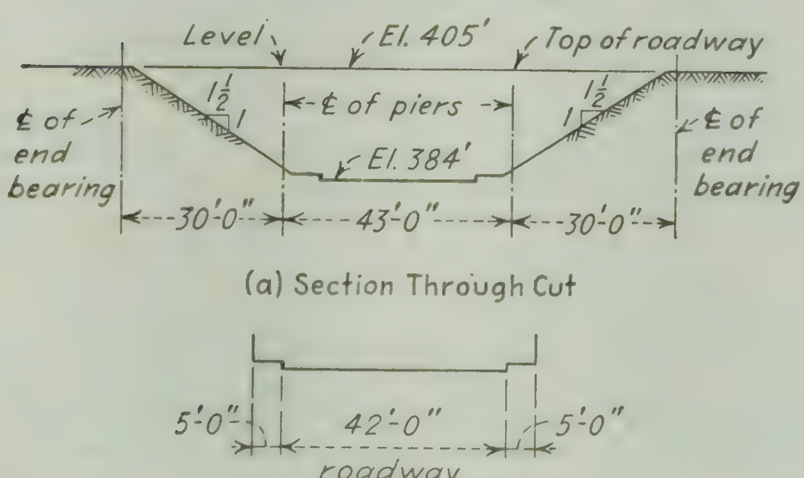
ments. The student may invent his own architectural and engineering features for the project.

Problem 15-8. Figure 15-12(*a*) shows a preliminary study of a highway bridge or trestle; (*b*) shows the dimensions of the proposed roadway and sidewalks. The superstructure is assumed to be a pair of two-span continuous sections of beam-and-slab type supported upon abutments



(a) Vertical Section (b) Inside Dimensions of Tunnel

FIG. 15-10. A conveyor tunnel under an ore pile.



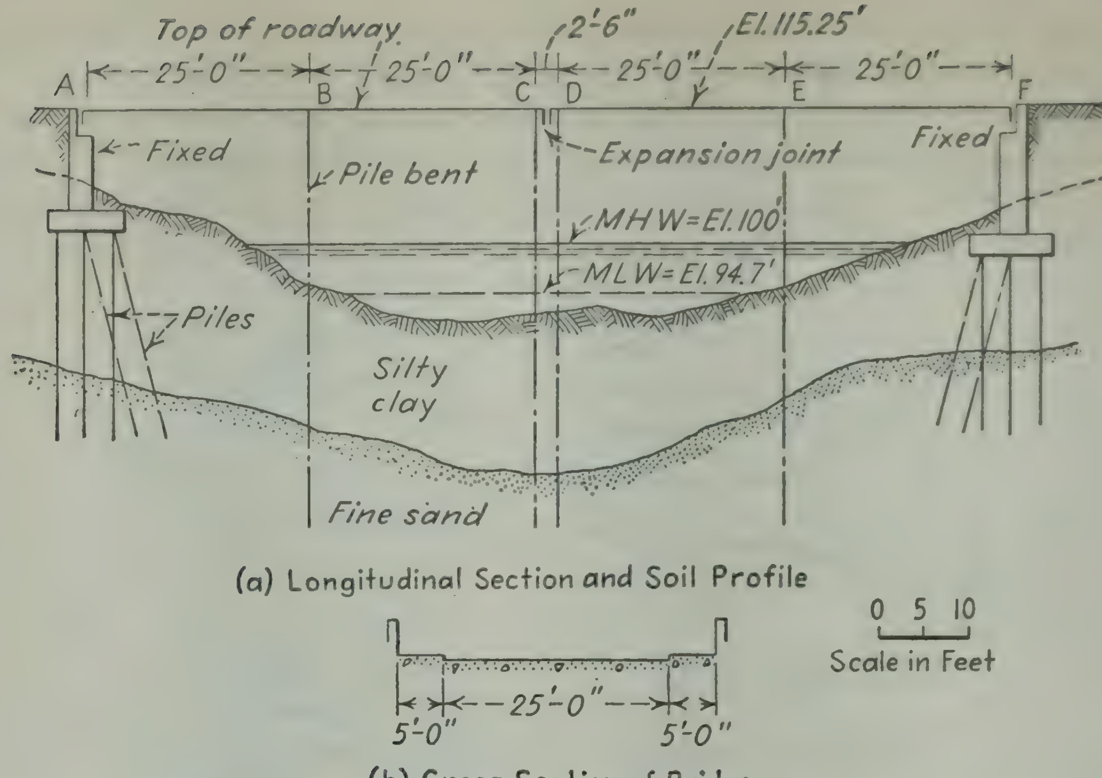
(a) Section Through Cut

(b) Dimensions of Overpass

FIG. 15-11. A highway grade-crossing elimination.

at A and F and upon piles and a cap or cross girder at B, C, D, and E. The abutments are 40 ft wide at the top, and they have wing walls parallel to the main abutment and sloping at $1\frac{1}{2}:1$ on top for a distance of 6 ft each side. Beyond that point, the embankment may extend around the ends of the wings. The piles are precast concrete. For the abutments, they are 12 in. square and good for 18 tons each. For the piers or pile bents, they are 16 in. octagonal and can safely support 22 tons apiece.

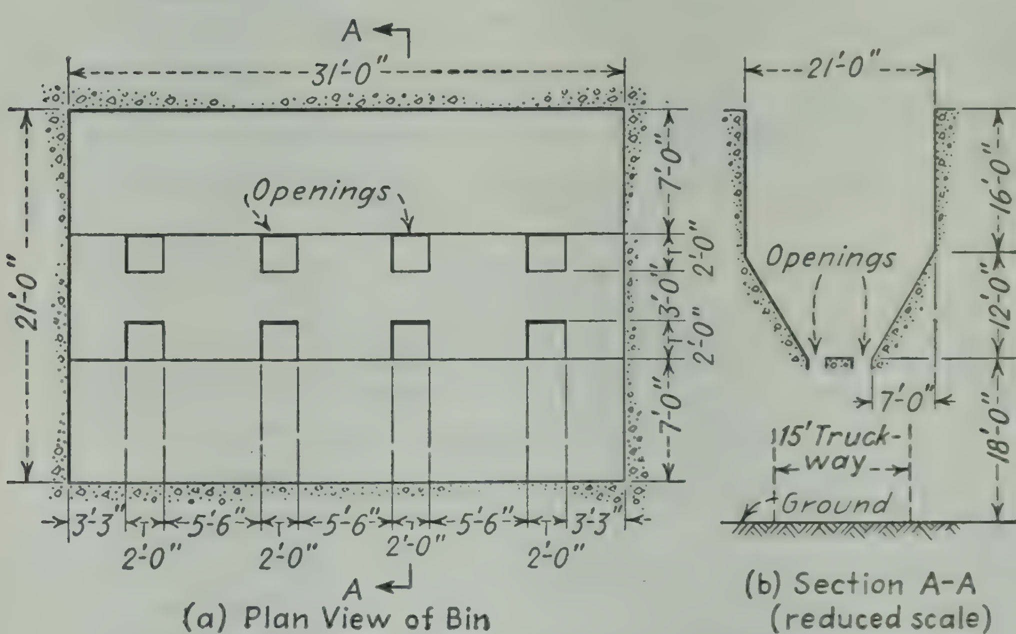
Assume a uniform live load of 250 psf on the roadway and 100 psf on the sidewalks. This includes impact. Design the superstructure and the bents, and at least make a sketch of one abutment.



(a) Longitudinal Section and Soil Profile

(b) Cross Section of Bridge

FIG. 15-12. Highway crossing of a small tidal stream.



(a) Plan View of Bin

(b) Section A-A (reduced scale)

FIG. 15-13. Inside dimensions of an ore bin.

Problem 15-9. Figure 15-13 pictures an elevated concrete bin for a granular product weighing 110 pcf. Assume $\phi = 30°$. Design the bin, assuming it to be filled level with the top. Assume that the lateral pressures are the same as they would be if the sides were retaining walls. Also design the supporting structure, assuming that a width of 15 ft and a height of 15 ft 6 in. must be maintained for trucks and gates under the openings. The soil is good for a pressure of 5 tons per ft².

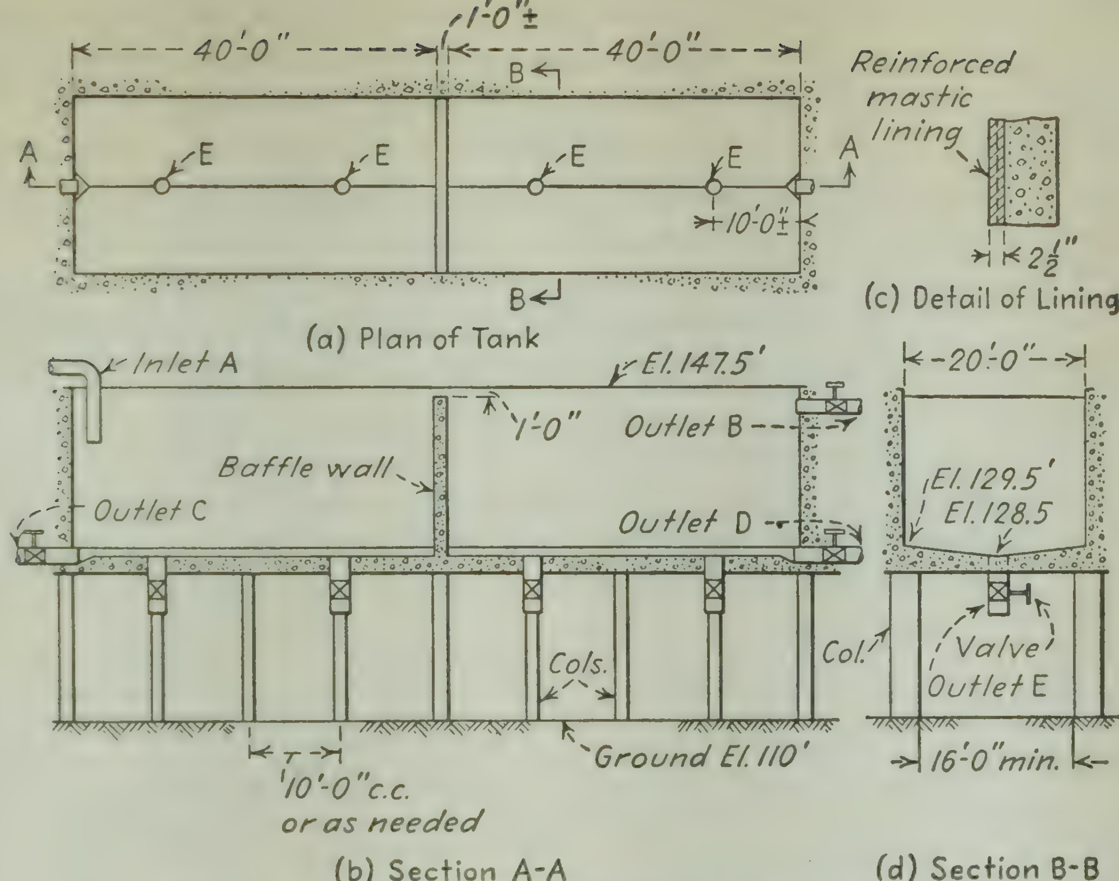


FIG. 15-14. A twin tank for chemicals at an industrial plant.

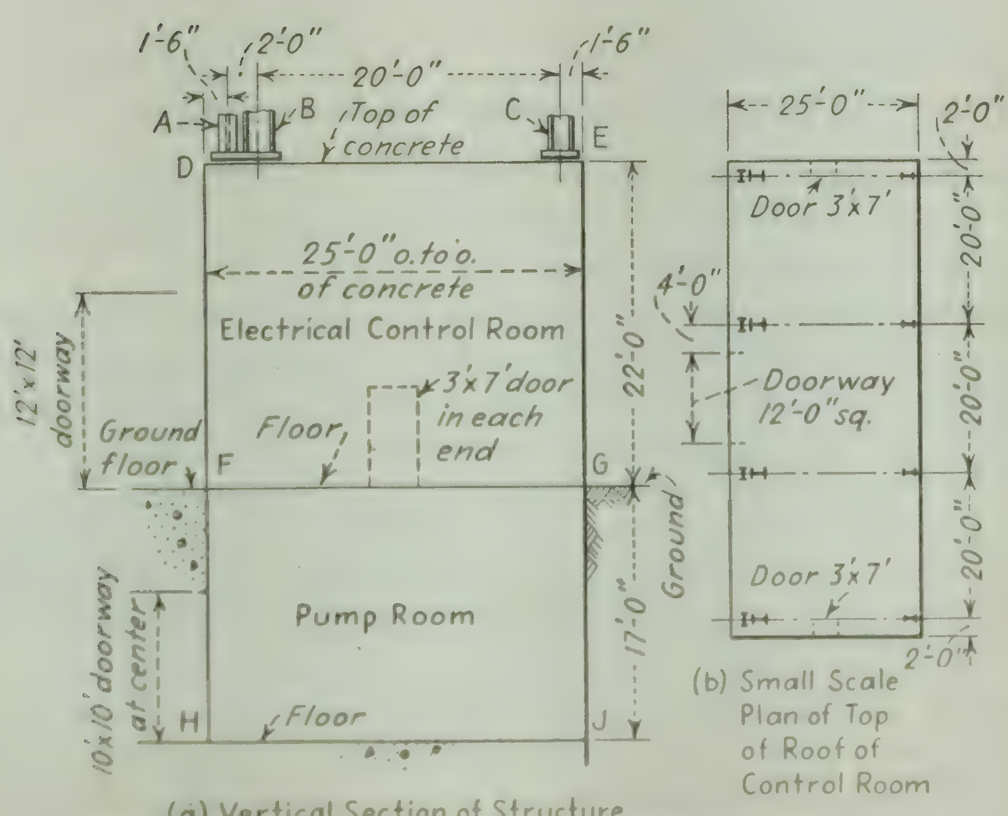
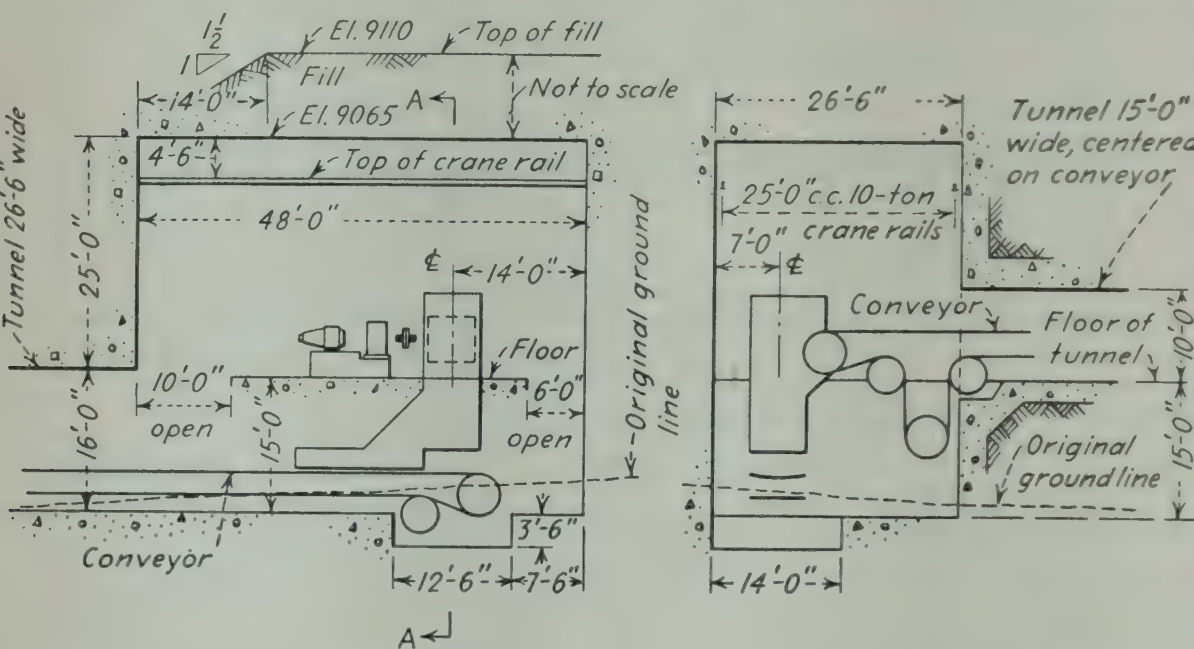


FIG. 15-15. Control and pump rooms at a large industrial plant.

Problem 15-10. Figure 15-14 pictures a two-compartment tank elevated above ground. It is generally filled at *A* and overflows the baffle into the right-hand compartment and thence out at *B*. The chemical can be drawn off through *C* and *D* when desired. Outlets *E* are used for disposing of sludge by emptying into tank trucks run under them.

Assume that the liquid weighs 70 pcf and that the compartments are filled up to the top. Design the tank and its supports for both compartments filled and for the left one full and the right one empty. The soil



(a) Longitudinal Section (b) Section A-A

FIG. 15-16. A conveyor junction house under a deep fill.

is safe for a bearing pressure of 4 tons per ft². The dimensions shown are for the concrete. Assume that the 2½-in. mastic lining covers all interior surfaces and weighs 25 psf.

Problem 15-11. Figure 15-15 shows a portion of an industrial plant. The upper part *DEGF* of this 25- by 64-ft structure contains electrical equipment. Assume a live load of 200 psf for the top, 150 psf for the control-room floor, and 200 psf for the pump room. The load on crane columns *A* may be assumed as 90 kips each. Columns *B* and *C* support a bin as well as the superstructure. Assume $B = 200$ kips each and $C = 160$ kips each. The soil can safely support a load of 8,000 psf. The access to the pump room *FGJH* is through a 10 ft square centrally located door in wall *FH*.

Problem 15-12. Figure 15-16 shows the plans for a conveyor junction house at a mine. It is under a very large embankment as indicated in (*a*). The crane rails are to be supported upon corbels projecting from the side walls.

Assume that the fill weighs 100 pcf, and that $\phi = 35°$. Assume $f'_c = 4{,}000$ psi, $f_s = 24{,}000$ psi, and other stresses as specified by the Code. Design the roof, bottom, side walls, and tunnels for the earth loads and lateral pressures only. The intermediate floor can be assumed to be a 12-in. slab with four beams across the 26.5-ft width. Assume that the maximum wheel loads of the crane at one end are two reactions of 18,000 lb at 9 ft c.c. The soil below the structure is hard gravel that can safely resist a pressure of 12,000 psf.

This structure involves some very difficult problems. Both bending and shear are very large. The locations and details of construction joints need special attention. The junction of the 15-ft tunnel at the right of Sketch (*b*) may crack if the tunnel settles on the fill under it. Probably both tunnels should be cut loose from the main structure.

APPENDIX

Explanation of Data. The tables and diagrams here are intended to be of special assistance to the designer of reinforced-concrete structures. They are not to take the place of the extensive information that is given in handbooks. The following instructions are given for each one:

Table 1. This contains data pertaining to the latest type of deformed bars, and it also gives information regarding the large square rods that were formerly prevalent and that may occasionally be encountered. The ¼-in. round bars are furnished in plain rods only.

TABLE 1. Table of Areas, Perimeters, Weights, and Other Data for Reinforcing Bars

All bars are deformed bars except ¼-in. rounds, which are plain

Type of bar	Bar No. or size*	Nominal dimensions			Weight, plf	Deformations		
		Diameter, in.	Net area, in.²	Perimeter, in.		Max avg spacing, in.	Min height, in.	Max gap, in.†
ASTM Designation: A-305-50T‡	3	0.375	0.11	1.178	0.376	0.262	0.015	0.143
	4	0.500	0.20	1.571	0.668	0.350	0.020	0.191
	5	0.625	0.31	1.963	1.043	0.437	0.028	0.239
	6	0.750	0.44	2.356	1.502	0.525	0.038	0.286
	7	0.875	0.60	2.749	2.044	0.612	0.044	0.334
	8	1.000	0.79	3.142	2.670	0.700	0.050	0.383
	9	1.128	1.00	3.544	3.400	0.790	0.056	0.431
	10	1.270	1.27	3.990	4.303	0.889	0.064	0.487
	11	1.410	1.56	4.430	5.313	0.987	0.071	0.540
Other types§	¼ in. round	0.250	0.05	0.78	0.17			
	1 in. square		1.00	4.00	3.44			
	1⅛ in. square		1.27	4.50	4.35			
	1¼ in. square		1.56	5.00	5.37			

* Numbers 3 to 11, inclusive, are based on number of ⅛ in. in nominal diameter of the bar section.

† Chord of 12½ per cent of nominal perimeter.

‡ Data given by American Society for Testing Materials. The ASTM Standards are subject to revision from time to time. ASTM headquarters in Philadelphia has copies and information on latest editions.

§ Bars used before introduction of A 305 type. Those between ¼ in. round and 1 in. square have same area and perimeter as corresponding size of A 305 type.

TABLE 2. Areas and Perimeters of Reinforcing Bars per Foot of Slab

Spacing, in.	Size of bar																			
	2		3		4		5		6		7		8		9		10		11	
	¼ in. ϕ		⅜ in. ϕ		½ in. ϕ		⅝ in. ϕ		¾ in. ϕ		⅞ in. ϕ		1 in. ϕ		1⅛ in. $+\phi$		1¼ in. $+\phi$		1$\frac{7}{16}$ in. $-\phi$	
	A_s	Σo	A_s	Σo	A_s	Σo	A_s	Σo	A_s	Σo	A_s	Σo	A_s	Σo	A_s	Σo	s	Σo	A_s	Σo
2	0.30	4.68	0.66	7.08	1.20	9.42	1.86	11.76												
2½	0.24	3.74	0.53	5.66	0.96	7.54	1.49	9.41	2.11	11.33										
3	0.20	3.12	0.44	4.72	0.80	6.28	1.24	7.84	1.76	9.44	2.40	11.00	3.16	12.56	4.00	14.18				
3½	0.17	2.67	0.38	4.05	0.69	5.39	1.06	6.72	1.51	8.09	2.06	9.43	2.71	10.77	3.43	12.15	4.35	13.68		
4	0.15	2.34	0.33	3.54	0.60	4.71	0.93	5.88	1.32	7.08	1.80	8.25	2.37	9.42	3.00	10.63	3.81	11.97	4.68	13.29
4½	0.13	2.08	0.29	3.15	0.53	4.19	0.83	5.23	1.17	6.29	1.60	7.33	2.11	8.37	2.67	9.45	3.39	10.64	4.16	11.81
5	0.12	1.87	0.26	2.83	0.48	3.77	0.74	4.70	1.06	5.66	1.44	6.60	1.90	7.54	2.40	8.51	3.05	9.58	3.74	10.63
5½	0.11	1.70	0.24	2.57	0.44	3.43	0.68	4.28	0.96	5.15	1.31	6.00	1.72	6.85	2.18	7.73	2.77	8.70	3.40	9.66
6	0.10	1.56	0.22	2.36	0.40	3.14	0.62	3.92	0.88	4.72	1.20	5.50	1.58	6.28	2.00	7.09	2.54	7.98	3.12	8.86
6½	0.09	1.44	0.20	2.18	0.37	2.90	0.57	3.62	0.81	4.36	1.11	5.08	1.46	5.80	1.85	6.54	2.34	7.37	2.88	8.18
7	0.09	1.34	0.19	2.02	0.34	2.69	0.53	3.36	0.75	4.05	1.03	4.71	1.35	5.38	1.71	6.08	2.18	6.84	2.67	7.59
7½	0.08	1.25	0.18	1.89	0.32	2.51	0.50	3.14	0.70	3.78	0.96	4.40	1.26	5.02	1.60	5.67	2.03	6.38	2.50	7.09
8	0.08	1.17	0.16	1.77	0.30	2.36	0.46	2.94	0.66	3.54	0.90	4.12	1.18	4.71	1.50	5.32	1.90	5.98	2.34	6.64
9	0.07	1.04	0.15	1.57	0.27	2.09	0.41	2.61	0.59	3.15	0.80	3.67	1.05	4.19	1.33	4.72	1.69	5.32	2.08	5.91
10	0.06	0.94	0.13	1.42	0.24	1.88	0.37	2.35	0.53	2.83	0.72	3.30	0.95	3.77	1.20	4.25	1.52	4.79	1.87	5.32
11	0.05	0.85	0.12	1.29	0.22	1.71	0.34	2.14	0.48	2.57	0.65	3.00	0.86	3.43	1.09	3.87	1.39	4.35	1.70	4.83
12	0.05	0.78	0.11	1.18	0.20	1.57	0.31	1.96	0.44	2.36	0.60	2.75	0.79	3.14	1.00	3.54	1.27	3.99	1.56	4.43

TABLE 3. Areas and Perimeters of Reinforcing Bars

No. of bars	Size of bar: 2 — ¼ in. φ — A_s	Σo	3 — ⅜ in. φ — A_s	Σo	4 — ½ in. φ — A_s	Σo	5 — ⅝ in. φ — A_s	Σo	6 — ¾ in. φ — A_s	Σo	7 — ⅞ in. φ — A_s	Σo	8 — 1 in. φ — A_s	Σo	9 — 1⅛ in. + φ — A_s	Σo	10 — 1¼ in. + φ — A_s	Σo	11 — 1$\frac{7}{16}$ in. − φ — A_s	Σo
1	0.05	0.78	0.11	1.18	0.20	1.57	0.31	1.96	0.44	2.36	0.60	2.75	0.79	3.14	1.00	3.54	1.27	3.99	1.56	4.43
2	0.10	1.56	0.22	2.36	0.40	3.14	0.62	3.92	0.88	4.72	1.20	5.50	1.58	6.28	2.00	7.09	2.54	7.98	3.12	8.86
3	0.15	2.34	0.33	3.54	0.60	4.71	0.93	5.88	1.32	7.08	1.80	8.25	2.37	9.42	3.00	10.6	3.81	12.0	4.68	13.3
4	0.20	3.12	0.44	4.72	0.80	6.28	1.24	7.84	1.76	9.44	2.40	11.0	3.16	12.6	4.00	14.2	5.08	16.0	6.24	17.7
5	0.25	3.90	0.55	5.90	1.00	7.85	1.55	9.80	2.20	11.8	3.00	13.8	3.95	15.7	5.00	17.7	6.35	20.0	7.80	22.2
6	0.30	4.68	0.66	7.08	1.20	9.42	1.86	11.8	2.64	14.2	3.60	16.5	4.74	18.8	6.00	21.3	7.62	23.9	9.36	26.6
7	0.35	5.46	0.77	8.26	1.40	11.0	2.17	13.7	3.08	16.5	4.20	19.2	5.53	22.0	7.00	24.8	8.89	27.9	10.9	31.0
8	0.40	6.24	0.88	9.44	1.60	12.6	2.48	15.7	3.52	18.9	4.80	22.0	6.32	25.1	8.00	28.4	10.2	31.9	12.5	35.4
9	0.45	7.02	0.99	10.6	1.80	14.1	2.79	17.6	3.96	21.2	5.40	24.8	7.11	28.3	9.00	31.9	11.4	35.9	14.0	39.9
10	0.50	7.80	1.10	11.8	2.00	15.7	3.10	19.6	4.40	23.6	6.00	27.5	7.90	31.4	10.0	35.4	12.7	39.9	15.6	44.3
11	0.55	8.58	1.21	13.0	2.20	17.3	3.41	21.6	4.84	26.0	6.60	30.2	8.69	34.5	11.0	39.0	14.0	43.9	17.2	48.7
12	0.60	9.36	1.32	14.2	2.40	18.8	3.72	23.5	5.28	28.3	7.20	33.0	9.48	37.7	12.0	42.5	15.2	47.9	18.7	53.2
13	0.65	10.1	1.43	15.3	2.60	20.4	4.03	25.5	5.72	30.7	7.80	35.8	10.3	40.8	13.0	46.1	16.5	51.9	20.3	57.6
14	0.70	10.9	1.54	16.5	2.80	22.0	4.34	27.4	6.16	33.0	8.40	38.5	11.1	44.0	14.0	49.6	17.8	55.9	21.8	62.0
15	0.75	11.7	1.65	17.7	3.00	23.6	4.65	29.4	6.60	35.4	9.00	41.2	11.8	47.1	15.0	53.2	19.0	59.8	23.4	66.4
16	0.80	12.5	1.76	18.9	3.20	25.1	4.96	31.4	7.04	37.8	9.60	44.0	12.6	50.2	16.0	56.7	20.3	63.8	25.0	70.9
17	0.85	13.3	1.87	20.1	3.40	26.7	5.27	33.3	7.48	40.1	10.2	46.8	13.4	53.4	17.0	60.2	21.6	67.8	26.5	75.3
18	0.90	14.0	1.98	21.2	3.60	28.3	5.58	35.3	7.92	42.5	10.8	49.5	14.2	56.5	18.0	63.8	22.9	71.8	28.1	79.7
19	0.95	14.8	2.09	22.4	3.80	29.8	5.89	37.2	8.36	44.8	11.4	52.2	15.0	59.7	19.0	67.3	24.1	75.8	29.6	84.2
20	1.00	15.6	2.20	23.6	4.00	31.4	6.20	39.2	8.80	47.2	12.0	55.0	15.8	62.8	20.0	70.9	25.4	79.8	31.2	88.6
21	1.05	16.4	2.31	24.8	4.20	33.0	6.51	41.2	9.24	49.6	12.6	57.8	16.6	65.9	21.0	74.4	26.7	83.8	32.8	93.0
22	1.10	17.2	2.42	26.0	4.40	34.5	6.82	43.1	9.68	51.9	13.2	60.5	17.4	69.1	22.0	78.0	27.9	87.8	34.3	97.5
23	1.15	17.9	2.53	27.1	4.60	36.1	7.13	45.1	10.1	54.3	13.8	63.2	18.2	72.2	23.0	81.5	29.2	91.8	35.9	101.9
24	1.20	18.7	2.64	28.3	4.80	37.7	7.44	47.0	10.6	56.6	14.4	66.0	19.0	75.4	24.0	85.1	30.5	95.8	37.4	106.3
25	1.25	19.5	2.75	29.5	5.00	39.2	7.75	49.0	11.0	59.0	15.0	68.8	19.8	78.5	25.0	88.6	31.8	99.8	39.0	110.8
26	1.30	20.3	2.86	30.7	5.20	40.8	8.06	51.0	11.4	61.4	15.6	71.5	20.5	81.6	26.0	92.1	33.0	103.7	40.6	115.2
27	1.35	21.1	2.97	31.9	5.40	42.4	8.37	52.9	11.9	63.7	16.2	74.2	21.3	84.8	27.0	95.7	34.3	107.7	42.1	119.6
28	1.40	21.8	3.08	33.0	5.60	44.0	8.68	54.9	12.3	66.1	16.8	77.0	22.1	87.9	28.0	99.2	35.6	111.7	43.7	124.0
29	1.45	22.6	3.19	34.2	5.80	45.5	8.99	56.8	12.8	68.4	17.4	79.8	22.9	91.1	29.0	102.8	36.8	115.7	45.2	128.5
30	1.50	23.4	3.30	35.4	6.00	47.1	9.30	58.8	13.2	70.8	18.0	82.5	23.7	94.2	30.0	106.3	38.1	119.7	46.8	132.9

Table 2. This contains information regarding cross-sectional areas and perimeters for use in designing and analyzing slabs. Except for the ¼-in. rounds, all bars are the modern A 305 type.

Table 3. This is a multiplication table to save time in computing areas and perimeters of groups of bars.

TABLE 4. Theoretical Minimum Embedment Length to Develop Bond—A 305 Reinforcing Bars

Length in inches

	f_s = 18,000				f_s = 20,000			
Bar No.	2,000	2,500	3,000	3,750*	2,000	2,500	3,000	3,750*
				$u = 0.10f'_c$				
3	8½	6¾	5¾	5	9½	7½	6½	5½
4	11½	9½	7½	6¾	12¾	10¼	8½	7½
5	14¼	11½	9½	8¼	15¾	13½	10½	9
6	17	13½	11¼	9¾	18¾	15	12½	10¾
7	19¾	15¾	13¼	11¼	22	17½	14¾	12½
8	22¾	18¼	15¼	13	25¼	20¼	16¾	14½
9	25½	20¾	17	14½	28¼	22½	19	16¼
10	28¾	23	19¼	16½	32	25½	21½	18¼
11	31¾	25½	21¼	18¼	35¼	28¼	23½	20¼
				$u = 0.07f'_c$				
3	12¼	9¾	8	7	12	10¾	9	7¾
4	16½	13¼	11	9½	18¼	14¾	12¼	10½
5	20½	16¼	13½	11¾	22¾	18	15	13
6	24¼	19½	16	13¾	26¾	21½	17¾	15¼
7	28¼	22½	18¾	16¼	31½	25¼	21	18
8	32½	26	21½	18½	36	28¾	24	20½
9	36½	29	24½	20¾	40½	32½	26¾	23
10	41¼	33	27½	23½	45¾	36½	30½	26
11	45½	36½	30¼	26	50½	40½	33½	28¾

* Max u = 350 psi (0.10 × 3,750 = 375 psi).

Table 4. This information is prepared to show the length of bar needed to develop various tensile unit stresses at lapped splices and anchorages of reinforcement by means of the allowable bond stresses specified in the Code for the strengths of concrete shown.

Figure 1. This contains data showing the bending moments at the ends of beams of constant section with both ends fixed and for some of the most common loading conditions. It is to be used when one must estimate the approximate bending in the interior spans of continuous beams.

$M = m \times W \times l$
$m =$ Coefficient taken from diagram
$W =$ Total load on beam
$l =$ Length of beam
$a =$ Length in terms of l

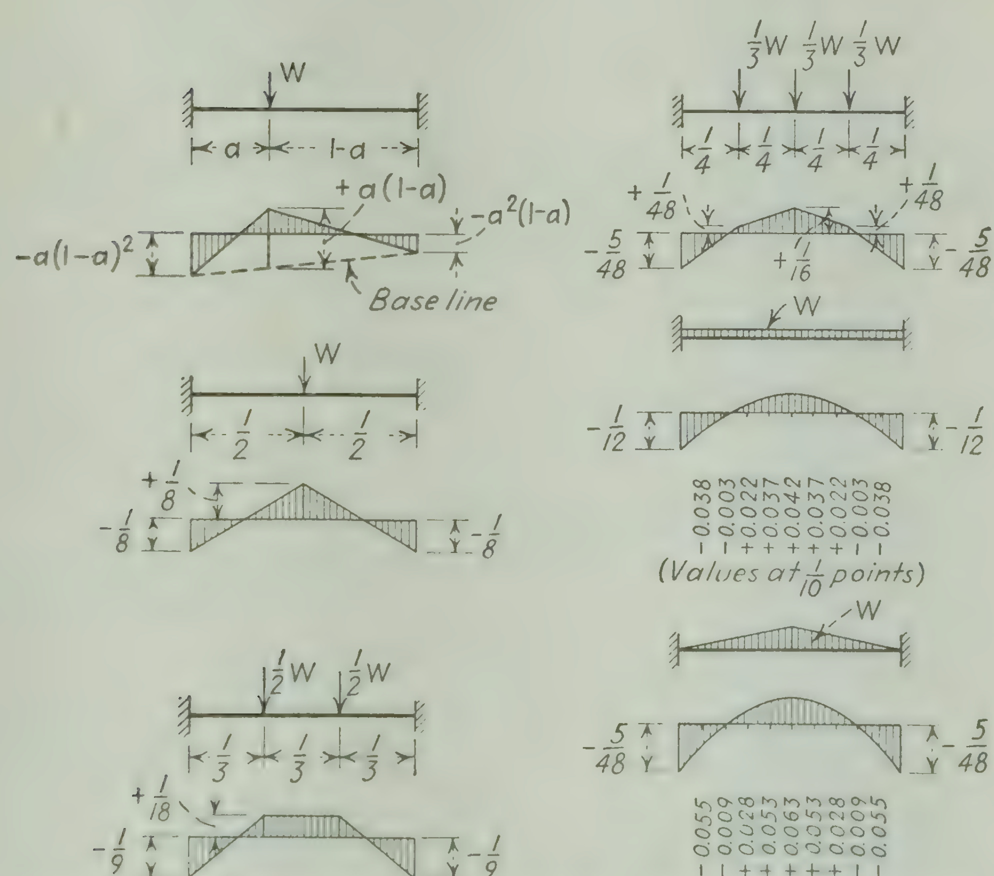


FIG. 1. Moments in beams of constant section with fixed ends. ***(Based upon data by Hardy Cross and the Portland Cement Association.)***

Figure 2. This information is for use in estimating the bending moments in the end spans of continuous beams.

$M = m \times W \times l$
m = Coefficient taken from diagram
W = Total load on beam
l = Length of beam

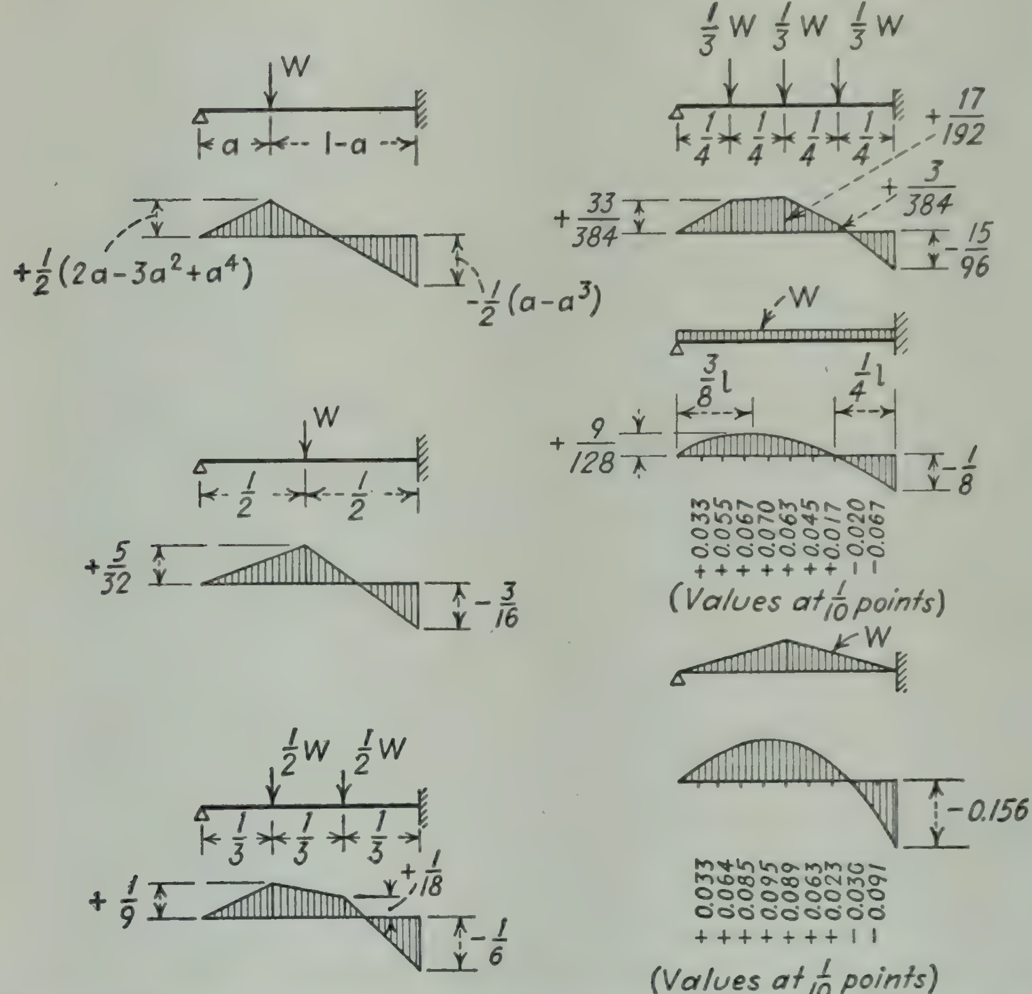


FIG. 2. Moments in beams of constant section with one end fixed and the other end simply supported.

Figure 3. By interpolating from this diagram one can determine the value of the coefficient in Coulomb's formula for the active pressure of earth against retaining walls when the soil is level or sloping behind the wall. The values are generally sufficiently accurate for the purpose.

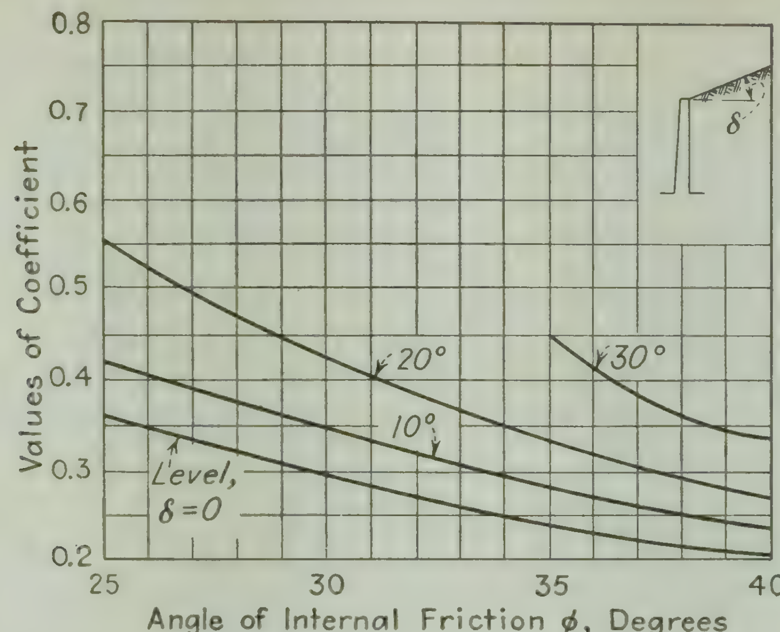


FIG. 3. Values of $\dfrac{\cos\phi}{(1+\sqrt{2\sin^2\phi - 2\sin\phi\cos\phi\tan\delta})^2}$ for Coulomb's equation for the magnitude of the active earth pressure inclined at angle ϕ when the surface is sloped at various angles δ.

Figures 4 *to* 9, *inclusive.* The analysis of rectangular beams can be greatly expedited by the use of these diagrams; they are also applicable for the analysis of T beams if the neutral axis lies within the flange or very close to it. In any case, the values of nA_s/b and $[(n-1)/b]A_s'$ per inch of width of the beam can be found, using b as the divisor for T beams as well as for rectangular ones when the bending moment is positive, or using b' for T beams when the bending moment is negative.

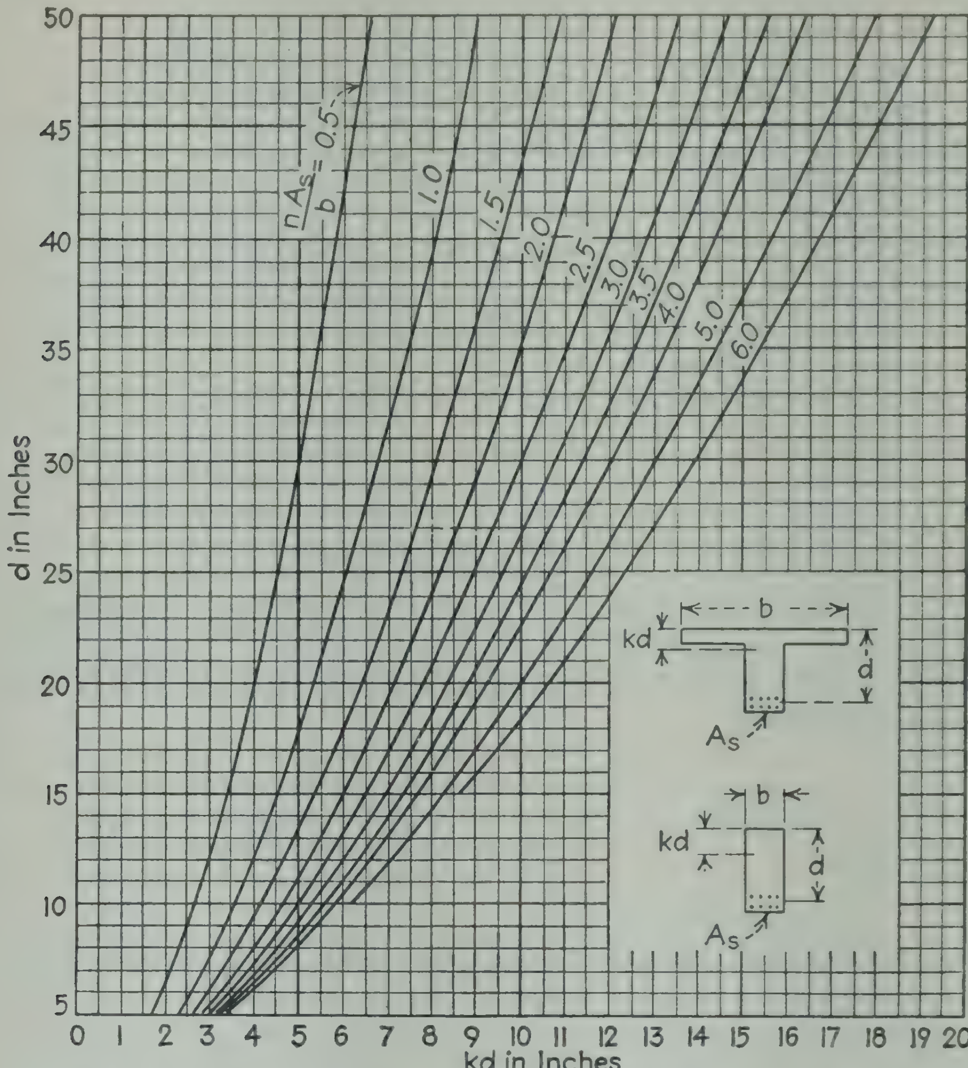


FIG. 4. Location of neutral axis of beam with tensile steel only.

The procedure in using the diagrams is as follows: When there is little or no steel in compression, use Figs. 4 and 5; otherwise use the curves that are prepared for the nearest value of $[(n - 1)/b]A_s'$; with d as the ordinate, cross horizontally to the proper (or interpolated) value of nA_s/b, then read the corresponding magnitudes of kd and S_c/b; multiply S_c/b by b (or b') to find S_c, then compute $S_s = S_c(kd)/n(d - kd)$ from the quantities already found. Results from two diagrams may be used for interpolation if greater accuracy is desired.

The advantage of these diagrams is the fact that they enable one to find the section moduli and thereby compute f_c and f_s. Of course, important members should be checked analytically after the diagrams have been used for approximate analysis.

In using these diagrams, it will generally be sufficient to assume that

$$j = 1 - \tfrac{1}{3}\, kd/d$$

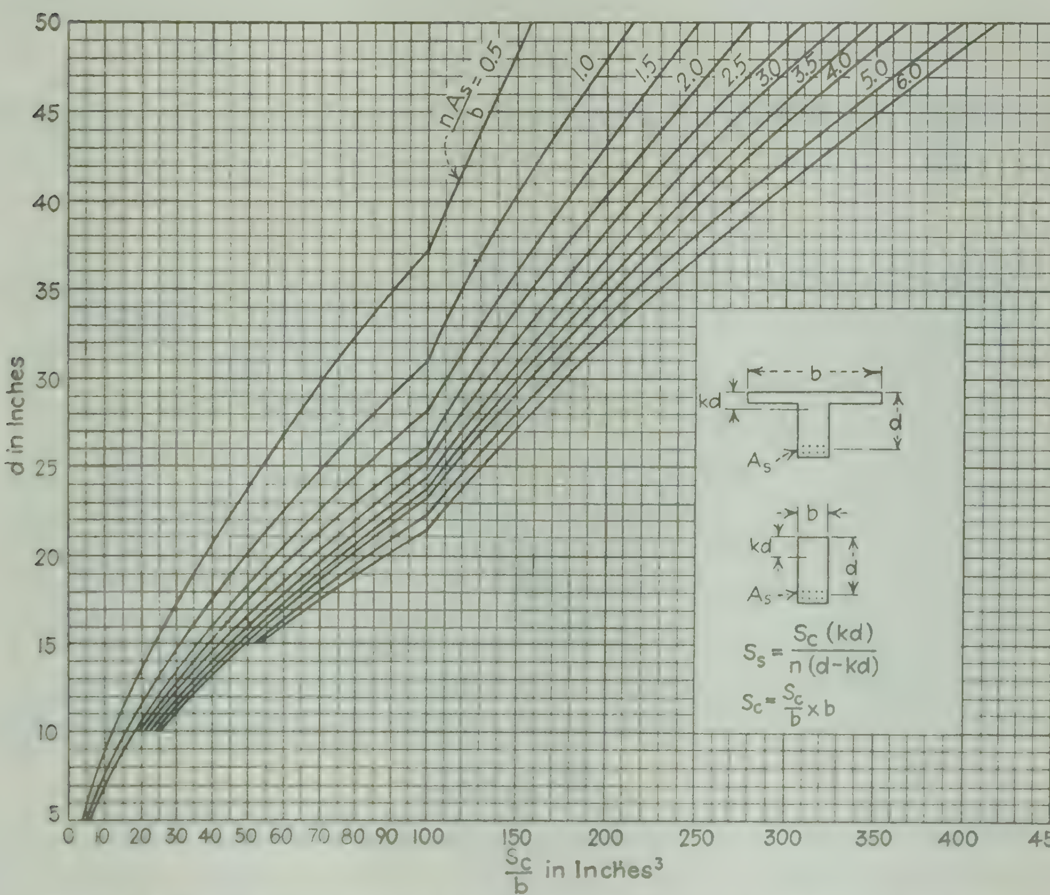


FIG. 5. Section modulus of a 1-in. width of a beam with tensile steel only.

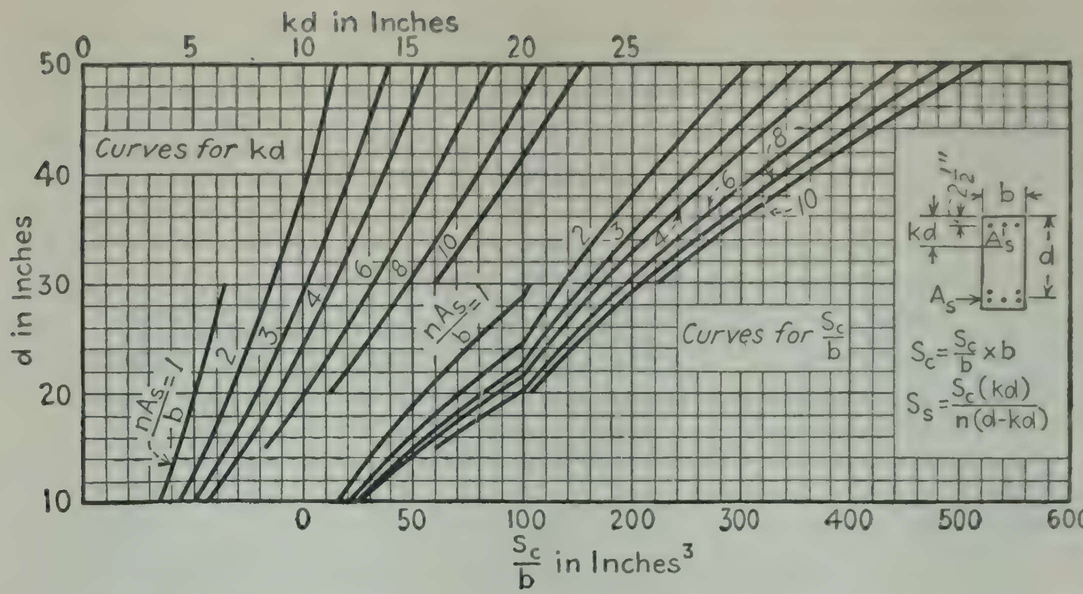


FIG. 6. Location of neutral axis, and magnitude of section modulus of a 1-in. width of a beam, when $[(n-1)/b]A'_s = 1.0$.

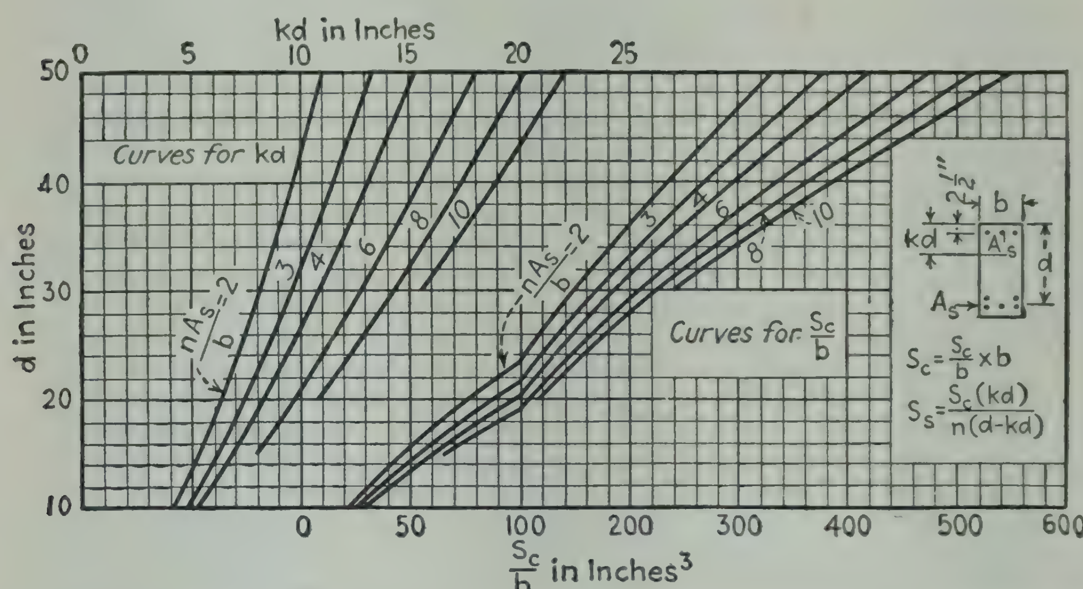


FIG. 7. Location of neutral axis, and magnitude of section modulus of a 1-in. width of a beam, when $[(n-1)/b]A'_s = 2.0$.

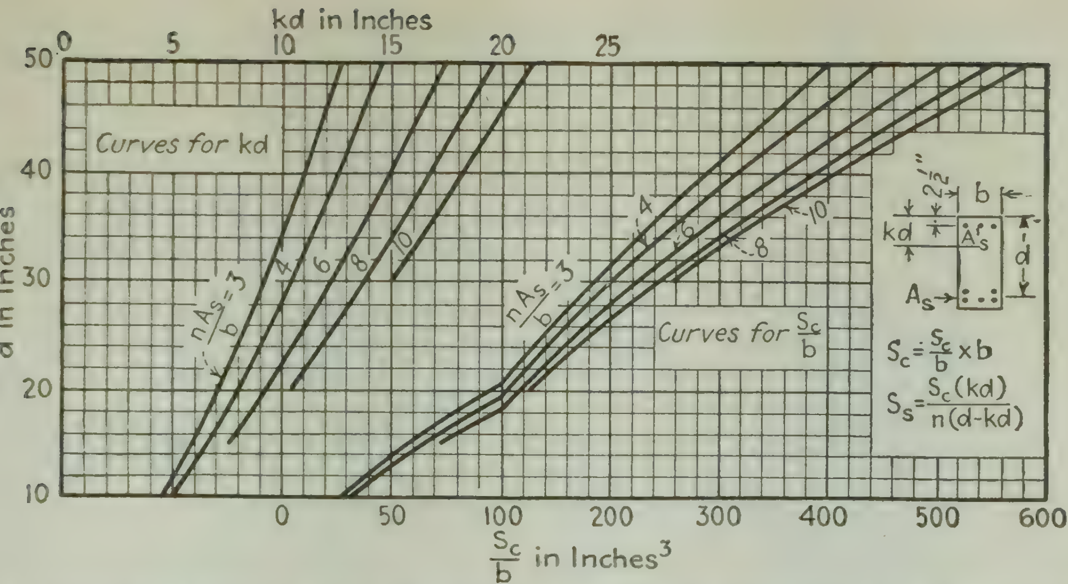


FIG. 8. Location of neutral axis, and magnitude of section modulus of a 1-in. width of a beam, when $[(n-1)/b]A'_s = 3.0$.

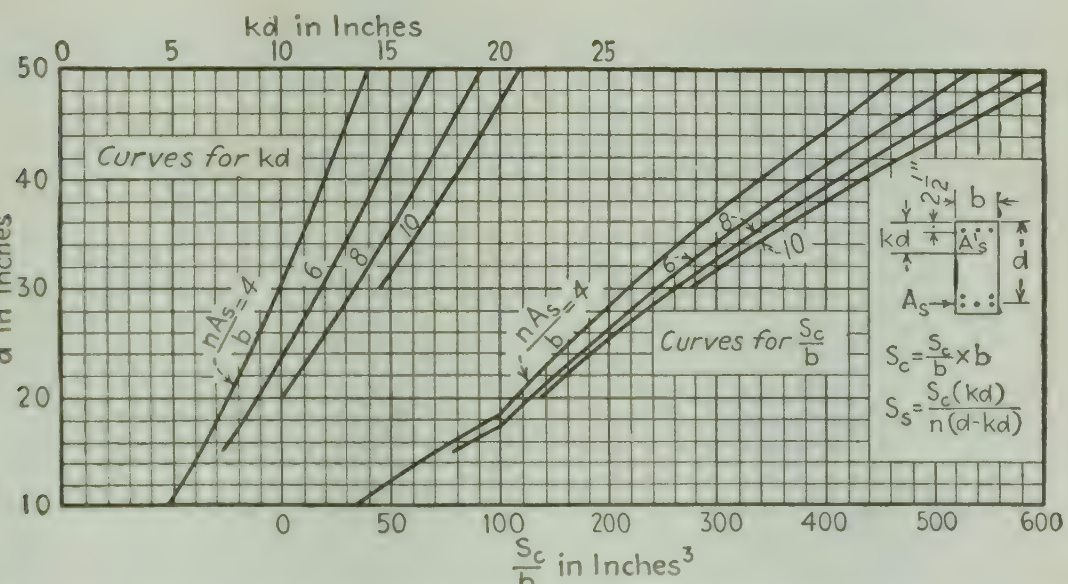
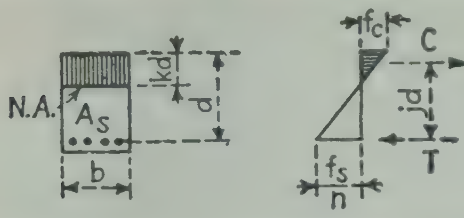


FIG. 9. Location of neutral axis, and magnitude of section modulus of a 1-in. width of a beam, when $[(n-1)/b]A'_s = 4.0$.

Tables 5 *and* 6. These tables are prepared for use in preliminary design to obtain the theoretically balanced design for a beam, the results then being modified by practical considerations if necessary.

TABLE 5. Coefficients p, k, j, and K for Rectangular Sections for Balanced Designs



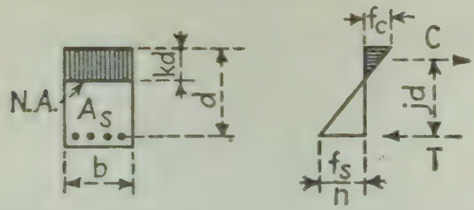
$$k = \frac{1}{1 + (f_s/nf_c)} \qquad j = 1 - \frac{1}{3}k$$

$$p = \frac{A_s}{bd} = \frac{f_c}{2f_s} \times k \qquad K = \frac{f_c}{2}kj = \frac{M}{bd^2}$$

f_c' and n	f_c	$f_s = 16{,}000$				$f_s = 18{,}000$			
		p	k	j	K*	p	k	j	K*
	650	0.0077	0.379	0.874	108	0.0063	0.351	0.883	101
	700	0.0087	0.396	0.868	120	0.0072	0.368	0.877	113
2000	750	0.0097	0.413	0.862	133	0.0080	0.385	0.872	126
15	800	0.0107	0.429	0.857	147	0.0089	0.400	0.867	139
	900	0.0129	0.458	0.847	175	0.0107	0.429	0.857	165
	1000	0.0151	0.484	0.839	203	0.0126	0.455	0.848	193
	700	0.0075	0.344	0.885	107	0.0062	0.318	0.894	100
	750	0.0084	0.360	0.880	119	0.0070	0.333	0.889	111
	800	0.0094	0.375	0.875	131	0.0077	0.348	0.884	123
2500	875	0.0108	0.396	0.868	150	0.0089	0.368	0.877	141
12	950	0.0124	0.416	0.861	170	0.0102	0.388	0.871	161
	1000	0.0134	0.429	0.857	184	0.0111	0.400	0.867	173
	1125	0.0161	0.458	0.847	218	0.0134	0.429	0.857	207
	1250	0.0189	0.484	0.839	254	0.0158	0.455	0.848	241
	750	0.0075	0.319	0.894	107	0.0061	0.294	0.902	99
	800	0.0083	0.333	0.889	118	0.0068	0.308	0.897	111
	900	0.0101	0.360	0.880	143	0.0083	0.333	0.889	133
3000	975	0.0115	0.379	0.874	161	0.0095	0.351	0.883	151
10	1050	0.0130	0.396	0.868	180	0.0107	0.368	0.877	169
	1125	0.0145	0.413	0.862	200	0.0120	0.385	0.872	189
	1200	0.0161	0.429	0.857	221	0.0133	0.400	0.867	208
	1350	0.0193	0.458	0.847	262	0.0161	0.429	0.857	248
	1500	0.0227	0.484	0.839	305	0.0190	0.455	0.848	289
	900	0.0087	0.310	0.897	125	0.0072	0.286	0.905	116
	1000	0.0104	0.333	0.889	148	0.0086	0.308	0.897	138
	1100	0.0122	0.355	0.882	172	0.0100	0.328	0.891	161
3750	1200	0.0141	0.375	0.875	197	0.0116	0.348	0.884	185
8	1300	0.0160	0.394	0.869	223	0.0132	0.366	0.878	209
	1400	0.0180	0.412	0.863	249	0.0149	0.384	0.872	234
	1500	0.0201	0.429	0.857	276	0.0167	0.400	0.867	260
	1700	0.0244	0.460	0.847	331	0.0203	0.430	0.857	313
	1875	0.0284	0.484	0.839	381	0.0237	0.455	0.848	362

* Sometimes denoted by R.

TABLE 6. Coefficients p, k, j, and K for Rectangular Sections for Balanced Designs



$$k = \frac{1}{1 + (f_s/nf_c)} \qquad j = 1 - \frac{1}{3}k$$

$$p = \frac{A_s}{bd} = \frac{f_c}{2f_s} \times k \qquad K = \frac{f_c}{2}kj = \frac{M}{bd^2}$$

f_c' and n	f_c	$f_s = 20{,}000$				$f_s = 22{,}000$			
		p	k	j	K*	p	k	j	K*
	650	0.0053	0.328	0.891	95	0.0045	0.307	0.898	90
	700	0.0060	0.344	0.885	107	0.0051	0.323	0.892	101
2000	750	0.0068	0.360	0.880	119	0.0058	0.338	0.887	112
15	800	0.0075	0.375	0.875	131	0.0064	0.353	0.882	125
	900	0.0091	0.403	0.866	157	0.0078	0.380	0.873	149
	1000	0.0107	0.429	0.857	184	0.0092	0.405	0.865	175
	700	0.0052	0.296	0.901	93	0.0044	0.276	0.908	88
	750	0.0058	0.310	0.897	104	0.0049	0.290	0.903	98
	800	0.0065	0.324	0.892	116	0.0055	0.304	0.899	109
2500	875	0.0075	0.344	0.885	133	0.0064	0.323	0.892	126
12	950	0.0086	0.363	0.879	152	0.0074	0.341	0.886	144
	1000	0.0094	0.375	0.875	164	0.0080	0.353	0.882	156
	1125	0.0113	0.403	0.866	196	0.0097	0.380	0.873	187
	1250	0.0134	0.429	0.857	230	0.0115	0.405	0.865	219
	750	0.0051	0.273	0.909	93	0.0043	0.254	0.915	87
	800	0.0057	0.286	0.905	104	0.0049	0.267	0.911	97
	900	0.0070	0.310	0.897	125	0.0059	0.290	0.903	118
3000	975	0.0080	0.328	0.891	142	0.0068	0.307	0.898	134
10	1050	0.0090	0.344	0.885	160	0.0077	0.323	0.892	151
	1125	0.0101	0.360	0.880	178	0.0086	0.338	0.887	169
	1200	0.0113	0.375	0.875	197	0.0096	0.353	0.882	187
	1350	0.0136	0.403	0.866	236	0.0117	0.380	0.873	224
	1500	0.0161	0.429	0.857	276	0.0138	0.405	0.865	263
	900	0.0060	0.265	0.912	109	0.0051	0.247	0.918	102
	1000	0.0072	0.286	0.905	129	0.0061	0.267	0.911	122
	1100	0.0084	0.306	0.898	151	0.0072	0.286	0.905	142
3750	1200	0.0097	0.324	0.892	173	0.0083	0.304	0.899	164
8	1300	0.0111	0.342	0.886	197	0.0095	0.321	0.893	186
	1400	0.0126	0.359	0.880	221	0.0107	0.337	0.888	209
	1500	0.0141	0.375	0.875	246	0.0120	0.353	0.882	234
	1700	0.0172	0.405	0.865	298	0.0148	0.382	0.873	283
	1875	0.0201	0.429	0.857	345	0.0173	0.405	0.865	328

* Sometimes denoted by R.

Figures 10 *and* 11. These diagrams are for the analysis of members. If k for any T beam lies below and to the right of the straight line in Fig. 11(a), it means that the neutral axis is within the flange and that it can be located by means of Fig. 10.

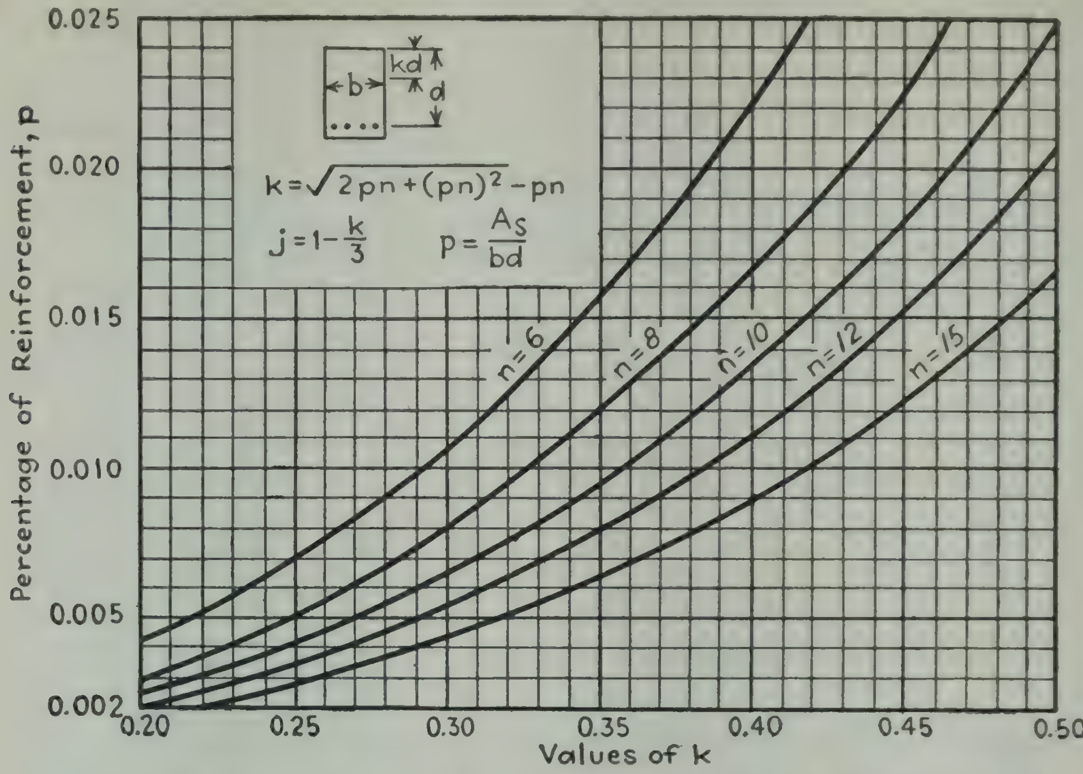
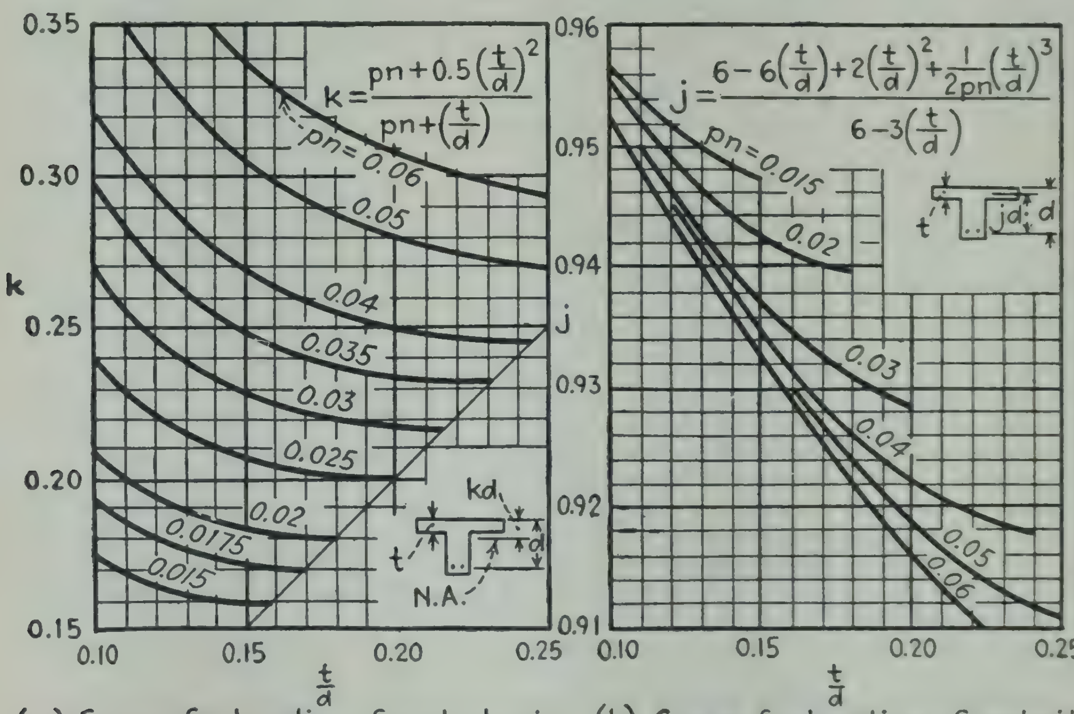


FIG. 10. Location of neutral axis of rectangular beams with tensile steel only.



(a) Curves for location of neutral axis (b) Curves for location of centroid of compression

FIG. 11. Magnitudes of k and j for T beams.

Figure 12 *and Table* 7. Knowing the longitudinal shear to be withstood by the stirrups per inch of length of the beam, varying combinations of sizes and spacings of stirrups can be secured from Fig. 12. It is then important to check the chosen size of stirrup in Table 7 to make sure that the beam is deep enough to provide adequate bond to develop the stirrups in the upper (or compression) half of the beam.

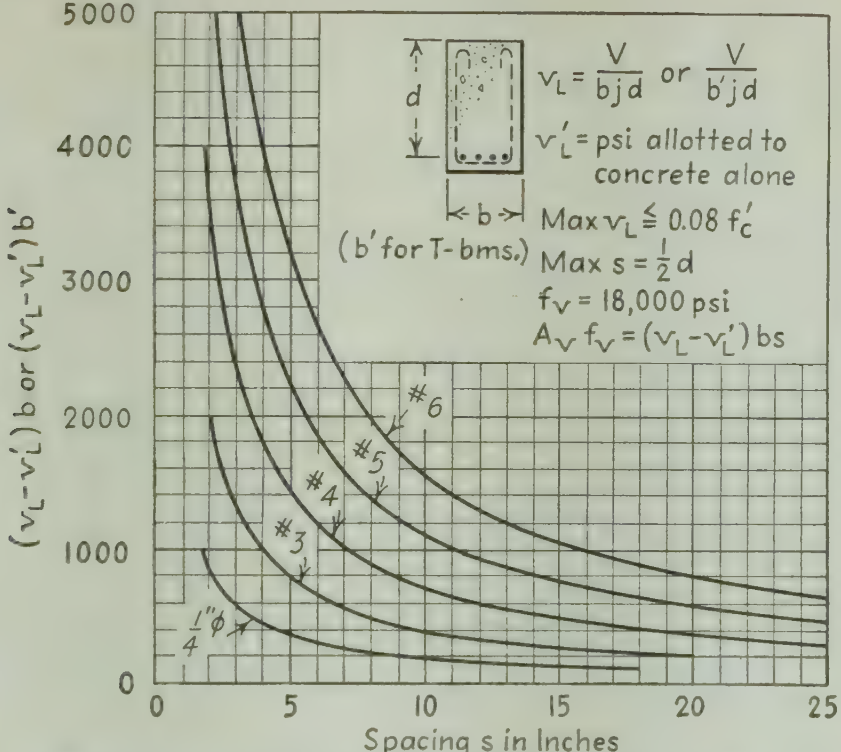


FIG. 12. Maximum spacing of vertical U-shaped stirrups.

TABLE 7. Recommended Minimum Effective Depth of Beam in Inches to Develop Single Vertical U-shaped Stirrups in One-half Depth Plus $1\frac{1}{2}$ In. of Cover, Using Standard Hooks

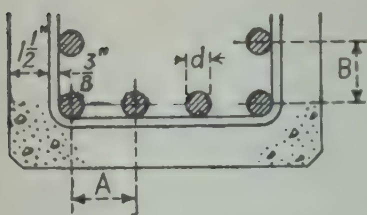
$u = 0.07f_c'$ for A 305 bars and $0.045f_c'$ for No. 2 plain bars

Size of stirrup, No.	f_c', psi				
	2,000	2,500	3,000	3,500	4,000
2	25	20	16	14	12
3	23	16	12	11	11
4	26	20	16	14	13
5	32	24	18	15	15
6	38	28	22	20	18

f_v = 18,000 psi.

Tables 8 *and* 8A. Although the Code permits the use of narrower beams than those shown in Table 8, the data given there have been prepared to provide generous space for thorough encasement of the steel. Table 8A shows the minimum widths for proper encasement.

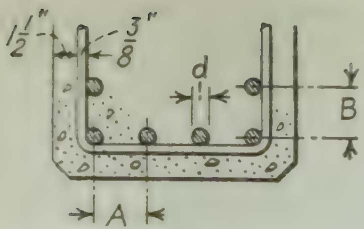
TABLE 8. Preferred Widths of Beams



Min $A = 3d$ or $d + 1\frac{1}{2} \times$ aggregate size.
Dimensions are in inches and are increased to nearest ½ in.
Allow extra for splices.

	Bar No.	No. of longitudinal rods									Min A or B	Preferred B
		2	3	4	5	6	7	8	9	10		
¾-in. aggregate	4	6	8	9½	11	12½	14	16	17½	19	1⅝	2
	5	6½	8½	10	12	14	16	17½	19½	21½	1⅞	2½
	6	7	9	11½	13½	16	18	20½	22½	25	2¼	2½
	7	7½	10	12½	15½	18	20½	23	26	28½	2⅝	3
	8	8	11	14	17	20	23	26	29	32	3	3
	9	8½	12	15	18½	22	25½	28½	32	35½	3⅜	3½
	10	9	12½	16½	20	24	27½	31½	35	39	3¾	4
	11	9½	14	18½	22½	27	31	35½	40	44	4¼	4½
1½-in. aggregate	4	7	10	12½	15½	18	21	23½	26½	29	2¾	3
	5	7½	10½	13	16	19	22	24½	27½	30½	2⅞	3
	6	7½	10½	13½	16½	19½	22½	25½	28½	31½	3	3
	7	8	11	14	17½	20½	23½	26½	30	33	3⅛	3½
	8	8	11½	14½	18	21	24½	27½	31	34	3¼	3½
	9	8½	12	15	18½	22	25½	28½	32	35½	3⅜	4
	10	9	12½	16½	20	24	27½	31½	35	39	3¾	4
	11	9½	14	18½	22½	27	31	35½	40	44	4¼	4½

TABLE 8A. Minimum Widths of Beams*



Min $A = 2d$ or $d + 1\frac{1}{3} \times$ aggregate size.
Dimensions are in inches and are increased to nearest ¼ in.
Allow extra for splices.

	Bar No.	No. of longitudinal rods									Min A or B	Preferred B
		2	3	4	5	6	7	8	9	10		
¾-in. aggregate	4	5¾	7¼	8¾	10¼	11¾	13¼	14¾	16¼	17¾	1½	2
	5	6	7¾	9¼	11	12½	14¼	15¾	17½	19	1⅝	2
	6	6¼	8	9¾	11½	13¼	15	16¾	18½	20¼	1¾	2¼
	7	6½	8½	10¼	12¼	14	16	17¾	19¾	21½	1⅞	2¼
	8	6¾	8¾	10¾	12¾	14¾	16¾	18¾	20¾	22¾	2	2½
	9	7¼	9½	11¾	14	16¼	18½	20¾	23	25¼	2¼	2½
	10	7¾	10¼	12¾	15¼	17¾	20¼	23	25¾	28¼	2⅝	3
	11	8	11	13¾	16½	19½	22¼	25	28	30¾	2⅞	3
1½-in. aggregate	4	6¾	9¼	11¾	14¼	16¾	19¼	21¾	24¼	26¾	2½	3
	5	7	9¾	12¼	15	17½	20¼	22¾	25½	28	2⅝	3
	6	7¼	10	12¾	15½	18¼	21	23¾	26½	29¼	2¾	3
	7	7½	10½	13¼	16¼	19	22	24¾	27¾	30½	2⅞	3¼
	8	7¾	10¾	13¾	16¾	19¾	22¾	25¾	28¾	31¾	3	3¼
	9	8	11¼	14¼	17½	20½	23¾	26¾	30	33	3⅛	3½
	10	8¼	11½	14¾	18	21¼	24½	27¾	31	34¼	3¼	3½
	11	8¾	12¼	15¾	19¼	22¾	26¼	29¾	33¼	36¾	3½	4

* Based in part on "Manual of Standard Practice for Reinforced Concrete Structures" (ACI 315-51).

Figures 13A and 13B. These diagrams are to assist in roughly checking the safe loads for short columns or in obtaining approximate sizes for design purposes. They are prepared for square tied columns and for round spirally reinforced ones.

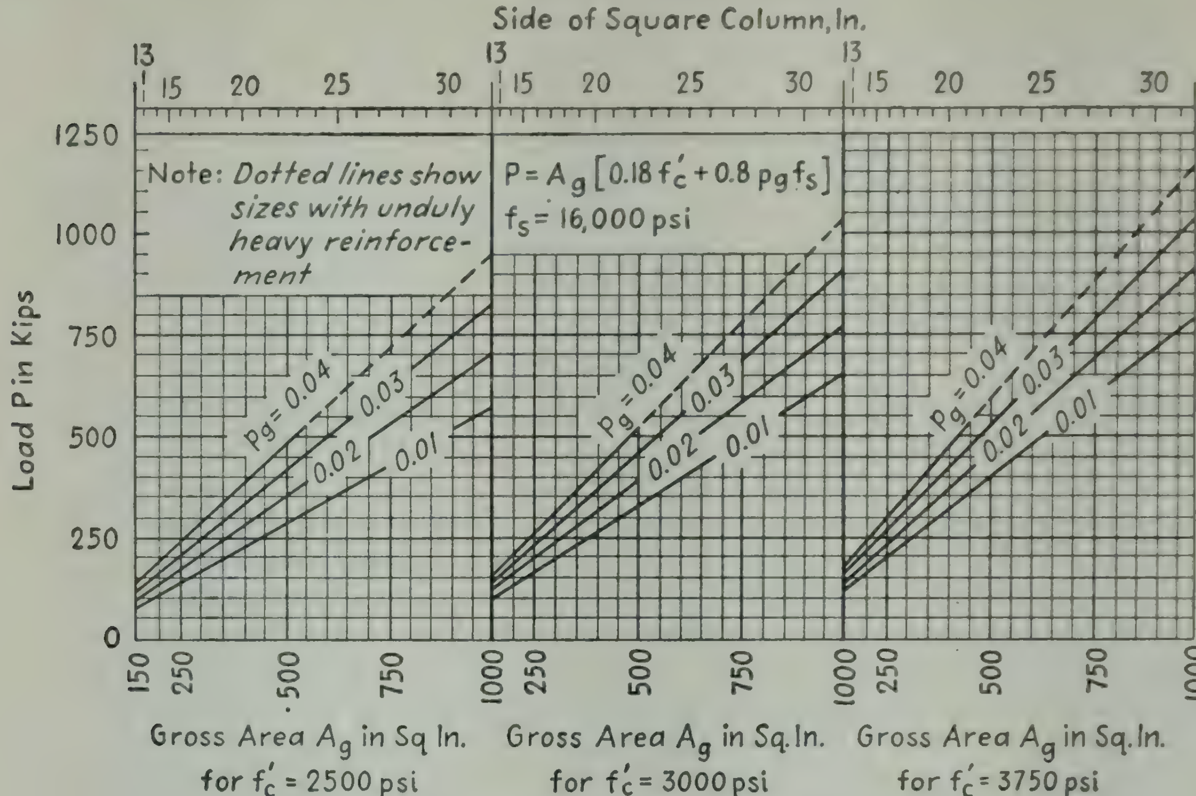


FIG. 13A. Safe loads *P* on short tied columns.

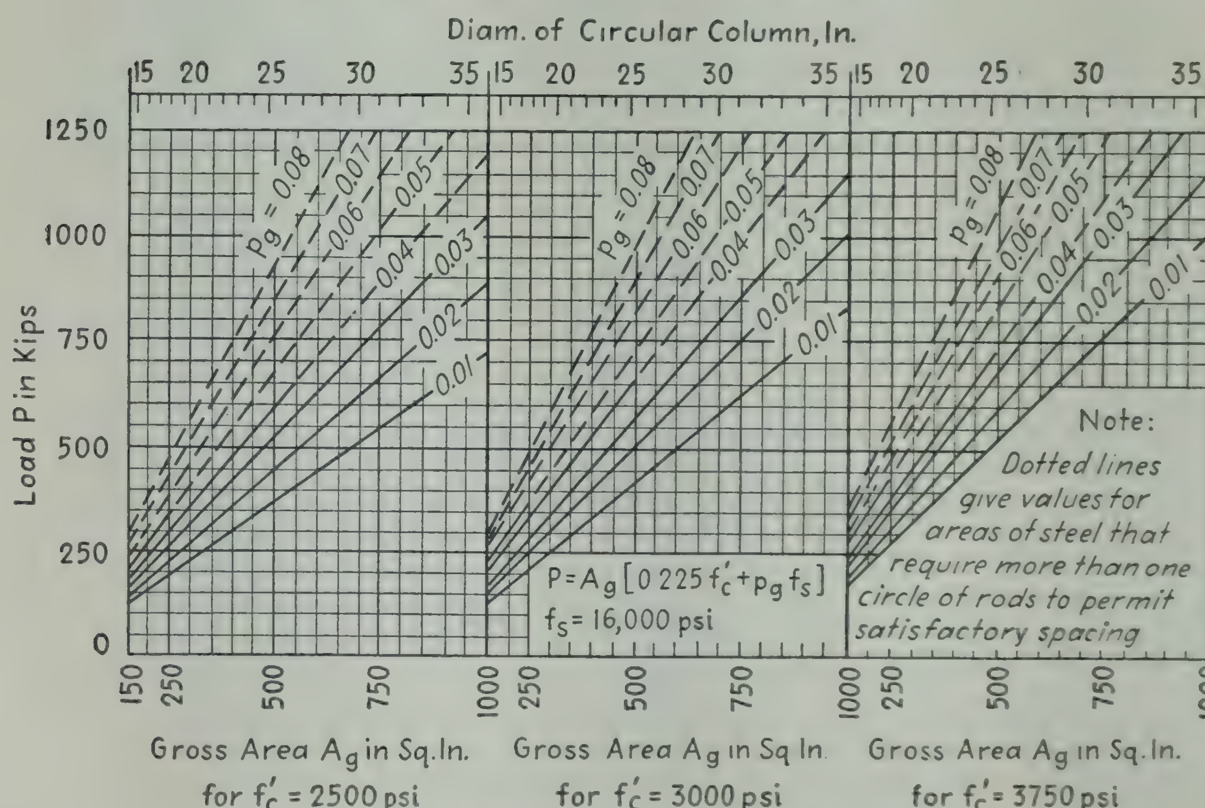


FIG. 13B. Safe loads *P* on short circular spirally reinforced columns.

Table 9 *and Figure* 14. Table 9 gives some suggested size and pitch for spirals in round and square columns. It is for a general guide. It is based upon data formerly given in the American Concrete Institute's "Reinforced Concrete Handbook." On the other hand, Fig. 14 gives the permissible size and pitch of spirals for any value of p' (volume of spiral ÷ volume of core).

TABLE 9. Size and Pitch of Spirals*

$p' \geqq 0.0112$; hot-rolled round rods; cover = $1\frac{1}{2}$ in.

Pitch in inches

Column size, in.	Core size, in.	Square columns f'_c			Round columns f'_c		
		2,500	3,000	3,750	2,500	3,000	3,750
14	11				No. 3—1¾	No. 3—1¾	
15	12	No. 4—2			No. 3—2	No. 3—1¾	No. 4—2
16	13	No. 4—2			No. 3—2	No. 3—1¾	No. 4—2
17	14	No. 4—2¼			No. 3—2¼	No. 3—1¾	No. 4—2¼
18	15	No. 4—2¼	No. 5—2½	No. 5—2¼	No. 3—2½	No. 3—1¾	No. 4—2½
19	16	No. 4—2¼	No. 5—2½	No. 5—2¼	No. 3—2¼	No. 3—1¾	No. 4—2½
20	17	No. 4—2	No. 5—2¾	No. 5—2¼	No. 3—2¼	No. 3—2	No. 4—2¾
21	18	No. 4—2	No. 5—2¾	No. 5—2	No. 3—2	No. 3—2	No. 4—2¾
22	19	No. 4—2	No. 5—2¾	No. 5—2	No. 3—2	No. 3—2	No. 4—2¾
23	20	No. 4—2	No. 5—2½	No. 5—2	No. 3—1¾	No. 3—1¾	No. 4—2¾
24	21	No. 4—2	No. 5—2½	No. 5—2	No. 4—3¼	No. 4—3¼	No. 4—2¾
25	22	No. 4—2	No. 5—2½	No. 5—2	No. 4—3¼	No. 4—3¼	No. 4—2¾
26	23	No. 5—3	No. 5—2½	No. 5—2	No. 4—3	No. 4—3	No. 4—2¾
27	24	No. 5—3	No. 5—2½	No. 5—2	No. 4—2¾	No. 4—2¾	No. 4—2¾
28	25	No. 5—2¾	No. 5—2¼		No. 4—2¾	No. 4—2¾	No. 4—2¾
29	26	No. 5—2¾	No. 5—2¼		No. 4—2¾	No. 4—2¾	No. 4—2¾
30	27	No. 5—2¾	No. 5—2¼		No. 4—2½	No. 4—2½	No. 4—2½
31	28	No. 5—2¾	No. 5—2¼		No. 4—2½	No. 4—2½	No. 4—2½
32	29	No. 5—2¾	No. 5—2¼		No. 4—2¼	No. 4—2¼	No. 4—2¼
33	30	No. 5—2½	No. 5—2¼		No. 4—2¼	No. 4—2¼	No. 4—2¼

* ACI "Reinforced Concrete Design Handbook."

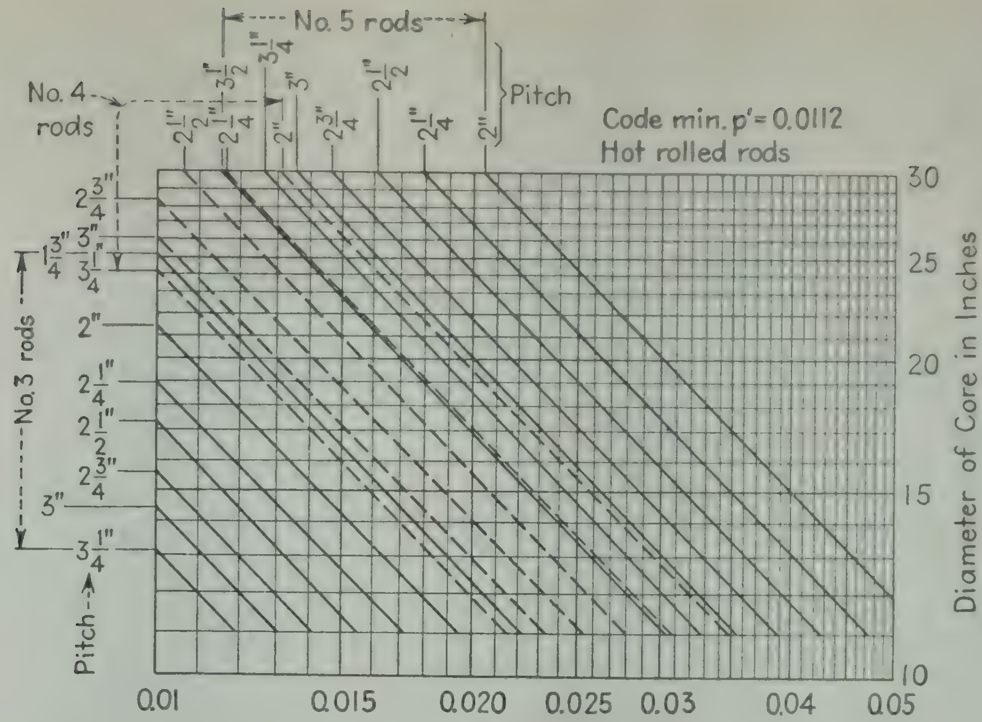
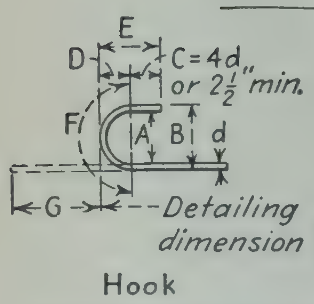
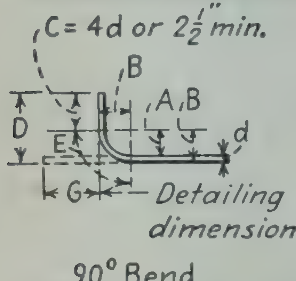


FIG. 14. Size and pitch of spirals for circular columns.

TABLE 10. Bending Details for Bars





	Size of bar									
Dim. / d, No.	2	3	4	5	6	7	8	9	10	11
Hook *A*	1½	2¼	3	3¾	4½	5¼	8	9	10	11½
B	2	3	4	5	6	7	10	11¼	1′½″	1′2½″
C	2½	2½	2½	2½	3	3½	4	4½	5	6
D	1	1½	2	2½	3	3½	5	5⅝	6¼	7¼
E	3½	4	4½	5	6	7	9	10¼	11¼	1′¾″
F	2¾	4⅛	5½	6⅞	8¼	9⅝	1′2 3/16″	1′3⅞″	1′5 11/16″	1′8 7/16″
G	4	5	6	7	8	10	1′1″	1′3″	1′5″	1′7″
90° Bend *A*	¾	1⅛	1½	1⅞	2¼	2⅝	4	4½	5	5¾
B	1	1½	2	2½	3	3½	5	5⅝	6¼	7¼
C	2½	2½	2½	2½	3	3½	4	4½	5	6
D	3½	4	4½	5	6	7	9	10	11¼	1′½″
E	1⅜	2 1/16	2¾	3 7/16	4⅛	4 13/16	7⅛	7 15/16	8⅞	10¼
G	3	3	3	4	4	5	6	7	8	9

Diameter of pin for bend: 6*d* for No. 2 to No. 7, inclusive; 8*d* for No. 8 to No. 11, inclusive.

G = approximate length to add to detailing dimension in order to make hook or bend.

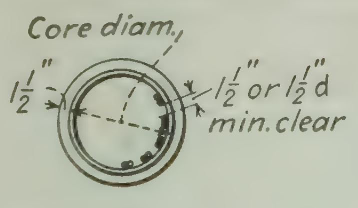
Dimensions are in inches except as shown.

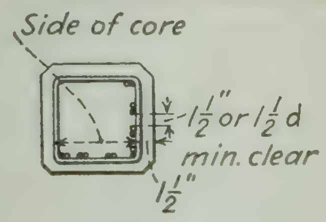
No. 2 bars are ¼-in. diameter plain bars.

Table 10. This gives the recommended dimensions for standard hooks and 90° bends of bars. It is suitable for hard-grade bars as well as for structural and intermediate grades. If necessary, the diameter of pin for structural and intermediate grades may be reduced, but not below $5d$.

Tables 11 *and* 12. These are tables giving recommendations for the maximum number of longitudinal bars to be used in columns. Table 12 shows the closest spacing

TABLE 11. Preferred Maximum Number of Longitudinal Bars in One Row in Columns





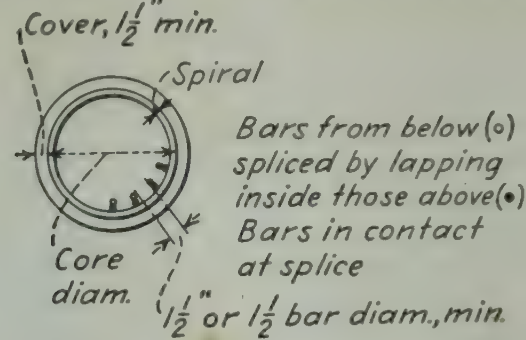
Diam. or side of core, in.	Size of tie or spiral, No.	Round spirally reinforced (p_g = 0.01 min, 0.08 max) Size of bar, No.							Square tied (p_g = 0.01 min, 0.04 max) Size of bar, No.						
		5	6	7	8	9	10	11	5	6	7	8	9	10	11
10	3	9	8	7	7	6			12	8	8	8	6*		
11	3	11	9	9	8	7	6		12	12	8	8	6*		
12	3	12	11	10	9	8	7	6	12	12	12	8	8	6*	
13	3	13	12	11	10	8	7	6	16	12	12	12	8	8	6*
14	3	14	13	11	11	9	8	7	16	16	12	12	10*	8	6*
15	3	15	14	12	11	10	9	8	16	16	16	12	12	10*	8
16	4	16	14	13	12	11	9	8	20	16	16	16	12	10*	8
17	4	17	15	14	13	11	10	9	20	20	16	16	12	12	8
18	4	18	17	15	14	12	11	9	20	20	16	16	16	12	10*
19	4	19	18	16	15	13	12	10	24	20	20	16	16	12	12
20	4	21	19	17	16	14	12	10	24	24	20	20	16	12	12
21	4	22	20	18	17	15	13	11	28	24	20	20	16	16	12
22	4	23	21	19	18	16	14	12	28	24	24	20	20	16	12
23	5	24	22	20	18	16	14	12	28	28	24	24	20	16	16
24	5	25	23	21	19	17	15	13	32	28	24	24	20	16	16
25	5	26	24	22	20	18	16	13	32	28	28	24	20	20	16
26	5	27	25	23	21	18	16	14	32	32	28	24	24	20	16
27	5	28	26	24	22	19	17	15	36	32	28	28	24	20	16
28	5	29	27	25	23	20	17	15	36	32	28	28	24	20	20
29	5	31	28	26	24	21	18	16	36	36	32	28	24	20	20
30	5	32	29	27	24	22	19	17	40	36	32	32	28	24	24

Max size of aggregate = 1 in.

* Limited by max p_g. Arrange symmetrically but unequally.

for round columns. Table 11 allows more room and is suitable for most cases even when the bars are spliced side by side instead of one inside the next bar.

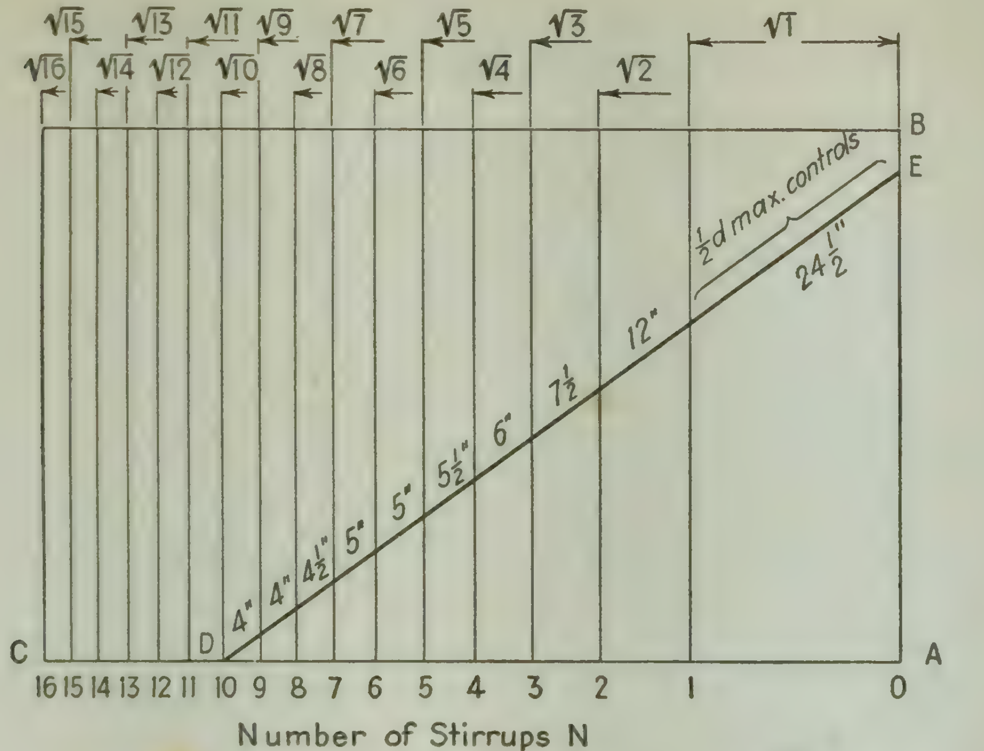
TABLE 12. Maximum Number of Longitudinal Bars in Round Columns*

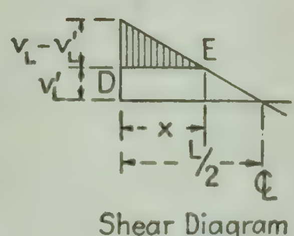


Diameter of core, in.	Size of spiral, No.	Size of bar, No.						
		5	6	7	8	9	10	11
11	3	12	11	10	9	7	6	
12	3	13	12	11	10	8	7	6
13	3	15	13	12	11	9	8	6
14	3	16	15	14	12	11	9	7
15	3	18	16	15	14	12	10	8
16	3	19	18	16	15	13	11	9
17	3	21	19	18	16	14	12	10
18	4	22	20	19	17	15	13	11
19	4	23	22	20	18	16	14	11
20	4	25	23	21	20	17	15	12
21	4	26	24	22	21	18	16	13
22	4	28	26	24	22	19	17	14
23	4	29	27	25	23	20	18	15
24	4	31	28	26	25	21	19	16
25	5	32	30	28	26	22	20	17
26	5	33	31	29	27	23	21	17
27	5	35	32	30	28	25	22	18
28	5	36	34	31	29	26	23	19
29	5	38	35	33	31	27	24	20
30	5	39	37	34	32	28	25	21
31	5	41	38	35	33	29	26	22

* Based on "Manual of Standard Practice for Detailing Reinforced Concrete Structures" (ACI 315-51).

Figure 15. This diagram is helpful in determining the theoretical spacing of stirrups in beams when the load is uniformly distributed.





Compute $v_L - v_L'$, x, and $\frac{A_v f_v}{b}$ for size of stirrups selected.

Number of stirrups in length x is $N = \dfrac{(v_L - v_L')x}{2\left(\dfrac{A_v f_v}{b}\right)}$

Take any convenient scale. Put one end at value of N on AC and rotate until distance x on this scale is on line AB. Read off spacing on this scale from end at N on AC to each division until $\frac{1}{2}$d maximum controls. Locate stirrup at midpoint of each space.

Example:

Let $\frac{L}{2} = 12'\text{-}6''$, $b = 16\frac{1}{2}''$, $d = 27''$, $v_L = 188$ psi, $v_L' = 90$ psi, stirrups $= \frac{1}{2}''\ \phi$ ⊔

$x = \dfrac{98 \times 150}{188} = 78''$, $\quad N = \dfrac{98 \times 78}{2\left(\dfrac{0.4 \times 16{,}000}{16.5}\right)} = 9.9$ (Call it 10)

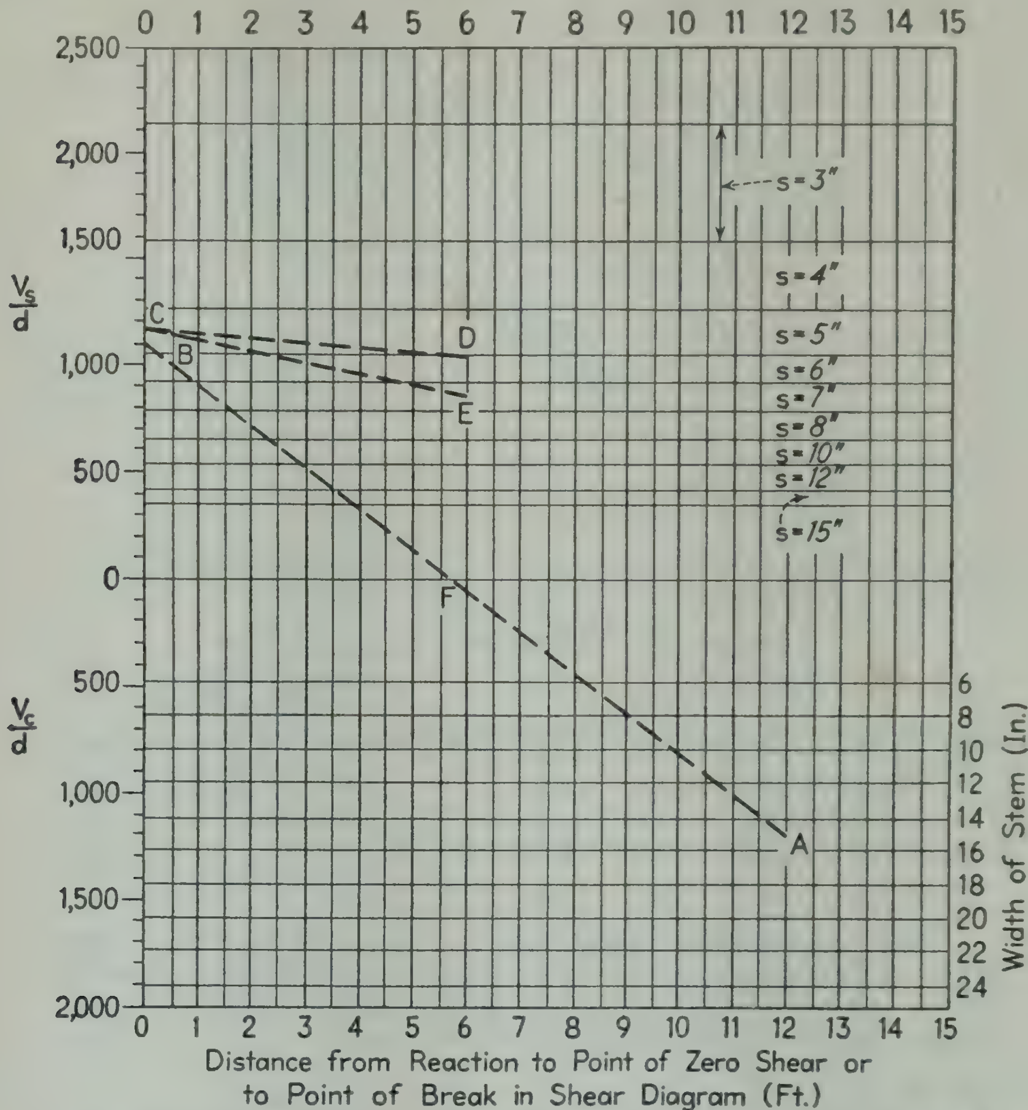
Measure 78" on some scale from D at 10 to E on AB. Put stirrups at near the middle of each space, using 13" maximum.

FIG. 15. Spacing of vertical U-shaped stirrups for uniform loading. *(Courtesy of Charles Macklin, Springfield, Ill.)*

Figure 16. This is a diagram to determine the spacing of vertical U-shaped stirrups in beams. Similar diagrams can be prepared as office standards for any combination of applicable data. To illustrate its use, the following problems are solved:

1. Assume a beam with $L = 24$ ft, $b = 15$ in., and $d = 21$ in. It supports a total uniformly distributed load of 4,000 plf. $V/d = 48{,}000/21 = 2{,}290$. $V_c/d = 90 \times 15 \times 0.88 = 1{,}190$. $V_s/d = 1{,}100$. The point of zero shear is at mid-span. Enter the

lower diagram at point A with the distance 12 ft and $b = 15$ in. Locate point B at $V_s/d = 1{,}100$. Draw the dotted line AB. Stirrups are needed only at the left of point F where AB crosses the zero line. The allowable spacing can be determined for various portions of the beam by the intercepts of AB with the horizontal lines denoting s. Start with a spacing of 5 in. in this case.



Data: s = spacing, $A_v = 2A_s$, $v_L' = 90$ psi., $j = 0.88$, $f_v = 18{,}000$ psi., max. $s = \frac{1}{2}d$

$$\frac{V_c}{d} = 90 \times 0.88b, \quad \frac{V_s}{d} = \frac{V}{d} - \frac{V_c}{d}, \quad \frac{V_s}{d} = \frac{A_v f_v j}{s}$$

FIG. 16. Diagram for determining spacing of vertical U-shaped stirrups. *(Courtesy of T. F. Collier, Westcott & Mapes, Inc., New Haven, Conn.)*

2. Assume a beam with $L = 24$ ft, $b = 16$ in., and $d = 27$ in. It supports a uniformly distributed load of 500 plf and concentrated loads of 40,000 lb at the center and quarter points. $V/d = 66{,}000/27 = 2{,}440$ at the end. $V'/d = 63{,}000/27 = 2{,}330$ at the left of the 6-ft point. $V_c/d = 1{,}270$. $V_s/d = 1{,}170$ at the end. $V_s'/d = 1{,}060$ at the quarter point. Enter the upper diagram at point C for $V_s/d = 1{,}170$. Find point D for $V_s'/d = 1{,}060$. Draw the line CD. The required stirrup spacing is 5 in. for the entire 6 ft. If the line had been CE, the spacing could have been 5 in. for 2 ft 6 in., 6 in. for another 2 ft 6 in., and 7 in. for the remainder. Since V_c/d will exceed the requirements to the right of D, no stirrups are needed in the central portion.

INDEX

A

Abeles, P. W., on prestressed concrete, 383*n*.
Abrams, Duff, fineness modulus, 19, 20
  water-cement ratio, 27–32, 40
Admixtures, 20–25
  air-entraining, 21, 24, 35, 44, 52
  aluminum, 22
  calcium chloride, 22
  fly ash, 23, 24
  pozzolan, 23
Aggregate, absolute volume, 41, 46
  alkali reactivity, 15, 16, 22
  apparent density, 16
  bulking, 38, 39
  cleanliness, 14
  coarse, 14
  definition, 13
  density, 16, 18
  expansion, 16, 17
  fine, 14
  fineness modulus, 19, 20
  freezing, 17
  grading, 18, 19
  handling, 49
  hardness, 14
  impurities, 14
  lightweight, 13, 14
  moisture, 44
  organic material, 14, 15*n*.
  properties, 14, 15
  proportioning, 18, 19
  reaction with alkali, 15, 16, 22
  segregation, 39, 49
  shape, 15, 17
  sieve analysis, 19
  size, 14, 17, 18
  soundness, 14, 15
  specific gravity, 46
  specifications, 14, 15
  storage, 49
  surface area, 17
  surface texture, 16
  tests, 14–16, 18
  thermal expansion, 16, 17
  voids, 17 18
Air-entraining admixtures, 21, 24, 44, 52
Allowable unit stresses, 62, 63, 74, 75
  safety factor for, 74
  variation in, 61–63, 75
Alpha system, 200–205
American Concrete Institute (ACI), Building Code Requirements for Reinforced Concrete, 28, 61–63
  Joint Committee, 37, 61
  symbols, 62, 63, 65–67
American Society of Civil Engineers (ASCE) specifications, 65
American Society for Testing Materials (ASTM) specifications, aggregate, 15*n*., 18*n*., 19*n*., 20*n*., 44*n*.
  bar, 58, 59*n*.
  cement, 6–10
  miscellaneous, 8*n*., 14*n*., 16*n*., 24*n*., 25*n*., 39*n*.
  steel, 58
Anchorage, of bent-up bars, 171, 172, 174
  longitudinal reinforcement, 115, 122, 123, 127, 134
  for prestressed concrete, 391, 392, 411
  for splices, 122, 468
  of stirrups, 154, 155, 479
Andersen, Paul, torsion formulas, 438, 439
Anderson, J., and P. Nerenst on wave velocity, 33*n*.
Angle of repose of soils, 272, 273
Angus, W. M., on precast construction, 376*n*.
Approximate beam formulas, method of design, of beams with compressive reinforcement, 95, 96
    combined compression and bending, 239–248
    rectangular columns with bending, 251–255
  rectangular beams, 80
Architectural treatment of concrete, 415 421
Areas of reinforcing bars, 465–467
Atlas Speed Forms, 361
Autogenous healing of concrete, 54

B

Baker, A. H., on curves for surcharge, 276*n*.
Balanced design of beams, 71, 476, 477
Bars (*see* Reinforcement)
Bases (*see* Footings; Pedestals)
Bauer, Edward E., on plain concrete, 1
Beams, analysis, 73–77
  approximate depths, 78, 79
  approximate formulas, 80, 95, 96
  coefficients for bending moments, 469, 470
  composite (*see* Composite beams)
  cracks, 69, 147, 158
  definition, 64
  deflection, 433, 434
  depth, effective, 65, 66
    vs. span, 77–79
    vs. width, 77
  design of, 77–81
    balanced, 71, 476, 477
    diagrams for, 472–475
  distribution, of loads, 105–107
    of stresses, straight-line theory, 67–73
      ultimate-load theory, 81–87
  economy, 71, 454
  elastic theory for, 69–81, 87–97
  fixed-end moments, 469
  irregular, 104, 105
  lateral supports, 77
  load factor, 85
  loads on, 105–107
  moment of inertia, 88
  neutral axis, definition, 68
    formulas for, 70–73, 88
    location, diagrams for, 472–475, 478
    by straight-line theory, 70, 72, 73
    tables for balanced design, 476, 477
    by transformed-section method, 88
    by ultimate-load theory, 82, 83
  neutral surface, 151
  overreinforced, 71
  prestressed, 382–414
  rectangular, 64–99
  reinforcement, compressive, 92–99
    percentage of, 66
    tensile, 64–92
      and compressive, 92–99
    web (*see* Web reinforcement)
  reinforcing details, 442–447
  resisting moments, 69–107
  section modulus, 88
  shear (*see* Shear; Web reinforcement)
  slab, 64
  span, 64
Beams, stiffness, 74, 380, 433, 434
  straight-line theory, 64, 69–81, 91–97
  stresses, distribution of, 67–69
  substitute, 87
  T, 99–104
  transformed-section method, 87–92
  trial depths, 78, 79
  ultimate-load theory, 64, 81–87, 97–99
  underreinforced, 71
  width, vs. depth, 77
    recommended, 77, 480, 481
    vs. span, 77
Bending and direct compression, 238–263
Bending coefficients for beams, 469, 470
Billner, K. P., on vacuum concrete, 52*n*., 364*n*.
Black, Erick M., approximate method for design of columns, 251*n*.
Blanks, R. F., on fly ash, 23*n*.
Blast-furnace slag cement, 8
Bleeding of mortar, 6, 21, 22, 45, 356
Bogue, R. H., on setting of cement, 11
Bogue and Lerch on increase of strength of concrete, 13
  on setting of cement, 11
Bogue, Lerch, and Taylor on disintegration, 11
Bond, allowable unit stresses, 62, 114, 115
  in composite beams, 207
  on compressive reinforcement, 128
  definition, 113
  development, of bars, 114, 468
    of splices, 122, 468
    of stirrups, 154, 155, 479
  distribution of, in beams, 123–143
  at ends, of continuous beams, 139–143
    of simply supported beams, 131–139
  impact effect, 143
  lengths for, 114, 468
  on longitudinal reinforcement, 123–143
  multiple layers of bars, 121
  on prestressing media, 386, 390, 394, 413
  in relation, to bending moment, 123–143
    to shear, 126
  reversal of stress, 143
  rigidly held horizontal bars, 114, 115
  structural plates and shapes, 143
  on tensile reinforcement, 62, 114, 115
  tests of, 114, 116, 117
  on top bars, 62, 114, 115
Bond creep, 217*n*.
Brackets, reinforced-concrete, 440
Breerwood patented flotation process, 10
Bronx-Whitestone Bridge, 2
Buildings, flat-slab construction, 339–350

Buildings, precast units for, 374–381
  ribbed floors, 339, 340
  tile-filled floors, 339, 340
  two-way slab floors, 334–339
Bulking of aggregate, 38, 39
Bureau of Standards, column tests, 5, 19, 228*n*.

C

Calcium chloride, 22
Calcium sulphate, 22
Cement, accelerators, 22
  admixtures for, 20–25
  air-entraining, 8–10, 21, 42, 44
  alumina, 4
  blast-furnace slag, 8
  chemical reactions, 12
  classifications, 8, 9
  colored, 9
  composition, 10, 11
  definition, 3, 10
  European, 9
  expansive, 9
  false set, 44
  Federal specifications, 8*n*., 9
  final set, 3
  flash set, 12
  free lime in, 11, 12
  grout, 2
  high-early-strength, 5, 9
  hydration and hydrolysis, 12
  hydraulic properties, 3, 12
  initial set, 3
  low-heat-of-hydration, 9
  Lumnite, 8
  manufacture, 7, 10
  moderate-heat-of-hardening, 9
  mortar, 2
  natural, 4, 8
  portland, 4
  pozzolan, 8, 23
  quick-setting, 5
  retarders, 11, 12, 22
  setting of, 3 12, 54
    heat evolved in, 6, 9, 12, 55
  specific gravity, 46
  specifications for, 4
  storage, 49
  sulphate-resisting, 9
  temperature, effect on setting, 54
  types, 8, 9
  unsoundness, 11
  waterproof, 9
  weight per bag, 46
Cement paste, 2
Cement-water ratio, 27–32, 40
Clark, A. P., on bond, 117
  on shear, 156, 159
Clay, in aggregate, 14, 15
  bearing value of, 270
Code, 62, 63, 65
Coefficient, bending moment for fixed-end beams, 469, 470
  friction of soil, 270
  shrinkage, 37
  thermal expansion, of concrete, 36
    of steel, 36
  of variation of tests, 26
Cofferdams, 359–363
Collier, T. F., stirrup spacing, 488
Columns, 212–237
  bending and compression in, 238–263
  code formulas, 218, 222, 228, 230–232
  code requirements, 215, 218, 222, 223, 227, 230, 232
  combination, 230, 231
  composite, 226–230
  core, 221, 222
  definition, 212
  diagrams, for design, 482
    safe-load, 482
  dowels, 216
  effective area, 214, 216
  elastic theory, 216, 217
  estimating loads on, 233–235
  footings, 310–325
  gross section, 216
  hoops for, 213, 218, 219
  long, 212, 231–233
  pedestals, 216, 307–310
  percentage of reinforcement, 217, 218, 222
  pipe, 231
  plastic flow, 217
  short, 212
  size of bars, 218, 485, 486
  spacing, of bars, 216, 220, 485, 486
    of spirals, 222, 223, 483, 484
    of ties, 218
  spirally reinforced, 215, 221–226
  spirals, 214, 222, 223, 483, 484
  splices, 215, 216
  tied, 215–221
  ties, 213, 218, 219
  types, 212
Combination columns, 230, 231
Combined bending and compression, 238–263
  (*See also* Compression and bending)
Combined footings, 325–332
Composite beams, Alpha system, 200–205
  bond stresses, 207

Composite beams, dead load supported by steel alone, 201
definition, 196
shear in, 198
thick slabs and I beams, 205–207
thin slabs and I beams, 196–205
trussed reinforcement, 207–211
Composite columns, 226–230
Compression and bending, approximate method of analysis, 251–255
code formulas, 240–241
in diagonal direction, 241
diagrams for, 243, 245
eccentricity, 240
with resulting tension, 248–251
without resulting tension, 239–248
straight-line theory, 239–251
ultimate-load theory, 255–261
Concept of design, 452–455
Concrete, aggregate, 2, 13–20
allowable unit stresses, 60–63
application of least squares, 31
autogenous healing, 54
batch, 45
bleeding, 6, 21, 22, 45, 356
cement factor, 46
compressive strength, 25–32
computations for proportioning, 41, 45–47
consistency, 37, 39
cooling, 424
cost, 43, 454
cracks in (*see* Cracks in concrete)
curing, 53–56
cyclopean, 264
definition, 1, 25
density, 33
depositing, 48–53
disintegration, 5, 11, 14–16, 33, 39, 50
durability, 6, 15, 21, 28, 33, 37, 43, 60
effect of varying components, 45
electrical heating, 55
exposure, 49, 50
false set, 44
forms for, 2, 56, 351–367
freezing, 10, 16, 17, 21, 38, 39, 55
gunite, 51
handling, 48–53
heat, excessive, 8
heating, 55
lightweight, 13, 35
materials embedded in, 364
maximum pours, 427
mixing, 47–53
modulus of elasticity, 32–34
permeability, 21, 23, 43
placing, 48–53
Concrete, plastic flow, 33, 34
plasticity, 17, 18, 27, 34
pour, 49, 239, 424, 427
precast, 363–381
prestressed, 382–414
proportioned, by volume, 2, 37, 38, 47
by weight, 38, 41, 45–47
protection, 53–56, 80
pumpcrete, 51
punching shear, 34, 149
in sea water, 5, 6, 11, 50
segregation, 39
setting, 3, 12
shearing strength, 34, 62, 149
shrinkage, 36, 37, 423, 424
slump, 39
spalling, 6, 52
specifications, 60–63
surface finishes, 3, 365, 419–421
tension in, 34, 67–73
testing, 25, 26, 30
thermal expansion, 36, 43
tremie, 49, 50
ultimate strength, 25, 26
unit stresses allowed, 60–63, 74, 75
unsoundness, 11
vacuum, 52, 364
vibration, 52, 355–357, 420
weight, 35
workability, 27, 40–45
yield, 45–47
Construction joints, 239, 424–427
Coulomb theory of earth pressures, 273, 274, 471
Cracks in concrete, 69
due, to diagonal tension, 147, 148
to longitudinal deformation, 67, 68
to shrinkage, 423
effect of, on deflection of beams, 433
on moment of inertia, 87–92
on shearing strength, 149
on torsional resistance, 439
Creep, bond, 217*n*.
of steel, 387
Cross, Hardy, deflected structures, 451
Curved bars, compression under, 174
radial component, 186
Cutoff points of reinforcement, 134–143
Cutoff walls, 270

D

Davis, R. E., on pozzolans, 23*n*.
Deflected structures, 451, 452
Deflection, of beams, 433, 434
of forms, 352–354
moment of inertia for, 433

Deflection, of prestressed concrete beams, 401
Deformation of structures, 69, 451, 452
Deformed bars, 115, 116
Design concepts, 452–455
Design problems, 450–463
Details, of bars, 442–447
  of concrete construction, 447–449
  of forms, 360–367
Diagonal compression, 153
Diagonal tension, definition, 148
  due to torsion, 438
  in prestressed beams, 392–394
  reinforcement for (*see* Web reinforcement)
Diagrams, column design, 482
  cutoff of bars, 134–143
  earth pressure, 471
  fixed-end moments, 469, 470
  neutral axis, 472, 474, 475, 478
  pressure on forms, 356–358
  section moduli, 473–475
  spirals, 484
  stirrup spacing, 479, 487, 488
  surcharge pressure, 275, 276
  T sections, 478
Dox floor system, 366, 378
Drainage details, 432, 433
Drawings, detail, 442–447

E

E (*see* Modulus of elasticity)
Earth, angle of repose, 272, 273
  coefficient of friction, 270
  lateral pressure caused by (*see* Lateral earth pressure)
  safe bearing capacity, 270
  surcharge pressures, 274–277
Economy of structures, 43, 71, 454
Elastic theory, for beams, 69–81, 87–97
  for columns, 216, 217
Elephant's trunk, 220
Encasement, of bars, 60, 214
  structural steel, 196, 435–437
Expansion joints, 37, 420, 424, 427–429
  of concrete, 36, 421
  of steel, 36

F

Fineness modulus, 19
Finish of concrete surfaces, 3, 365, 419–421
Fire protection, 60, 214
Fixed-end moments, 469, 470
Flashing, 428
Flat slabs, code for design of, 341–346
  column capital, 342, 345
  drop panel, 342, 344, 345
  exterior panels, 343, 344
  interior panels, 342, 343
  with marginal beams, 343, 344
  negative bending moments, 341
  positive bending moments, 342
  shear, 345
  slab thickness, 344
  strips for design, 340, 341
Flexure, in beams (*see* Beams)
  and direct stress (*see* Compression and bending)
Floors (*see* Slabs)
Fluid pressure, equivalent, 274
Fly ash, 23, 24
Footings, code requirements, 306, 307, 312, 313
  of columns, 310–325
  combined, 325–332
  notched, 322–325
  with overturning, 318–325
  pedestals, 307–310
  on piles, 317, 318, 326–332
  plain concrete, 307–310
  of retaining walls, 265, 287–289
  sloped, 311, 313–316
  spread, 304
  stopped, 311, 316, 317
  unsymmetrical, 322–325
Forms, Atlas Speed, 361
  beam-and-slab, 362, 363
  bracing, 354
  in cofferdams, 359–363
  deflection, 352–354
  design, 352, 355–363
  details, 360–367
  Dox, 366, 378
  effect of vibration on, 355–357
  failure of, 352, 354, 366
  height of pours, 353, 355–358
  lateral pressure on, 355–363
  lining, 365
  loads, 351, 355–358
  modular, 361, 363
  movable, 363
  for precast members, 370
  prefabricated, 367
  pressures on, 57, 355–362
  purpose of, 2
  removal of, 56, 364
  shoulders, 359, 426
  simplification, 363
  slip, 363
  standard sizes, of lumber, 363
    of plywood, 417

Forms, steel, 361, 363
  stiffness of, 352–354
  strength of, 352, 355–363
  striking of, 56, 364
  for surface finishes, 365
  ties, 359
  time left in place, 56
  tremie concrete, pressure on, 355
  uplift on, 359, 366
  vacuum, 363, 364
  with vertical loads, 351–355
  wooden, 362–367
Foundations, allowable pressure, 270
  combined footings, 325–332
  definition, 304
  failure, 305
  mat, 304
  net pressure, 306
  with overturning, 318–322
  partly on rock, 271, 272
  pedestals, 307–310
  pile, 317, 318, 326–332
  resultant pressures, 305
  retaining wall, 267–272
  (*See also* Footings)
Friction coefficient for soils, 270
Frost depth, 271
Fuller's rule, volumetric proportioning, 38

G

George Washington Bridge, 33, 34
Gilkey, Chamberlin, and Beal on curing and bond, 116*n*.
Goldbeck A. T., on bulking, 38*n*.
Gonnerman and Lerch on characteristics of cement, 28*n*.
Gravel, bearing value, 270
  grading, 18–20
  properties, 13–20
Grout, 2
Gunite, 51
Gyration radius, 240

H

Halloran and Talbot on concreting under water, 355*n*.
Hansen, W. C., on alkali-aggregate reaction, 15*n*.
High-early-strength cement, 5, 9
Holt, Ralph, on slip forms, 363*n*.
Hooks, in compression, 120
  for reinforcement, 118–121
  standard, 484
  for stirrups, 154, 155
Hoops for columns, 213, 218, 219
Horizontal shear (*see* Longitudinal shear)

I

I (*see* Moment of inertia)
I beams, encasement, 196–207
  of prestressed concrete, 393–411
  and thick slabs, 205–207
  and thin slabs, 196–205
Ice pressure on walls, 278
Itakura, C., on electric heating, 55*n*.

J

j, for beams, with compressive reinforcement, 94, 152
    with tensile reinforcement, 70, 152
  definition, 66
  diagrams, for rectangular beams, 472–475, 478
    for T beams, 478
  for T beams, 101
  tables for balanced design, 476, 477
Jensen, V. P., on ultimate-load design, 64*n*.
Joint Committee membership, 37*n*.
Joints, construction, 424–427
  contraction, 37, 420–423, 427–429
  dummy, 420, 429
  expansion, 37, 420, 424, 427–429

K

k, definition of, 66
  (*See also* Neutral axis)
Kelly, Schuman, and Hornibrook on alkali-aggregate reaction, 15*n*.
Kern radius, 252
Keys, 424–426
  intermittent, 424
Kinnane, R. F., on tests for bond strength, 124*n*.
Kip, 66
Komendant, A. E., on prestressed concrete, 383*n*., 413*n*.

L

Laitance, 48
Lateral earth pressure, Coulomb's theory of, 272–274, 471
  direction of, 273, 274
  point of application of resultant, 274
  surcharge, 277
Lateral pressure on forms, 355–363
Lateral support for beams, 77
Least squares, 31
LeChatelier, Michaelis, and Tornebohm on cement, 5*n*.

Lerch, W., on durability, 11
Lerch and Taylor on heat-treatment, 11
Limit design (*see* Ultimate-load design)
Load factor, 85
Loads, on beams, 105–107
  on columns, 233–235
Long columns, 212, 231–233
Longitudinal shear, 150
  in composite beams, 198
  in concrete beams, 146–191
  in relation, to bond, 126
    to diagonal tension, 150
    to transverse shear, 149
Lowy, Frederick C., approximate method for design of columns, 251*n*.
Luminite cement, 8, 423
Lyse, Inge, on fineness and mixing of aggregates, 28*n*., 40

M

Macklin, Charles, on forms, 356
  on stirrup spacing, 487
Magnel, G., on prestressed concrete, 383*n*.
Mains, R. M., on bond, 113*n*.
Masonry, strength of, 295, 296
  walls faced with, 293–298
Maurer and Withey torsion formula, 439
Miller, A. L. and H. F. Faulkner on high temperatures, 423*n*.
Moderate-heat-of-hardening cement, 9
Modular ratio (*see* *n*)
Modulus of elasticity, for concrete in compression, 32–34
  definition, 32
  for masonry, 295
  secant, 32, 33
  for steel, 59, 60
  for stone masonry, 295
  tangent, 33
Moldings, 418, 419
Moment of inertia, by transformed-section method, 87–89
Moore, O. L., on scaling, 21*n*.
Moran, W. T., on admixtures, 23*n*.
Mortar, definition, 2
  safe stress, 296
Moving loads, web reinforcement for beams supporting, 187–190

N

n, $\frac{E_s}{E_c}$, for concrete, 72, 73
  definition, 66
  effect of change, 90
  for masonry, 296

Navy specifications, 65
Net line for excavation, 297
Neutral axis, definition, 67
  diagrams for determining, 472, 474, 475, 478
  formula, for analysis, 73
    for design, 72
  tables for balanced design, 476, 477
  transformed-section method of computation, 87–89
Neutral surface, definition, 151
Notations and their definitions, 65–67

O

Olsen, P. E., on tests for bond strength, 124*n*.
Openings, reinforcement around, 437
Organic impurities in aggregate, 14
Overreinforced beams, 71
Overturning moments on footings, 318–325

P

Parapets, 281, 290, 437
Paste, cement, 2
Payment line for excavation, 297
Pearson, J. C., on cement specifications, 8*n*.
Pedestals, 307–310
Percentage of reinforcement, longitudinal,
    in beams, 66
    in columns, 217, 218
  web, 155
Perimeters of bars, 465–467
Pile foundations, 317, 325–332
Pipe columns, 231
Placing of concrete, 48–53
Plastic flow, in concrete, 17, 27, 34
  effect on columns, 217
Port of New York Authority, 34, 251
Portland cement, definition of, 3, 10
  high-early-strength, 5, 9
  moderate-heat-of-hardening, 9
  quick-setting, 5, 9
  sulphate-resisting, 9
Portland Cement Association, on Joint Committee, 5
  on prestressed concrete, 394*n*.
Pour, definition, 239
  height of, 49
  sequence of, 424
  volume of, 427
Powers, T. C., on admixtures, 21*n*.
  on bleeding of concrete, 6
  on dispersion of cement, 22*n*.

Powers and Brownyard on properties of cement paste, 40*n*.
Pozzolan, 3, 23
Precast concrete, 368–381
  blocks for, 379
  bridges, 371–374
  building construction, 374–381
  design features, 369–371
  Dox system, 366, 378
  economics of, 368, 369
  forms for, 370
  panels, 378, 379
  prestressed, 389
  stiffness, 380
  vacuum as aid, 369, 390
Pressures, allowed, on foundations, 270
    on masonry, 291, 295
  of concrete on forms, 355–363
  lateral, due, to earth, 272–274
      to ice, 278
      to surcharge, 274–277
      to water, 277, 278
Prestressed concrete, 382–414
  anchorages, 391, 392, 411
    end, 391, 392, 411
    Freyssinet, 391, 392
    Magnel, 391
    Roebling, 411
      cables for, 405
  bond for, 386, 390, 394, 413
  compressive deformation in, 387
  computations for, 394–411
  concrete for, 388
  continuous members, 413, 414
  cracking moment, 399
  deflection of beams, 401
  design principles, 383–389
  diaphragms for, 413
  economics, 382
  end details, 391–394, 411
  methods of tensioning, 389–392
  miscellaneous construction, 411–414
  partial prestressing, 388, 389
  plastic flow in, 386, 387
  posttensioned, 386, 390, 402–411
  with precast blocks, 411, 412
  pretensioned, 386, 389, 394–402
  principles of, 383–389
  reinforcement for, 388
  safety factor, 399
  shear in, 392–394
  shrinkage, 386, 387
  stiffness, 382
  web reinforcement of, 388
Problems, design, 450–463
Proportioning of concrete, Fuller's rule, 38
Proportioning of concrete, general, 2, 37, 38, 41, 45–47
  variation of components, 45
  volumetric method, 2, 37, 38, 47
  weight method, 38, 41, 45–47
Pumpcrete, 51
Punching shear, 34, 149

R

Radius, of gyration, 240
  Kern, 252
Reinforcement, anchorage of bars, 115, 122, 123, 127, 134, 171–174
    prestressing wires and cables, 386, 390–394, 413
  areas, perimeters, and weights for, table, 465–467
  bending, 174, 186, 484
  bends, 174, 186, 484
  bent-up, 167–190
  bond, 62, 114, 115
  compressive, in beams, 92–99
  cover over, 60, 214
  curved, 174, 186
  cutoff points, 134–143
  deformed, 115, 116
  details of, 442–447
  development, 114, 122, 468
  dowels, 216, 308, 426
  effective areas, 69
  embedment, 60, 214
  fireproofing, 60
  hooks, 118–121, 154, 155, 484
  laps, 122, 468
  maximum number, in beam, 480, 481
    in column, 485, 486
  mesh, 59
  multiple layers, 121
  multiplication tables, 466, 467
  not parallel to tension, 69
  numbering system, 59
  packing at beam intersections, 180
  percentage, 66, 155, 217, 218, 222
  perimeters, 465–467
  planning, 440–442
  posttensioned, 386
  prestressed, 382–414
  pretensioned, 386
  protection, 60, 214
  radii of bends, 174, 186, 484
  savings by varying allowable stresses, 63, 74
  screen effect, 19
  shrinkage, 423, 424
  spacing, in beams, 480, 481
    in columns, 216, 220, 485, 486

Reinforcement, specifications, 57–60
spiral, 214, 222, 223
splices, 122, 123, 215
stirrups (*see* Web reinforcement)
stresses, unit, 63
temperature, 421–423
tensile, in beams, 57, 67–69
trussed, 207–211
web (*see* Web reinforcement)
weights of, 465
wire-mesh, 59
Retaining walls, 264–303
buttresses, 266, 294
cantilevered, 264
copings, 297
counterforts, 266
cutoff walls, 270
definitions, 265
design, of heel, 287, 288
of stem, 282–284
of toe, 288, 289
details, 290–294
expansion joints, 291
flashing, 290
foundations, 268–272, 284–286
gravity, 264
inclined stems, 266
masonry-faced, 294–298
miscellaneous data, 298–300
overturning, 267, 268, 284–286
parapets, 281, 290
parts, 265
preliminary design data, 278–282
pressures, earth, 272–274, 471
equivalent fluid, 274
on foundations, 267–272, 284–286
ice, 278
surcharge, 274–277
water, 277, 278
proportions, 265–267
reinforcement, 284, 292
safety factor, 267, 268, 284–286
sliding, 268, 270
stability, 267, 268, 284–286
stepped footings, 271
stone-faced, 294–298
tapered, 271
types of, 265–267
Richart, F. E., on footings, 307*n.*
on shear in beams, 148*n.*, 155*n.*
on tests of beams, 157, 158
Roberts, S. B., on precast construction, 379*n.*
Robin, Olsen, and Kinnane on bond, 124*n.*
Rock, allowable pressures on, 270
Rods (*see* Reinforcement)

S

Safety factor, 60–63, 74, 267
Sand, angle of repose, 270
bearing value, 270
coefficient of friction, 270
properties, 13–20
Sea water, concrete exposed to, 5, 6, 11, 60
Section modulus, 88
diagrams for determining, 473–475
Segregation, 49
Setting time of concrete, 3
Shank, J. R., on bond and shrinkage, 217*n.*
Shaw, Edmund, on grading aggregate, 19*n.*
Shear, allowable, in concrete, 62, 157, 160
allowable unit stresses, 62
in beams, composite, 198
continuous, 175–179
cracked, 148
prestressed, 392–394
rectangular, 149, 150
T, 150
tests of, 155*n.*, 156*n.*, 157, 159
uncracked, 147, 148
code requirements, 62, 157, 160–164, 172, 174
diagonal tension due to (*see* Diagonal tension)
diagram for stirrup spacing, 479, 487, 488
formulas, 149, 150
general principles, 146–191
longitudinal (*see* Longitudinal shear)
multiple layers of bars, 121
punching, 34, 149
reinforcement (*see* Web reinforcement)
torsion, 439
transverse, 149
Shideler, J. J., on calcium chloride, 22*n.*
Shrinkage, coefficient of, 37
of concrete, 35–37, 423, 424
joints, 423, 427–429
in prestressed members, 386, 387
wet, 30, 353
Sinnickson, W. B., on properties and manufacture of concrete, 1–63
Slabs, flat (*see* Flat slabs)
one-way, 334
as rectangular beams, 64
ribbed, 339, 340
thick, and I beams, 205–207
thin, and I beams, 196–205
tile-filled, 339, 340
two-way, 334–339

Slump test, 39
Span lengths of beams, 64
Spangler, M. G., on surcharge, 275, 276
Specifications, for aggregates, 14, 15
  ACI, 28, 65
  ASTM, 8
  for cement, 8, 9
  Federal, 8, 9
  Joint Committee, 37
  Navy, 65
  Portland Cement Association, 5, 30*n*.
  for steel, 57–60
Spirally reinforced columns, 215, 221–226
Spirals, for columns, 214, 222, 223
  diagrams, 483, 484
  for torsion, 438
Stanton, T. E., on expansion of aggregate, 15*n*.
Stiffness of beams, 74, 380, 433, 434
Stirrups (*see* Web reinforcement)
Strength, of concrete, in compression, 25–32
    in prestressed construction, 388
    in shear, 34, 62, 149
    in tension, 34, 67–73
  of forms, 351–363
  of soil, 270
  of steel, 58, 63
Stresses (*see* Unit stresses)
Structural steel, bond on, 143
  in composite beams, 196–207
  in composite columns, 226–231
  encasement of, 196–207, 366, 435–437
Sulphate-resisting cement, 9
Surcharge, concentrated loads, 274–277
  uniform loads, 277
Surface finishes, 3, 365, 419–421
Surface spalling, 6, 52
Swaze and Gruenwald on concrete mixes, 41*n*.
Symbols, 65–67

T

T beams, analysis, 99–104
  diagrams for *k* and *j*, 478
  effective section at interior supports, 101
  prestressed, 394–411
  proportions specified, 101
  shear, 150
Talbot and Richart on strength of concrete, 40*n*.
Temperature, coefficient of expansion due to, 36
  effect of, on setting of concrete, 53–55
  ice pressure on walls, 278
Temperature, Lumnite cement, 8
Temperature reinforcement, 421–423
Tension, in concrete, 34, 67–73
  diagonal, 148
  in rods not parallel to force, 69
Terzaghi, Karl, on pressures due to saturated sands, 277*n*.
Thunman, C. E., forms, 358
Tied columns, 216–221
Ties, column, 213, 218
  for forms, 359
  spacing in columns, 218
Torsion in concrete, 438–440
Transformed-section methods, 87–92
Transverse shear, 149
Tremie concrete, 49, 50
  pressure from, 355
Trussed reinforcement, 207–211
Two-way slabs, 334–339
Tyler Company, W. S., sieves, 19*n*.
Types, of cement, 9
  of reinforcement, 116

U

Ultimate-load design of beams, 64, 81–87, 97–99
  combined bending and direct stress, 255–261
  compressive reinforcement, 97, 98
Underreinforced beams, 71
Unit stresses (allowable), in concrete, A.C.I. code, 62
  in reinforcement, 63
  ultimate loads, 67–69, 81–87
  variations allowable in, 63, 74, 75
U.S. Bureau of Standards, column tests, 5, 19, 228*n*.
U.S. Geological Survey, 5*n*.
Universal Form Clamp Company, 57

V

V cuts, 418
Vacuum concrete, 52, 364, 369
Vibration of concrete, 52, 420
  effect on forms, 355–357
Volumetric proportioning of concrete, 2, 37, 38, 47

W

Walls, inside cofferdams, 359–363
  retaining (*see* Retaining walls)
Water, effect of, 27–32, 40
  gallons per cubic foot, 46

Water, measurement, 2, 47
  pressure of, on walls, 277, 278
  weight of, 46
Water-cement ratio, 27–32, 40
Water table, 278
Waterproofing, membrane, 429–432
  without membrane, 432
Waterstops, 428
Web reinforcement, bent-up bars, 167–175
  combined, 179–187
  diagonal tension, 148
  diagrams for spacing of vertical stirrups, 479, 487, 488
  general principles, 146–191
  inclined stirrups, 164–167
  for moving loads, 187–190
  for prestressed concrete, 388
Web reinforcement, vertical stirrups, 152–164, 175–179
Weight, of concrete, 35
  proportioning of concrete by, 38, 41, 45–47
Welded splices of bars, 123, 216
Wernisch, George R., on bond, 116*n*.
Whitney, Charles S., on distribution of stresses, 82
  on ultimate-load design, 64*n*., 81–87, 97–99, 255–261
Workability of concrete, 27, 40–45
Workmanship, importance of, 1, 43, 63

Y

Yield of concrete, 45–47
Yield point of steel, 58
Young, A. M., on crack control, 420*n*.

Asian Students' Editions of McGraw-Hill Textbooks

The following titles are McGraw-Hill textbooks reprinted for the benefit of students and teachers throughout Asia.

Production in Japan makes it possible to offer these books at prices within the reach of students in all Asian countries. Each reprint maintains standards of printing and binding comparable with the American original, of which it is an exact and complete reproduction.

☆Leighou: CHEMISTRY OF ENGINEERING MATERIALS 4/e
☆Darken: PHYSICAL CHEMISTRY OF METALS
☆Slater: MODERN PHYSICS
☆Morse: VIBRATION AND SOUND 2/e
☆Mitchell: A TEXTBOOK OF GENERAL PHYSIOLOGY 5/e
☆Spangenberg: VACUUM TUBES
☆Doan: PRINCIPLES OF PHYSICAL METALLURGY 3/e
☆Lichty: INTERNAL COMBUSTION ENGINES 6/e
☆Daniels, et al.: EXPERIMENTAL PHYSICAL CHEMISTRY 5/e
☆Jenkins & White: FUNDAMENTALS OF OPTICS 3/e
☆Richtmyer, et al.: INTRODUCTION TO MODERN PHYSICS 5/e
☆Stephenson: INTRODUCTION TO NUCLEAR ENGINEERING
☆Weber, et al.: COLLEGE PHYSICS 3/e
☆Badger & Banchero: INTRODUCTION TO CHEMICAL ENGG.
☆McAdams: HEAT TRANSMISSION 3/e
☆McCabe & Smith: UNIT OPERATIONS OF CHEMICAL ENGG.
☆Perry: CHEMICAL ENGINEERS' HANDBOOK 3/e
☆Shreve: CHEMICAL PROCESS INDUSTRIES 2/e
☆Ryder: ENGINEERING ELECTRONICS
☆Terman: ELECTRONIC AND RADIO ENGINEERING 4/e
☆Timoshenko & Young: ENGINEERING MECHANICS 4/e
☆Zemansky: HEAT AND THERMODYNAMICS 4/e
☆Marcus & Levy: ELEMENTS OF RADIO SERVICING 2/e

[These editions are available only in Asia.]